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New Controllers design method for FOPDT/SOPDT processes

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Abstract— ***This paper extends writer's previous work and proposes a new simple and efficient P-, PI, PD- and PID-controllers design methods to cope with a wide range of FOPDT/SOPDT processes to achieve an important design compromise; acceptable stability, and medium fastness of response, the proposed method is based on relating and selecting controllers' parameters based on process's parameters. Simple set of formulas are proposed for calculating and soft tuning of controllers' gains, the proposed controllers design methods was tested using MATLAB/Simulink for different FOPDT and SOPDT processes***

Index Terms— **Controller, FOPDT, SOPDT, Controller design.**

I. INTRODUCTION

The term control system design refers to the process of selecting feedback gains (poles and zeros) that meet design specifications in a closed-loop control system. Most design methods are iterative, combining parameter selection with analysis, simulation, and insight into the dynamics of the plant (Katsuhiko Ogata,1997)(Ahmad A. Mahfouz,et al,2013). An important compromise for control system design is to result in acceptable stability, and medium fastness of response, one definition of acceptable stability is when the undershoot that follows the first overshoot of the response is small, or barely observable. Beside world wide known and applied controllers design method including Ziegler and Nichols known as the "process reaction curve" method (J. G. Ziegler,1943) and that of Cohen and Coon (G. H. Cohen,1953) Chiein-Hrones-Reswick (CHR), Wang–Juang–Chan, many controllers design methods have been proposed and can be found in different texts including (Katsuhiko Ogata,1997)(Astrom K.J et al1994)(Ashish Tewar, et all,2002)(Norman S. Nise,2011)(Gene F. Franklin, et all, 2002)(Dale E. Seborg, et all, 2004)(Dingyu Xue,et all, 2004)(Chen C.L, 1998) (R. Matousek,2012)(K. J. Astrom,2001)(Susmita Das, et all,2012)(L. Ntogramatzidis, et all,2010)(M.Saranya, et all, 2012)(Fernando G. Martons,2005)(Saeed Tavakoli, et all, 2003)(Juan Shi , et all,2004)(Farhan A. Salem,2014), each method has its advantages, and limitations. (R. Matousek, 2012) presented multi-criterion optimization of PID controller by means of soft computing optimization method HC12. (K. J. Astrom, et all, 2001) Introduced an improved PID tuning approach using traditional Ziegler-Nichols tuning method with the help of simulation aspects and new built in function. (L. Ntogramatzidis, et all, 2010) a unified approach has been presented that enable the parameters of PID, PI and PD controllers (with corresponding approximations of the derivative action when needed) to be computed in finite terms given appropriate specifications expressed in terms of steady-state performance, phase/gain margins and gain crossover frequency. (M.Saranya, et all, 2012) proposed an IMC tuned PID controller method for the DC motor for robust operation. (Fernando G. Martons,2005) proposed a procedure for tuning PID controllers with Simulink and MATLAB. (Saeed Tavakoli, et all, 2003) presented using dimensional analysis and numerical optimization techniques, an optimal method for tuning PID controllers for FOPDT systems. (Juan Shi, et all, 2004) presented some derivation of IMC controllers and tuning procedures when they are applied to SOPDT processes for achieving set-point response and disturbance rejection tradeoff. (Farhan A. Salem,2013) proposed a new and simple controllers efficient model-based design method, based on relation controller's parameters and system's parameters. Many tuning formulas for PID controllers have been obtained for FOPDT processes (B.A. Ogunnaike, et all, 1989)(J.Shi, et all,2002)(F.G. Shinskey, 1998), by optimizing some time-domain performance criteria. (Juan Shi, et all, 2004) proposed set point response and disturbance Rejection tradeoff for second-order plus dead time processes. (Jan Cvejn,2011) presented simple PI/PID controller tuning rules for FOPDT plants with guaranteed closed-loop stability margin.

This paper extend previous work (Farhan A. Salem,2013) and proposes simple and efficient P, PI, PD and PID controllers design method for FOPDT and SOPDT processes, the proposed method is to be a good starting point to get a process under control and is based on relating controller(s)' parameters and plant's parameters to result in meeting an important design compromise; acceptable stability, and medium fastness of response. To achieve smoother response with in terms of minimum *PO%, 5T*, T_S, and E_{SS}, soft tuning parameters with recommended ranges are to be introduced. To achieve approximate desired output response or a good start design point, expression for calculating corresponding controller's gain are to be proposed.

