A METHEMATICAL MODEL FOR EPIDEMICS SPREAD IN A COMMUNITY

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***Abstract*— A model is a simplified representation of a real-life situation. This paper proposes a mathematical model Epidemics spread e.g. HIV/AIDS. The peculiar applicability and limitations of this model with reference to Nigeria were examined. We also elaborated why modeling of epidemics is one of the most needed statistical supports for meeting the millennium development goals (MDGs), particularly in Nigeria. We concluded by proposing a disease transmission predictive mode which could be used to project the spread of epidemics in our society.**

***Index Terms*—Model, Epidemics, Predictive, disease**

I. INTRODUCTION

The goal of any statistical /mathematical modeling exercise is to extract as much information as possible from available data and provided an accurate representation of both the knowledge and uncertainty about the epidemic Solomon and Murray (2001). In proposing such model however, care should be taken to ensure that the assumptions of such model are not only explicitly defined, but should also be realistic and unambiguous. The very first general epidemic model was developed by Kendrick M.C (1926). For any meaningful Millennium Development Goal (MDGs) in the Nigeria health sector, vis-à-vis the HIV/AIDS issues, mathematical modeling is inevitable to checkmate the spread of epidemics. Contributions from non-governmental organizations such as WHO, UNAIDS, etc towards the fight against epidemics like HIV/AIDS in Nigerian cannot be over emphasized, with all this epidemiology on ground mathematical modeling of epidemics which is being proposed in this article will be of immense help in making some projections about the spread of most dreaded diseases in our society.

II. Methodology

Suppose there is an out-break of an epidemics in a closed community with a fixed population of N individuals, such that N can be grouped into S_t and I_t denoting S_t as number of people susceptible at time t and I_t denotes the number of people infected at time t respectively.
Such that $S_t + I_t = N$, with both S_t and I_t large enough to be regarded as continuous variables.

III. Assumptions:

1) Let the proportion of infected people at the initial time (t=0) be X_0. i.e. $X(t=0) = X_0$.
2) The disease spreads only by contact between infected and susceptible members of the community.
3) Infected and susceptible members of the community move about freely among each other.

In the study of infectious disease, the susceptible individual having been infected is said to be in a latent period from infection until the obvious manifestation of the disease e.g. the period from HIV infection until AIDs diagnosis, is termed the incubation period Oyewale (2005)

IV. Model Fitting

Let: N = number of people in the community
I_t = number of people infected in the community at time t
S_t = Number of people susceptible in the community at time t

:. $I_t + S_t = N$ ………………………………………… (i)
t = time
let x = the proportion of infected individuals I_t in the community at time t.
Y = The proportion of susceptible individuals S_t, in the community at time t.

:. $X = I_t / N$, $Y = S_t /N$ and $x + y = 1$ ……………………(ii)
Since N is large, we take x and to be continuous random variables.
The rate at which the diseases spreads is dx/dt.
According to Inyama and Osuagwu (1999), from assumption (2) above, the rate of spread dx/dt is proportional to the number of contacts, and the number of contacts is proportional to xy by assumption (3)

Hence $dx/dt \propto xy = dx/dt = \beta xy$ ……………………… (iii)
Where β is constant of proportionality.
From equation (ii) $y = 1 - x$ ……………… …………. (iv)

Substituting equation (iv) into equation (iii) gives $dx/dt = \beta x (1 - x)$

$$\therefore \frac{dx}{x(1-x)} = \beta dt \quad \text{……………………………. (v)}$$

Resolving $\frac{1}{x(1-x)}$ into the partial fractions gives

$$\left[\frac{1}{x} + \frac{1}{(1-x)}\right]$$

Equation (v) therefore becomes

$$\left[\frac{1}{x} + \frac{1}{(1-x)}\right] dx = \beta dt$$

Integrating both sides gives
$In(X) - In(1 - X) = \beta t + a$

$$\frac{In\,(x)}{In\,(1-x)} = \beta t + a \text{ (where a is constant of integration)}$$

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$$\frac{X}{1-X} = e^{\beta t + a} = e^{\beta t} \cdot e^{a} = Ke^{\beta t} \text{ .... (vi)}$$

(Where K = e^a is a constant)

Using Assumption (1) we obtain

$\frac{X_0}{1 \quad X_0}$ = K (proportion of people infected at the first time)

From equation (vi)

$$\frac{X}{1-X} - \frac{X_o \; e^{\beta t}}{(1-X_0)}$$

$X (1 – X_0) = X_0 (1 - X) e^{\beta t}$

$X - X_o X = X_o e^{\beta t} - X_o X e^{\beta t}$

$X (1 - X_o e^{\beta t} t) = X_o e^{\beta t} t$

$$\therefore X = \frac{X_0 e^{\beta t}}{1 - X_0 + X_0 e^{\beta t}} =$$

$$\frac{X_0}{(1-X_0)e^{-\beta t} + X_0} =$$

$$\frac{1}{1 + \left[\frac{1}{X_0} - 1\right]^{e-\beta t}}$$

$$\therefore = \frac{1}{1 + \left[\frac{1}{X_0} - 1\right] e - \beta t} \text{ .... (vii)}$$

This model represents the portion of those infected by the disease at the time t.

β is the infection rate, usually, a positive constant if $X_o > 0$, then X_t tends to 1, as t tends to ∞, $e^{-\beta t}$ tends to zero and eventually X_t tends to 1.

Numerical illustration / results

Let X_0 = 10,000, t = 0, e = 2.7178

$$\therefore X_t - \frac{1}{1 + \left[\frac{1}{X_0} - 1\right] e - \beta t}$$

t(Months)	β= 0.01	β= 0.02	β= 0.05
0	10,000	10,000	10,000
1	9990	9802	9512
5	9950	9048	7788
10	9901	8187	6066
20	9802	6703	3679
45	9560	4066	1055
52	9493	3535	744
100	9048	1354	68
250	7788	68	1.037
300	7408	25	1.0031
520	5946	1.3	1

The table above shows three different increasing rates of the disease spread, β= 0.01, β= 0.02 and β= 0.05, with assumed initial population (X_o) of 10,000 at t = 0

As t (time) in months approach infinity (increases) $e^{-\beta t}$ approaches zero (decreases) which means that the value of X_t from our model will tend to 1, (see last column on the table).

This means that the last susceptible person in the will eventually become infected by the epidemics if the increasing rate of spread is not checked.

CONCLUSION

In this paper a mathematical model was developed for disease transmission prediction which could be use to project the spread of epidemics in a community.

If a substantive and workable preventive policies must be put in place, then modeling must form an integral part of the devices or means that reveals the dynamics of spread of infectious diseases our communities and if the Millennium Development Goals (MDGs) must be achieved in the fight against HIV/AIDS in Nigeria, then it is time for statisticians and mathematicians all over the country to wake up to this clarion calls. For mathematical modeling further researches can be done in the area of sensitivity analysis and simulation.

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