Common fixed point theorem using the Property (E.A.) and Implicit Relation in Fuzzy Metric Spaces

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Abstract— **In this paper we prove a common fixed point theorem in fuzzy metric spaces by extending the use of a common property (E.A.) and implicit relation for four self maps. Our results generalized the results of Kumar S. et al. [20].**

Index Terms— **Fuzzy metric spaces, fixed point, Weak compatible map, Property (E.A.) and implicit relation.**

I. INTRODUCTION

Zadeh [1] introduced the concept of fuzzy sets. Kramosil et al. [2] introduced the notion of a fuzzy metric space by generalized the concept of the probabilistic metric space to the fuzzy situation. George et al. [3] modified the concept of fuzzy metric space. Application of fuzzy mathematics played an important role in all discipline of applied sciences such as mathematical programming, modeling theory, neural network theory, control theory, communication, image processing, medical sciences etc. Some authors Schweizer et al. [4], Kaleva et al. [5], Grabiec et al. [6], Jungck [7], Singh et al. [8], Vasuki [9] have applied various form of the fuzzy sets from topology and modified the concept of fuzzy metric space. A number of fixed point theorems have been obtained by various authors in fuzzy metric spaces by using the concept of compatible mappings, weakly compatible mappings and R-weakly compatible mappings. Popa [10] prove theorem for weakly compatible non continuous mapping using implicit function. Pant et al. [11] extended the common fixed points of a pair of non-compatible mapping and the common property (E.A.) to fuzzy metric spaces. Mishra et al. [12] extended the notion of compatible maps under the name of asymptotically commutating maps. The property (E.A.) initiated by Aamri et al. [13] have been generalized the concept of non-compatible in metric spaces. Pathak et al. [14], Mihet [15], Imdad et al. [16], Aabbas et al. [17] have been obtained several results by using the concept of property (E.A.). Implicit relations are used as a tool for finding common fixed point of contraction mapping. Aalam et al. [18] have been proved a common fixed point theorem generalized the result of Singh et al. [19] without completeness condition in the spaces and continuity of involved mappings in fuzzy metric spaces. In this paper fixed point theorem has been established using the concept of property (E.A.) and implicit relation which generalized the results of Kumar et al. [20].

Manuscript received Sep 12, 2014

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II. PRELIMINARIES

Definition 2.1[1] Suppose that X be any non- empty set. A fuzzy set M in X is a function with domain X and values in$[0,1]$.

Definition 2.2 [2] The three- tuple $(X,M,*)$ is called a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0,\infty)$ satisfying the following conditions for all $x,y,z \in X$ and $t,s>0$.

[2.3.1] $M(x,y,t)>0,$

[2.3.2] $M(x,y,t)=1$ for all t > 0 if and only if $x=y$,

[2.3.3] $M(x,y,t)=M(y,x,t)$,

[2.3.4] $M(x,y,t)*M(y,z,s) \le M(x,z,t+s)$,

[2.3.5] $M(x,y,.):[0,\infty) \to [0,1]$ is left continuous,

Then M is called a fuzzy metric on X. Note that, M(x, y, t) can be thought as degree of nearness between x and y with respect to t. It is known that $M(x,\ y,.)$ is nondecreasing for all $x,y \in X$.

Definition 2.3[4] A mapping $*:[0,1]\times[0,1]\to[0,1]$ is called a continuous t-norm if $([0,1],*)$ is an abelian topological monoid with unit 1such that $a*b \ \le c*d$, whenever $a,b,c,d \in [0,1]$.

Definition 2.4[6] Suppose that $(X,M,*)$ be a fuzzy metric space then

[a] A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ (denoted by $\lim_{n\to\infty} x_n = x$) if $\lim_{n\to\infty} M(x_n,x,t)=1$ for all $t>0$.

[b]A sequence $\{x_n\}$ in X is called a Cauchy sequence if $\lim_{n\to\infty} M(x_{n+q},x,t)=1$ for all $t>0$ and $q>0$.

