A Novel 2/2 Second-Order Compensator used with a Highly Oscillating Second-Order Process

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Abstract—Compensators are used in place of classical PID controllers for possible achievement of better performance. Un satisfactory dynamics of some industrial processes represent an engineering problem that has to be solved. Highly oscillating processes and very slow processes are examples.

In this paper a novel 2/2 second-order compensator based is proposed and applied to control a process having 85% overshoot and 6 seconds settling time. The proposed compensator is capable of controlling the steady-state characteristics of the closed-loop control system and its dynamic characteristics. The advantage of the proposed compensator is its simple tuning using MATLAB without need to the optimization toolbox. It is possible with the second-order compensator to satisfy a system performance without any overshoot with a settling time of only 0.2 seconds and steady-state error as low as 0.01 for a unit step input.

Index Terms—Two/two second-order forward compensator, compensator tuning, highly oscillating second-order process, control system performance.

I. INTRODUCTION

Feedforward compensators find wide application in both linear and nonlinear dynamic systems. The design of classical compensators such as lag, lead, lag-lead, PID and pre-filter are investigated in automatic control textbooks [1-5].

Randall and Sinencio (1985) discussed the application of the operational transconductance amplifier (OTA) in first-order and second-order filters structures [6]. Angulo and Snencio (1994) studied the low pass, high pass and band pass second-order filters using the multiple output nonlinearized operational transconductance amplifiers [7]. Edwards and Cauwenberghs (1998) described the synthesis of a second-order log-domain band pass filter and showed experimental results [8]. Ilehenko, Savvchenkov, Handley and Maleki (2003) demonstrated an approach for fabricating a photonic filter with second-order response function consisting of two Germany-doped silica microtoroidal cascaded in series [9]. Wilamowski and Gottiparthi (2005) described an educational MATLAB tool simplifying the process of analog filter design for several types of filters [10]. Menekay, Tarcan and Kuntman (2007) designed a current-mode square-root circuit with reduced short channel effect proposing a second-order low pass filter [11]. Zambon and Lehtonen (2008) presented a sustain-pedal effect simulation algorithm for piano synthesis using parallel second-order filters [12]. Crump (2010) studied the first-order, second-order and biquadratic filters examining their responses to simplify the filter design task for design engineers [13]. Ibarra (2011) offered the basics on active filter design by introducing the Butterworth approach as well as some practical examples [14]. Pandey, Singh, Kumar, Dubey and Tyagi (2012) presented a current mode second-order filter employing single current differenting transconductance amplifiers using current mode approach providing high pass, band pass and low pass responses [15]. Vishal, Saurabh, Singh and Chauhan (2012) studied the implementation of second-order low pass, high pass and band pass filters by using the current-controlled current differencing buffered amplifier [16]. Hassaan, Al-Gamil and Bashin (2013) used a lag-lead second-order compensator to control a first-order plus an integrator process. They tuned the compensator through minimizing the sum of square of error objective function and could reduce the maximum overshoot to 2.43% and the settling time to 0.65 s [17]. Jung and Kim (2014) used a second-order harmonic current reduction compensator for a two-stage DC-DC-AC grid connected inverter system. They built a 1 kW hardware to verify the effecting of the proposed compensator [18].

II. ANALYSIS

Process:
The process considered in this analysis has the transfer function, \( G_p(s) \):

\[
G_p(s) = \frac{\omega_{np}^2}{s^2 + 2z\omega_{np}s + \omega_{np}^2}
\]

Where:

\( \omega_{np} \) = process natural frequency = 10 rad/s.

\( z \) = process damping ratio = 0.05

The time response of this process in a unit feedback loop without compensation to a unit step input is shown in Fig.1 as generated by MATLAB:

![Fig.1 Step response of the uncompensated process.](image-url)
The performance of the process is measured by its maximum percentage overshoot and its settling time. It has a maximum overshoot of 85.45% and about 6 seconds settling time.

**The Proposed Compensator:**

The proposed compensator is a second-order one having the transfer function, \( G_c(s) \) given by [5]:

\[
G_c(s) = K_e \left( s^2 + 2 \zeta \omega_n s + \omega_n^2 \right) / \left( s^2 + 2 \zeta \omega_n s + \omega_n^2 \right)
\]

(2)

It has the 5 parameters:
- Gain, \( K_e \).
- Natural frequencies, \( \omega_n1 \) and \( \omega_n2 \).
- Damping ratios, \( \zeta_1 \) and \( \zeta_2 \).