1.1 Controllers Mathematical Modeling

The controllers that will be considered are P-, PD-, PI- and PID controllers; where P- term gives control system an instant response to an error, the I- term eliminates the error in the longer term, and D-term have the effect of reducing the maximum overshoot and making the system more stable by giving control system additional control action when the error changes consistently, it also makes the loop more stable (up to a point) which allows using a higher controller

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gain and a faster integral-terms (shorter integral time or higher integral gain) .
The transfer function of P-Controller is given by Eq.(1). The transfer function of PD-Controller is given by Eq.(2). Equation (2) can be rewritten, in terms of derivative time constant T_D. The transfer function of PI-Controller is given by Eq.(3), Equation (3) can be rewritten, in terms of integral time constant T_I, to have the from given by (4). The transfer function of PID-controller is given by Eq.(5), This equation is second order system, with two zeros and one pole at origin, and can be expressed to have the form given by Eq.(6), which indicates that PI and PD controllers are special cases of the PID controller. The ability of PI and PID controllers to compensate many practical processes has led to their wide acceptance in many industrial applications. The transfer function of PID controller, also, can also be expressed to have the manipulated form given by Eq.(7), since PID transfer function is a second order system, it can be expressed in terms of damping ratio ζ and undamped natural frequency ω_n to have the form given by Eq.(8). The transfer function of PID control given by Eq.(8) can, also, be expressed in terms of derivative time T_D and integral time T_I to have the form given by Eq.(9), since in Eq. (9) the numerator has a higher degree than the denominator, the transfer function is not causal and can not be realized, therefore this PID controller is modified through the addition of a lag to the derivative term, to have the form given by Eq.(10), where: N: determines the gain K_{HF} of the PID controller in the high frequency range, the gain K_{HF} must be limited because measurement noise signal often contains high frequency components and its amplification should be limited. Usually, the divisor N is chosen in the range 2 to 20. All these controllers and their form are simulated in MATLAB/Simulink as shown in Figure 2. Lead and lag Compensators are soft approximations of PI, PD-controllers, and used to improve systems transient and steady state response by presenting additional poles and zeros to the system, the transfer function of compensator are given by Eq.(11), where lag compensator is soft approximations of PI, with $Z_o > P_o$, and Z_o small numbers *near* zero , $Z_o = K_I/K_P$. The smaller we make P_o, the better this controller approximates the PI controller. Lead compensator is soft approximations of PD, with Z < P, The larger the value of P, the better the lead controller approximates PD control. PD-controller is approximated to lead controller as given by Eqs.(12)

$$G_p(s) = \frac{U(s)}{E(s)} = K_p \quad (1)$$

$$G_{PD}(s) = K_P + K_D s = K_D(s + \frac{K_P}{K_D}) = K_D(s + Z_{PD})$$

$$G_{PD}(s) = K_P + K_D s = K_P(1 + \frac{K_D}{K_P}s) \quad (2)$$

$$G_{PD}(s) = K_P(1 + T_D s) =$$
$$K_P(1 + \frac{T_D s}{1 + T_D s}) = K_P(1 + \frac{T_D s}{1 + T_D s / N})$$

$$G_{PI}(s) = K_P + \frac{K_I}{s} = \frac{K_P s + K_I}{s} = \frac{K_P(s + \frac{K_I}{K_P})}{s} = \frac{K_P(s + Z_{PI})}{s} \quad (3)$$

$$G_{PI}(s) = K_P + \frac{K_I}{s} = K_P(1 + \frac{K_I}{K_P s}) = K_P(1 + \frac{1}{T_I s}) = K_P\left(\frac{T_I s + 1}{T_I s}\right) \quad (4)$$

$$u(t) = K_P(e(t) + \frac{1}{T_I}\int e(t)dt) \Rightarrow u(t) = K_P(e + \frac{e}{Ti})$$

$$G_{PID}(s) = K_P + \frac{K_I}{s} + K_D s = \frac{K_D s^2 + K_P s + K_I}{s} = \frac{K_D\left[s^2 + \frac{K_P}{K_D}s + \frac{K_I}{K_D}\right]}{s} \quad (5)$$

$$G_{PID} = \frac{K_D(s + Z_{PI})(s + Z_{PD})}{s} =$$
$$K_D(s + Z_{PI})\frac{(s + Z_{PD})}{s} = G_{PD}(s)G_{PI}(s) \quad (6)$$

