STEADY STATE SOLUTION OF M/M/2/N QUEUING SYSTEM WITH CATASTROPHES AND RESTORATIONS

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Abstract— **A steady-state solution is obtained for the system is an M/M/2/N queue with catastrophes and restorations. Queuing models have their place in modeling the real life phenomena. In fact their utility gets enhanced when one is not able to get the probability distribution for either the arrival times or for the service times. The service times are assumed to have general service time distribution in the case of computer networks modeling. Recently, the emphasis is put on the catastrophe modeling and its applications in real life situations**

Index Terms— **Catastrophes, Homogeneous servers, Markovian queue, Restoration, Steady state analysis**
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I. INTRODUCTION

In the study of catastrophe modeling and analysis has been playing a vital and domestic role in various areas of science and technology like as computer networks, telecommunication networks, supermarkets, banking industries, etc. Computer networks with virus may be modeled by queueing networks with catastrophes. Studies have been made on stochastic models for the growth of population subject to catastrophes. These include works by Tuckwell [12] ,Chao[2,3]. In recent years, queueing systems with catastrophes have been studied by Boucherie and Boxma [1], Gelenbe and Pujolle [5], Pavai Madheswari and Sadiq Basha [10] and Kumar, B.K. [7]. The notion of catastrophes occurring at random, leading to the annihilation of all the customers there and momentarily inactivation of service facility until the new customers enter in the system, is common in many practical problems. Jain and Kumar [6] introduced the concept of restoration in catastrophic queues. According to them, any system suffering from catastrophe will always require some time to functioning in normal state, which is taken some sort of time as restoration time. During this time, no customer was allowed to join the system. Kumar [8, 9] studied some queueing models with catastrophic and restorative effects. Di Crescenzo et al [4] studied a double ended queuing model with catastrophes and repair and obtained the transient and steady-state solutions. Tarabia [11] performed the transient and steady-state analysis of an M/M/1 queue with balking, catastrophes, server failures and repairs using the generating function and direct approach. Let us first give an example to illustrate the applicability of our queuing system with catastrophes and restorations in the context of shared buses local area networks. We consider a local area network (LAN) with N stations connected by a two buses. Messages arrive at the stations according to a Poisson stream. Upon receiving messages, the station checks the state of the buses. If the buses are free, then messages are immediately transmitted to the destination stations. On the other hand, if the buses are busy or virus may cause catastrophe, annihilates all the messages (packets) are stored in the buffer and the station will try it recovery times luck again after some sort of time i.e. restorations time. The steady-state solution of the model has been obtained with catastrophes and restorations. The rest of the paper is organized as follows: in the next section, the queuing model has been formulated, section 3 deals with the steady-state solution of the model and model has been concluded in section 4.

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II. Formulation of queuing model

The queuing model under consideration is based on the following assumptions:

A. The system having two homogenous servers.
B. The queue discipline is FCFS.
C. The customers arrive at a counter in accordance with a Poisson process with mean arrival rate $\lambda>0$.
D. The service time distribution of a customer is negative exponential with mean rate $\mu>0$.
E. The catastrophes occur according to Poisson process with mean rate ξ only when the system is not empty. The occurrence of a catastrophe destroys all the customers in the instants and affects the system as well. The system starts working after the restorations time is over. The arrivals are not allowed during restoration periods.
F. The restoration times are independently, identically exponentially distributed with parameter $\pi>0$.
G.

III. Steady-state solution of the queuing model

We define

$P_{n,m}(t)$ = The joint probability that there are n customers in the system at time t and system has reaches its capacity in the time interval (0,t]

$P_{00,m}(t)$ = the prob. that there are zero customers in the system at time t without the occurrence of catastrophe.

$Q_{00,m}(t)$= the prob. that there are zero customers in the system at time t with the occurrence of catastrophe.
In the steady state,

$$P_{n,m} = \underset{t\to\infty}{Lt}\, P_{n,m}(t), 0 \le n \le N$$

$$P_{00,m} = \underset{t\to\infty}{Lt}\, P_{00,m}(t) \quad Q_{00,m} = \underset{t\to\infty}{Lt}\, Q_{00,m}(t) \qquad [1]$$

Theorem 1: For ξ >0, the steady-state distribution {1} of the M/M/2/N queue with catastrophes and restorations corresponds to

$$P_{n,m} = \frac{\lambda P_{00,m} + \gamma^2 Q_{00,m}}{[\lambda(1-\xi)+\beta^{-2}+\xi]}\left[\frac{\gamma^2 T(N-1)}{(\lambda+\xi)^2+2\mu(\lambda+\xi)}\right]P_{0,m} \qquad [2]$$

$$P_{00,m} = \frac{\{\lambda\xi+\beta+\lambda\xi^2\}P_{0,m}}{\lambda-\xi+\beta^{-2}} - \left[\begin{matrix}(\lambda\xi+1)P_{0,m} \\ \left\{\dfrac{2\mu\gamma^2 T(n)+(\xi+\lambda)T(N-1)}{\lambda\{1-T(n)\xi-\xi^2\beta^{-2}(\lambda+\beta+\xi)\}}\right\}\end{matrix}\right] \qquad [3]$$

$$Q_{00,m} = \frac{\xi\left[1-\gamma^2 P_{00,m}\{2\mu\gamma^2(\alpha_1^1-\alpha_2^1)^{-1}\}T(N-1)\right]}{(\lambda+\beta-\xi)^2} \qquad [4]$$