[c]A fuzzy metric space in which every Cauchy sequence is convergent is called complete fuzzy metric spaces.

Definition 2.5[7] Two self-mappings P and U of a fuzzy metric space $(X,M,*)$ are said to be compatible if $\lim_{n\to\infty} M(PUx_n,UPx_n,t)=1$, whenever $\{x_n\}$ is a sequence in X such that

$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Ux_n = a$ for some $a \in X$ and for all t > 0.

Definition 2.6[9] Two self-mappings P and U of a fuzzy metric space $(X, M, *)$ are said to be weakly commuting if $M(PUx, UPx, t) \geq M(Px, Ux, t)$, for each $x \in X$ and for all $t > 0$.

Definition 2.7[9] Suppose that P and U be self maps on a fuzzy metric space .The mappings P and U are said to be non compatible if $\lim_{n\to\infty} M(PUx_n, UPx_n, t) \neq 1$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Ux_n = a$, for some $a \in X$ and for all $t > 0$.

Definition2.8[13] Suppose that two self maps P and U of a metric space (X, d) are said to satisfy property (E.A.) if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Ux_n = c$, for some $c \in X$. Two self-maps U and V of a fuzzy metric space $(X, M, *)$ satisfy property (E.A.), if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} M(Ux_n, Vx_n, t) = 1$. Property (E.A.) allows replacing the completeness requirement of the space with a more natural condition of closeness of the range space.

Definition2.9 [17] Suppose that a pair of mappings P and U from a fuzzy metric space $(X, M, *)$ into itself are weakly compatible if they commute at their coincidence points. i.e. $Px = Ux$ implies that $PUx = UPx$.

Definition 2.10[17] The maps P, Q, U and V from a fuzzy metric space $(X, M, *)$ into itself are said to satisfies the property (E.A.) if there exists sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Ux_n = \lim_{n\to\infty} Qy_n = \lim_{n\to\infty} Vy_n = a$$

for some $a \in X$.

Definition 2.11[20] Suppose that ξ_5 be the set of all real continuous functions $\xi : (R^+)^5 \to R^+$ satisfying the following condition:

[i] $\xi(a, b, a, b, a) \geq 0 \Rightarrow a \geq b$ for all $a, b \in [0,1]$.

[ii] $\xi(a, a, b, a, b) \geq 0 \Rightarrow a \geq b$ for all $a, b \in [0,1]$.

[iii] $\xi(a, 1, a, 1, a) \geq 0$ Implies that $a \geq 1$.

Example: $\xi(t_1, t_2, t_3, t_4, t_5) = t_1 - min\{t_2, t_3, t_4, t_5\}$

Lemma 2.12 [3] Let $(X, M, *)$ be a fuzzy metric space. Then M is a continuous function on $X^2 \times (0, \infty)$.

III. Main Results

Theorem 3.1 Suppose that $(X, M, *)$ be a fuzzy metric space with $a * b = \min\{a, b\}$. Suppose that P, Q, U and V is a self maps of X satisfying the following:

[3.1.1] $P(X) \subseteq V(X)$ and $Q(X) \subseteq U(X)$,

[3.1.2] Pairs (P, U) and (Q, V) are weak compatible maps,

[3.1.3] Pairs (P, U) and (Q, V) satisfies the common property (E.A.),

[3.1.4] One of $P(X), Q(X), U(X)$ and $V(X)$ is closed subset of X,

[3.1.5] for some $\xi \in \xi_5$ and for every $x, y \in X$ and $t > 0$,

$$\xi\left[\begin{array}{l} M(Px, Qy, t), M(Qy, Vy, t), \\ M(Px, Vy, t), M(Qy, Ux, t), \\ \min\left\{\begin{array}{l}\dfrac{M(Ux, Vy, t) + M(Px, Ux, t)}{2}, \\ M(Ux, Vy, t), M(Px, Ux, t)\end{array}\right\}\end{array}\right] \geq 0$$

Then P, Q, U and V have a unique common fixed in X.