$$G_{PID} = \frac{K_D(s + Z_{PI})(s + Z_{PD})}{s} =$$
$$\frac{K_D s^2 + (Z_{PI} + Z_{PD})K_D s + (Z_{PI}Z_{PD}K_D)}{s} \quad (7)$$

$$G_{PID} = \frac{K_D s^2}{s} + \frac{K_D s(Z_{PI} + Z_{PD})}{s} + \frac{K_D(Z_{PI}Z_{PD})}{s} =$$
$$(Z_{PI} + Z_{PD})K_D + \frac{(Z_{PI}Z_{PD}K_D)}{s} + K_D s$$

$$G_{PID}(s) = \frac{K_D\left[s^2 + \frac{K_P}{K_D}s + \frac{K_I}{K_D}\right]}{s} = \frac{K_D\left[s^2 + 2\xi\omega_n s + \omega_n^2\right]}{s} \quad (8)$$
$$\text{where: } 2\xi\omega_n = \frac{K_P}{K_D}, \quad \omega_n^2 = \frac{K_I}{K_D}$$

$$G_{PID} = K_P\left(1 + \frac{1}{T_I s} + T_D s\right) = K_P\frac{T_I T_D s^2 + T_I s + 1}{T_I s} \quad (9)$$
$$\text{where: } T_I = \frac{K_P}{K_I}, \text{Integral time}, T_D = \frac{K_D}{K_P}, \text{derivative time}.$$

$$G_{PID} = K_P\left(1 + \frac{1}{T_I s} + \frac{T_D s}{1 + \frac{T_D s}{N}}\right),$$
T_D/N - time constant of the added lag (10)
$$G_{PID} = K_P\left(1 + \frac{1}{T_I s}\right)\frac{1 + T_D s}{1 + \frac{T_D s}{N}}$$

$$G(s) = K_c\frac{(s + Z_o)}{(s + P_O)} \quad (11)$$

$$G_{Lead}(s) = K_P + K_D\frac{Ps}{s + P} = \frac{K_P(s + P) + K_D Ps}{s + P} =$$
$$(K_P + K_D P)\frac{s + \left[\frac{K_P P}{K_P + K_D P}\right]}{s + P} = K_c\frac{(s + Z_o)}{(s + P_O)} \quad (12)$$

2. Plants' standard forms

a wide variety of ways in which processes with time delay may be modeled, Common models are; (a) Stable first order lag plus time delay (FOPD) model,(a) integral plus delay model (IPD),(c) First order lag plus integral plus delay (FOIPD) ,(d) Second order plus delay time (SOPDT), this paper is most applied for (FOPD) and (SOPDT) models , with transfer function given by Eqs.(14),(15), meanwhile, transfer functions of other models are given next:

$$\frac{C(s)}{R(s)}=\frac{Ke^{-Ls}}{s} \qquad \text{(IPD)}$$

$$\frac{C(s)}{R(s)}=\frac{Ke^{-Ls}}{s\left(Ts+1\right)} \qquad \text{(FOIPD)}$$

2.1 First order plus delay time (FOPDT) process.

A large number of industrial plants can approximately be modeled by FOPTD process (Katsuhiko Ogata,1997),(Saeed Tavakoli,et all, 2003). FOPDT models are a combination of a first-order process model with dead-time, it can be represented by the block diagram shown in Figure 1(a), with transfer function given by Eq.(13) and response curve shown in Figure 2(a), this s-shape curve with no overshoot is called reaction curve, it is characterized by three parameters ; the delay time *L*, time constant *T* and steady state level *K*. As shown in Figure 1, these three parameters can be determined by drawing a tangent line at the inflection point of the s-shaped curve, and finding the intersection of the tangent line with time axis and steady state level *K* (Farhan A. Salem, 2013) .Three examples of FOPDT process are given by Eq(15).

2.2 Second order plus delay time (SOPDT) process:

A second order plus dead-time transfer function given by Eq.(14), these are a combination of a second-order process model with dead-time, and can be represented by the block diagram shown in Figure 2(a). However, if used controller's gains are chosen such that the closed-loop system exhibits an under-damped response, then the closed-loop response can be approximated (Mamat, R et all, 1995) by a second order plus dead-time transfer function given by Eq.(14) (Tony Kealy, et all, 2002). SOPDT can be characterized by constants *L, T, K,* ζ, and ω_n . Three examples of second order plus delay time (SOPDT) process are given by Eq(15), and step response of SOPDT is shown in Figure 1(b). MATLAB codes given by Eq.(16) can be used to define FOPDT and SOPDT in MATLAB and return step response.