The five parameters are function of the values of the resistance and capacitance of the resistor and capacitor components encountered in the filter electronic circuit.

The configuration of this compensator as defined by Eq.2 has the advantages:
- Easy cancellation of the undesired process poles.
- Easy replacement of the bad poles by better ones.
- Flexibility of setting the compensator poles and zeros through using potentiometers.

The quadratic zero of the compensator is used to cancel the bad quadratic pole of the process resulting in its highly oscillating time response. That is:

\[
\zeta_1 = \zeta_p
\]

and

\[
\omega_n1 = \omega_{np}
\]

(3)

**Control System Transfer Function:**

Assuming that the control system is a unit feedback one, its transfer function with \( G_c(s) \) of Eq.2 and \( G_f(s) \) of Eq.1 considering the conditions in Eq.3 is:

\[
M(s) = b_0 / \left( s^2 + a_1 s + a_2 \right)
\]

(4)

where:

\[
b_0 = K \omega_{np}^2
\]

\[
a_1 = 2 \zeta \omega_n \omega_{np}
\]

\[
a_2 = \omega_n^2 + K \omega_{np}^2
\]

**System step response and performance:**

A unit step response is generated by MATLAB using the numerator and denominator of Eq. 4 providing the system response \( c(t) \) as function of time for a set of compensator parameters [20].

The characteristics of the compensated control system quantifying its performance are:
- Steady-state error, \( e_{ss} \):

\[
e_{ss} = 1 / \left\{ 1 + (K \omega_{np}^2 / \omega_n^2) \right\}
\]

(5)

- Maximum percentage overshoot, \( OS_{max} \):

Using the time response of the control system to a unit step input, the maximum percentage overshoot is:

\[
OS_{max} = 100 (c_{max} - c_{ss}) / c_{ss}
\]

(6)

Where: \( c_{max} \) = maximum time response to a step input. 
\( c_{ss} \) = steady state response of the control system to the unit step input.

- Settling time, \( T_s \):

The time response of the system enters a band of \( \pm 5\% \) of the steady-state response and remains inside this band.

**III. COMPENSATOR TUNING**

The compensator proposed in this work is tuned without need to sophisticated optimization techniques. The procedure is as follows:

- A desired steady-error \( e_{ssdes} \), maximum percentage overshoot \( OS_{des} \) and settling time \( T_{des} \) are assigned as a measure for the performance of the compensated control system.

- The following three nonlinear equations are formulated:

\[
e_{ss} - e_{ssdes} = 0
\]

(7)

\[
OS - OS_{des} = 0
\]

(8)

\[
T_s - T_{des} = 0
\]

(9)

- Eqs.7-9 are functions of the compensator parameters \( K, \zeta_2 \) and \( \omega_{np} \).

- The 3 equations are solved using the MATLAB command "fsolve" revealing the compensator parameters [20,21].

**1. Tuning Results**

Eqs.7-9 are solved using MATLAB for a desired control system performance defined by:

\[
e_{ssdes} = 0.01
\]

\[
OS_{des} = 0
\]

\[
T_{des} = 0.2 \text{s}
\]

Resulting in the compensator parameters:

\[
K = 4.792
\]

\[
\zeta_2 = 9.531
\]

\[
\omega_{np} = 2.202 \text{ rad/s}
\]

The time response of the compensated system to a unit step input is shown in Fig.2.

![Fig.2 Step response of the second-order compensated process](image-url)
Performance of the control system incorporating the second-order compensator:
- Steady-state error: 0.01
- Maximum percentage overshoot: 0 %
- Settling time: 0.2 s

Conclusion
- The suggested tuning technique of the second-order compensators is superior in handling a difficult process such as the highly oscillating process.
- Through using a the proposed tuning technique, it was possible to reduce the maximum percentage overshoot of uncompensated process with unit feedback from 85.4 % to zero.
- Using the proposed tuning technique, it was possible to reduce the settling time from about 6 seconds to only 0.2 seconds.
- A new tuning technique is presented without need to optimization.
- The formulated nonlinear equations were easily solved using one MATLAB command.

REFERENCES

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