Proof: Suppose that (Q, V) satisfy the property (E.A.) then $\exists$ a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Qx_n = \lim_{n\to\infty} Vx_n = c$ for some $c \in X$. Since $Q(X) \subseteq U(X)$, $\exists$ a sequence $\{y_n\}$ in X such that $\lim_{n\to\infty} Qx_n = \lim_{n\to\infty} Uy_n = c$.

Now we prove that $\lim_{n\to\infty} Py_n = c$

Step 1

We put $x = y_n$ and $y = x_n$ in 4.1.5

$$\xi\left[\begin{array}{l} M(Py_n, Qx_n, t), M(Qx_n, Vx_n, t), \\ (Py_n, Vx_n, t), M(Qx_n, Uy_n, t), \\ \min\left\{\begin{array}{l}\dfrac{M(Uy_n, Vx_n, t) + M(Py_n, Uy_n, t)}{2}, \\ M(Uy_n, Vx_n, t), M(Py_n, Uy_n, t)\end{array}\right\}\end{array}\right] \geq 0$$

Taking limit as $n \to \infty$

$$\xi\left[\begin{array}{l} M(Py_n, c, t), M(c, c, t), \\ M(Py_n, c, t), M(Py_n, c, t), \\ \min\left\{\begin{array}{l}\dfrac{M(c, c, t) + M(Py_n, c, t)}{2}, \\ M(c, c, t), M(Py_n, c, t)\end{array}\right\}\end{array}\right] \geq 0$$

$$\xi\begin{bmatrix} M(Py_n,c,t),1, \\ \mathrm{M}(Py_n,c,\mathrm{t}),\mathrm{M}(Py_n,c,\mathrm{t}), \\ \min\left\{\dfrac{1+\mathrm{M}(Py_n,c,\mathrm{t})}{2}, 1,\mathrm{M}(Py_n,c,\mathrm{t})\right\} \end{bmatrix}\geq 0$$

$$\xi\begin{bmatrix} M(Py_n,c,t),1,\mathrm{M}(Py_n,c,t), \\ \mathrm{M}(Py_n,c,\mathrm{t}),\mathrm{M}(Py_n,c,t) \end{bmatrix}\geq 0$$

By the definition of ξ_5

$M(Py_n,c,t)\geq 1$ for all $t>0$

Hence $M(Py_n,c,t)=1$

i.e. $Py_n=c$

Since $U(X)$ is a closed subset of X, therefore $c=Ua$ for some $a\in X$,

We have, $\lim_{n\to\infty} Qx_n=\lim_{n\to\infty} Vx_n = \lim_{n\to\infty} Uy_n=\lim_{n\to\infty} Py_n=Ua=c.$

Step 2

We put $x=a, y=x_n$ in 4.1.5

$$\xi\begin{bmatrix} M(Pa,Qx_n,t),M(\mathrm{Q}x_n,\mathrm{V}x_n,\mathrm{t}), \\ \mathrm{M}(\mathrm{P}a,\mathrm{V}x_n,\mathrm{t}),\mathrm{M}(\mathrm{Q}x_n,\mathrm{U}a,\mathrm{t}), \\ \min\left\{\dfrac{M(\mathrm{U}a,\mathrm{V}x_n,\mathrm{t})+\mathrm{M}(\mathrm{P}a,\mathrm{U}a,\mathrm{t})}{2}, M(\mathrm{U}a,\mathrm{V}x_n,\mathrm{t}),\mathrm{M}(\mathrm{P}a,Ua,\mathrm{t})\right\} \end{bmatrix}\geq 0$$