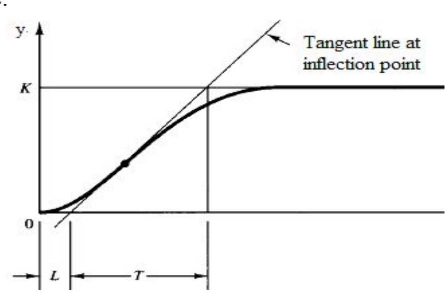


Figure 1(a) FOPDT S-shaped curve with terminology (Farhan A. Salem, 2013)

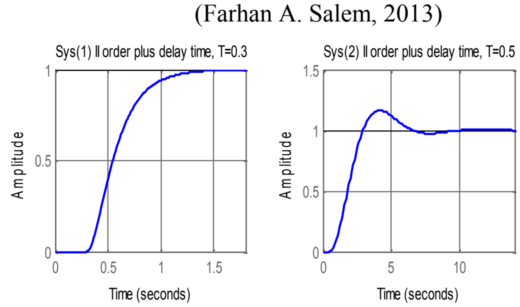
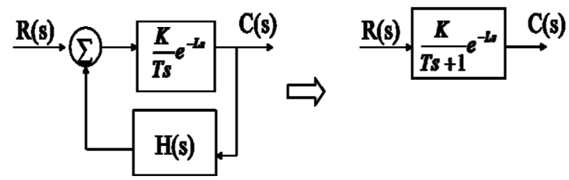
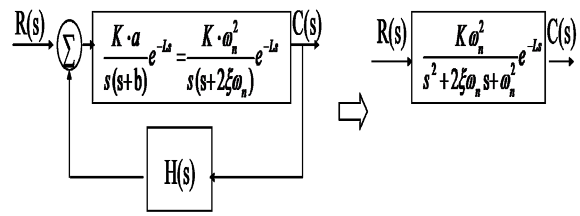


Figure 1(b) SOPDT S-shaped curve



(a) Block diagram representation of FOPDT process



(b) Block diagram representation of SOPDT process

Figure 2 FOPDT and SOPDT processes

$$\frac{C(s)}{R(s)}=\frac{Ke^{-Ls}}{Ts+1} \qquad (13)$$

$$\frac{C(s)}{R(s)}=\frac{Ke^{-Ls}}{\left(T_1s+1\right)\left(T_2s+1\right)}=\frac{Ke^{-Ls}}{(s^2/\omega_n^2)+(2\xi\, s/\omega_n)+1}=\frac{Ke^{-Ls}}{(\tau^2\omega_n^2)+(2\xi\tau\, s)+1} \qquad (14)$$

$$G_a(s)=\frac{1}{s+1}e^{-0.3s} \quad G_b(s)=\frac{5}{10s+1}e^{-0.5s}$$
$$,G_c(s)=\frac{0.005}{5s+1}e^{-0.9s} \quad G_d(s)=\frac{0.1}{2s+10}e^{-0.2s}$$
$$G_{sys1}(s)=\frac{50e^{-0.3s}}{\left(0.1s+1\right)\left(0.2s+1\right)}=\frac{50e^{-0.3s}}{s^2+15s+50} \qquad (15)$$
$$G_{sys2}(s)=\frac{e^{-0.3s}}{s^2+s+1} \quad , \quad G_{sys3}(s)=\frac{0.05}{2s^2+9s+1}e^{-0.5s}$$

```
G = tf(1,[1 15  50],'InputDelay',2.1),Step(G)
s = tf('s'), G = exp(-L*s)/(Ts+1), Step(G)
```
(16)

3. Proposed Controllers design method.

In basic terms, for processes with dead time, if the time constant is greater than the dead time (T>L), control system design is not difficult, however, if the time constant is less than the dead time (T<L), then design of a satisfactory control system may be difficult. The proposed (P, PI, PD, and PID) controllers design method is based on relating and selecting controllers' parameters based on process's parameters (ζ, ω_n *T, K* and *L*), as given by Eq.(16b), these function to be reduced in terms of variables and based on required controller type and process order, to result in expressions for calculating controllers' parameters to meet acceptable stability, and medium fastness of response. The methods applied for derivation proposed expressions include dimensional analysis to simplify a problem by reducing the number of variables to the smallest number of essential ones (M. Zlokarni, 1991), mathematical modeling and solving step response for tracking acceptable acceptable stability and medium fastness of response in terms of minimum *PO%, 5T,* T_S, and E_{SS}, analysis of results based on relating processes parameters to controllers parameters, considering effects of each parameter on overall system response, e.g. relating