Where

$$P_{0,m} = \left[\frac{\begin{matrix}(\alpha_1^N-\alpha_2^N)(\alpha_1^{N+1}-\alpha_2^{N+1}) \\ -\beta^{-2}T(N)\{\lambda T(N)\}^{m-1}\end{matrix}}{(\beta^{2m}\mu^m T(N)\}^{1+m}}\right], m \ge 1$$

$$T(k) = \left[\{\alpha_1^{k+1}-\alpha_2^{k+1}\} - 2\mu\lambda^{-1}\{\alpha_1^k-\alpha_2^k\}\right]$$

$$\alpha_1\alpha_2(=\beta^{-2}) = 2\mu\lambda^{-1}$$

Proof

The steady state equations governing the model are

$$0 = -\lambda P_{00,m} + \mu P_{1,m} + \pi Q_{00,m}, n = 0$$

$$0 = -(\lambda+\pi)Q_{00,m} + \xi\sum_{n=1}^{N} P_{n,m}, n = 0$$

$$0 = -(\lambda+\mu+\xi)P_{1,m} + \lambda P_{0,m} + 2\mu P_{2,m}, n = 1, m \ge 0 \qquad [5]$$

$$0 = -(\lambda+2\mu+\xi)P_{n,m} + \lambda P_{n-1,m} + 2\mu P_{n+1,m}, 1 < n < N, m \ge 0$$

$$0 = -(2\mu+\xi)P_{N,m} + \lambda P_{N-1,m}, n = N, m \ge 1$$

Define the generating functions by

$$G_n(x) = \sum_{m=0}^{\infty} P_{n,m}x^m, |x| \le 1$$

$$H(x,y) = \sum_{n=0}^{N} G_n(x)y^n \qquad [6]$$

Multiplying (5) by suitable powers of x, summing over m and using (6). After then multiplying by suitable powers of y, summing over n and using (6), we have

$$H(x,y) = \frac{\begin{matrix}(2\mu+\xi)G_N(x)\left[\dfrac{(x-1)}{x}+\dfrac{\lambda(1-y)}{2\mu+\xi}\right] \\ y^{N+1}-(1-y) \\ \left[2\mu+y(\lambda+\xi)\right]G_0(x) - y\xi(y-2)\end{matrix}}{y(\lambda+\xi)-2\mu(1-y)-\lambda y^2} \qquad [7]$$

Particular case when ξ =0 i.e. no catastrophes then the zeros of the denominator of [7] are given by

$$\alpha_i = \frac{(\lambda+\mu)\pm\sqrt{(\lambda+\mu)^2-4\lambda\mu}}{2\lambda}, i = 1,2$$

Here the value of the square root with positive real part is taken i.e α_i has Re>0 has the positive sign before the radical. The existence of H(x, y) is only possible if the numerator vanishes for α_i , i=1, 2 the zeros of the denominator. This will give rise two equation and solving them, we get

$$G_0(x) = \frac{D_1(x)}{D(x)}$$

$$G_{N-1}(x) = \frac{D(.)}{D(x)} \qquad [8]$$

Where

$$D(.) = \alpha_1 - \alpha_2$$

$$D_1(x) = \{\alpha_1^{N+1} - \alpha_2^{N+1} - \alpha_1^N + \alpha_2^N\}\lambda\mu x + \{\alpha_1^N - \alpha_2^N\}(1-x) \qquad [9]$$

$$D(x) = \{\alpha_1^{N+1} - \alpha_2^{N+1} - \beta^{-2}(\alpha_1^N - \alpha_2^N)\}\lambda(1-x), \beta^{-2} = 2\mu\lambda^{-1}$$

Comparing the coefficients of y^n using (6) and setting x=0, we have

$$G_n = \frac{\beta^{2n}}{\lambda}\left[\frac{\begin{Bmatrix}2\mu T(n)+\lambda+ \\ \xi T(n-1)\end{Bmatrix}\xi T(N-1)}{\begin{matrix}\{2\lambda+\lambda T(N)+(\lambda+\xi)T(N-1)\} \\ -\xi T(n-1)\end{matrix}}\right] \qquad [10]$$

Taking into the account the complexity of the solution obtained in Theorem 1, it would be interesting to find simple equations $P_{n,m}$, $P_{00,m}$ and $Q_{00,m}$, and investigate the effect of the parameters on the system performance in particular case when no catastrophes and no restoration i.e. $\xi = 0 \; and \; \pi = \infty$.

$$H(x,y) = P_{00}(x) + \sum_{n=1}^{N} G_n(x)y^n$$

Which is the steady state probability generating function.

Conclusions

This paper discusses an M/M/2/N queueing system with catastrophes and restorations effects. The steady state solution of the model has been derived. This model finds its application in computer network communication and tele-communication.

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