Taking limit as $n\to\infty$

$$\xi\begin{bmatrix} M(Pa,c,t),M(c,c,\mathrm{t}), \\ \mathrm{M}(\mathrm{P}a,c,\mathrm{t}),\mathrm{M}(c,c,\mathrm{t}), \\ \min\left\{\dfrac{M(c,c,\mathrm{t})+\mathrm{M}(\mathrm{P}a,c,\mathrm{t})}{2}, M(c,c,\mathrm{t}),\mathrm{M}(\mathrm{P}a,c,\mathrm{t})\right\} \end{bmatrix}\geq 0$$

$$\xi\begin{bmatrix} M(Pa,c,t),1, \\ \mathrm{M}(\mathrm{P}a,c,\mathrm{t}),1, \\ \min\left\{\dfrac{1+\mathrm{M}(\mathrm{P}a,c,\mathrm{t})}{2}, 1,\mathrm{M}(\mathrm{P}a,c,\mathrm{t})\right\} \end{bmatrix}\geq 0$$

$$\xi\begin{bmatrix} M(Pa,c,t),1,\mathrm{M}(\mathrm{P}a,c,\mathrm{t}), \\ 1,M(Pa,c,t) \end{bmatrix}\geq 0$$

By the definition of ξ_5

$M(Pa,c,t)\geq 1$ for all $t>0$

Hence $M(Pa,c,t)=1$

i.e. $Pa=c$

We have $Pa=Ua$ .The weak compatibility of P and U implies that $PUa=UPa$ and then $Pc=PUa=Uc=UUa=Uc$.

Since $P(X)\subseteq V(X)$, Therefore $\exists$ a point $b\in X$ such that $Pa=Vb$.

Step 3

We claim that $Vb=Qb$, put $x=a$ and $y=b$ in 4.1.5

$$\xi\begin{bmatrix} M(Pa,Qb,t),M(\mathrm{Q}b,\mathrm{V}b,\mathrm{t}), \\ \mathrm{M}(\mathrm{P}a,\mathrm{V}b,\mathrm{t}),\mathrm{M}(\mathrm{Q}b,\mathrm{U}a,\mathrm{t}), \\ \min\left\{\dfrac{M(\mathrm{U}a,\mathrm{V}b,\mathrm{t})+\mathrm{M}(\mathrm{P}a,\mathrm{U}a,\mathrm{t})}{2}, M(\mathrm{U}a,\mathrm{V}b,\mathrm{t}),\mathrm{M}(\mathrm{P}a,Ua,\mathrm{t})\right\} \end{bmatrix}\geq 0$$

Taking limit as $n\to\infty$

$$\xi\begin{bmatrix} M(Pa,Qb,t),M(\mathrm{Q}b,Pa,\mathrm{t}), \\ \mathrm{M}(\mathrm{P}a,Pa,\mathrm{t}),\mathrm{M}(\mathrm{Q}b,Pa,\mathrm{t}), \\ \min\left\{\dfrac{M(Pa,Pa,\mathrm{t})+\mathrm{M}(\mathrm{P}a,Pa,\mathrm{t})}{2}, M(\mathrm{U}a,Pa,\mathrm{t}),\mathrm{M}(\mathrm{Pa},Pa,\mathrm{t})\right\} \end{bmatrix}\geq 0$$

$$\xi\begin{bmatrix} M(Pa,Qb,t),M(\mathrm{Q}b,Pa,\mathrm{t}),1, \\ \mathrm{M}(\mathrm{Q}b,Pa,\mathrm{t}),\min\{1,1,1\} \end{bmatrix}\geq 0$$

$$\xi\begin{bmatrix} M(Pa,Qb,t),M(Pa,\mathrm{Q}b,\mathrm{t}), \\ 1,\mathrm{M}(\mathrm{Pa},\mathrm{Q}b,\mathrm{t}),1 \end{bmatrix}\geq 0$$

By the definition of ξ_5

$M(Pa,\mathrm{Qb},t)\geq 1$ for all $t>0$

Hence $M(Pa,Qb,t)=1$

i.e. $Pa=Qb$

We get $Pa=Qb=Vb$

Thus $Pa=Ua=Vb=Qb=c.$

The weak compatibility of Q and V implies that $QVb=VQb$ and $VVb=VQb=QVb=QQb$,

i.e. $Vc=Qc$

Step 4

Now we prove $Pa(=c)$ is a common fixed point of P,Q,U and V

We put $x = c$ and $y = b$ in 4.1.5 we get

$$\xi\left[\begin{array}{l} M(Pc,Qb,t), M(Qb,Vb,t), \\ M(Pc,Vb,t), M(Qb,Uc,t), \\ \min\left\{\dfrac{M(Uc,Vb,t)+M(Pc,Uc,t)}{2}, M(Uc,Vb,t), M(Pc,Uc,t)\right\} \end{array}\right] \geq 0$$

Taking limit as $n \to \infty$

$$\xi\left[\begin{array}{l} M(Pc,c,t), M(c,c,t), \\ M(Pc,c,t), M(c,Pc,t), \\ \min\left\{\dfrac{M(Pc,c,t)+M(Pc,Pc,t)}{2}, M(Pc,c,t), M(Pc,Pc,t)\right\} \end{array}\right] \geq 0$$

$$\xi\left[\begin{array}{l} M(Pc,c,t), 1, \\ M(Pc,c,t), M(Pc,c,t), \\ \min\left\{\dfrac{M(Pc,c,t)+1}{2}, M(Pc,c,t), 1\right\} \end{array}\right] \geq 0$$

$$\xi\left[\begin{array}{l} M(Pc,c,t), 1, \\ M(Pc,c,t), M(Pc,c,t), \\ \min\left\{\dfrac{M(Pc,c,t)+1}{2}, M(Pc,c,t), 1\right\} \end{array}\right] \geq 0$$

$$\xi\left[\begin{array}{l} M(Pc,c,t), 1, M(Pc,c,t), \\ M(Pc,c,t), M(Pc,c,t) \end{array}\right] \geq 0$$

By the definition of ξ_5

$M(Pc,c,t) \geq 1$ for all $t > 0$

Hence $M(Pc,c,t) = 1$

i.e. $Pc = c$

We have $Pc = c$. Hence $c = Pc = Uc$ and c be a common fixed point of P and U. We can also prove that $Qb = c$ is also a common fixed point of Q and V. Thus we conclude that c is a common fixed point of P, Q, U and V. Similarly suppose that $V(X)$ is a closed subset of X. In this cases in which $P(X)$ or $Q(X)$ be a closed subset of X are similar to the cases in which $U(X)$ or $V(X)$ respectively is closed.

Step-5

Uniqueness: Suppose that c and d be two common fixed points of maps P, Q, U and V.

We put $x = c$ and $y = d$ in 4.1.5 we get

$$\xi\left[\begin{array}{l} M(Pc,Qd,t), M(Qd,Vd,t), \\ M(Pc,Vd,t), M(Qd,Uc,t), \\ \min\left\{\dfrac{M(Uc,Vd,t)+M(Pc,Uc,t)}{2}, M(Uc,Vd,t), M(Pc,Uc,t)\right\} \end{array}\right] \geq 0$$

$$\xi\left[\begin{array}{l} M(c,d,t), M(d,d,t), \\ M(c,d,t), M(d,c,t), \\ \min\left\{\dfrac{M(c,d,t)+M(c,c,t)}{2}, M(c,d,t), M(c,c,t)\right\} \end{array}\right] \geq 0$$

$$\xi\left[\begin{array}{l} M(c,d,t), 1, \\ M(c,d,t), M(c,d,t), \\ \min\left\{\dfrac{M(c,d,t)+1}{2}, M(c,d,t), 1\right\} \end{array}\right] \geq 0$$

$$\xi\left[\begin{array}{l} M(c,d,t), 1, M(c,d,t), \\ M(c,d,t), M(c,d,t) \end{array}\right] \geq 0$$

By the definition of ξ_5

$M(c,d,t) \geq 1$ for all $t > 0$

Hence $M(c,d,t) = 1$

i.e. $c = d$

Thus c is the unique common fixed point of P, Q, U and V

Theorem 3.2 Let $(X, M, *)$ be a fuzzy metric space with continuous t-norm. Let P, Q, U and V be self mappings of X satisfying 4.1.5.Then P, Q, U and V have a unique common fixed point in X, the pairs (P, U) and (Q, V) satisfy the property (E.A.). $V(X)$ and $U(X)$ are closed subsets of X and the pairs (P, U) and (Q, V) are weakly compatible.

Proof: Suppose that (P, U) and (Q, V) satisfy property (E.A.), then $\exists$ two sequences $\{x_n\}$ and $\{y_n\}$, such that $Qx_n = Vx_n = Uy_n = Py_n = c$ for some c in X. Since $V(X)$ and $U(X)$ are closed subsets of X, therefore $c = Ua = Vb$ for some $a, b \in X$.

We claim that $Pa = c$

We put $x = a, y = x_n$ in 4.1.5,

$$\xi\begin{bmatrix} M(Pa,Qx_n,t), M(Qx_n,Vx_n,t), \\ M(Pa,Vx_n,t), M(Qx_n,Ua,t), \\ \min\left\{\dfrac{M(Ua,Vx_n,t)+M(Pa,Ua,t)}{2}, M(Ua,Vx_n,t), M(Pa,Ua,t)\right\}\end{bmatrix} \geq 0$$

Taking limit $n \to \infty$

$$\xi\begin{bmatrix} M(Pa,c,t), M(c,c,t), \\ M(Pa,c,t), M(c,c,t), \\ \min\left\{\dfrac{M(c,c,t)+M(Pa,c,t)}{2}, M(c,c,t), M(Pa,c,t)\right\}\end{bmatrix} \geq 0$$

$$\xi\begin{bmatrix} M(Pa,c,t), 1, \\ M(Pa,c,t), 1, \\ \min\left\{\dfrac{1+M(Pa,c,t)}{2}, 1, M(Pa,c,t)\right\}\end{bmatrix} \geq 0$$

$$\xi\begin{bmatrix} M(Pa,c,t), 1, \\ M(Pa,c,t), \\ 1, M(Pa,c,t)\end{bmatrix} \geq 0$$

By the definition of ξ_5

$M(Pa,c,t) \geq 1$ for all $t > 0$

Hence $M(Pa,c,t) = 1$

i.e. $Pa = c$

We have $Pa = c = Ua$.

We show that $Vb = Qb$.

We put $x = a, y = b$ in 4.1.5,

$$\xi\begin{bmatrix} M(Pa,Qb,t), M(Qb,Vb,t), \\ M(Pa,Vb,t), M(Qb,Ua,t), \\ \min\left\{\dfrac{M(Ua,Vb,t)+M(Pa,Ua,t)}{2}, M(Ua,Vb,t), M(Pa,Ua,t)\right\}\end{bmatrix} \geq 0$$

$$\xi\begin{bmatrix} M(Vb,Qb,t), M(Qb,Vb,t), \\ M(Vb,Vb,t), M(Qb,Vb,t), \\ \min\left\{\dfrac{M(Vb,Vb,t)+M(Vb,Vb,t)}{2}, M(Vb,Vb,t), M(Vb,Vb,t)\right\}\end{bmatrix} \geq 0$$

$$\xi\begin{bmatrix} M(Vb,Qb,t), M(Vb,Qb,t), \\ 1, M(Vb,Qb,t), 1\end{bmatrix} \geq 0$$

By the definition of ξ_5

$M(Vb,Qb,t) \geq 1$ for all $t > 0$

Hence $M(Vb,Qb,t) = 1$

i.e. $Vb = Qb$

Thus $Pa = Ub = Qb = Vb = c$

Therefore P, Q, U and V have a unique common fixed point c in X.

Conclusion

Main results generalized and improve results of Kumar S. and Fisher B. [20]. This result is proved for common fixed point theorem using the property (E.A.) and Implicit relation. We conclude that common property (E.A.) permit replacing the completeness requirement of the space with a more natural condition of the closeness of the space.

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