controller's derivative time constant T_D to process's dead time L ,and integral time constant T_I to process's time constant T, and finally trial and error. for some cases the proposed method present a good starting point to get a process under control, to achieve desired and/or smoother response, soft tuning parameters (α, β and ε) with recommended ranges are to be introduced. Simulink to be used to test the proposed design method for different FOPDT and SOPDT processes, to achieve desired output of 10 for 10 input value, controllers will be used and tested in different forms given by Eqs. (1) by (10), manual switches are used to switch between controllers' types and forms, as well as, controlled systems, the block diagram representation of the model used is shown in Figure 2. First Ziegler-Nichols (ZN) controller tuning method with expressions listed in Table 1, will be used to compare the result of proposed design, since ZN still widely used (Katsuhiko Ogata, 1997).

$$K_x = f_y(\zeta, \omega_n, T, K, L) \qquad (16b)$$

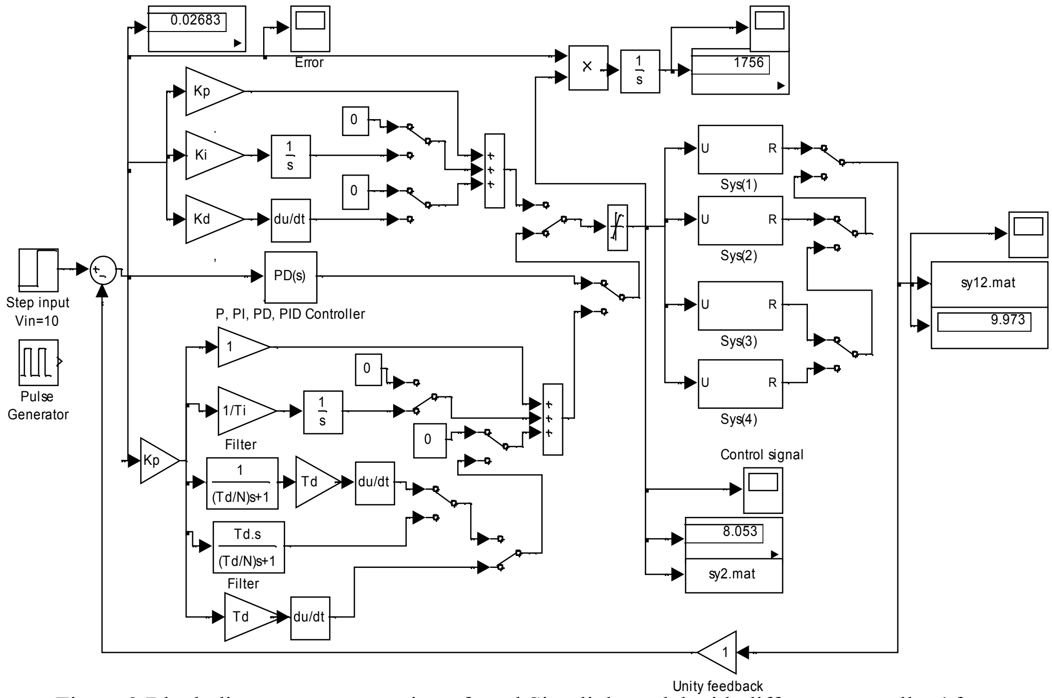


Figure 2 Block diagram representation of used Simulink model with different controllers' forms.

Table 1: Ziegler-Nichols controller tuning method

Controller type	K_P	T_I	T_D
P	T/L	∞	0
PI	0.9 T/L	L/0.3	0
PID	1.2 T/L	2L	0.5L

4. P-Controller design

4.1 P-Controller for first order plus delay time (FOPDT) processes.

P-Controller gives control system an instant response to an error. Based on FOPDT process's L, T, and *K*, expressions given by Eq.(17) are proposed to calculate P-controller gain K_P, a long with tuning parameter α. For processes with small DC gain and/or small time constant, to speed up response and to minimize time of design process, the expression given by Eq.(18) can be applied directly, where each increase of parameter α will increase K_P by 10. The expressions are applied first assigning α=1, and then parameter α can be tuned to soften resulted response. It is important to notice that, applying P-only controller shows offset error, and oscillatory response, moreover increasing α will result in increasing overshoot and reducing steady state error, also, assigning α big values may result in undamped response up to unstable response.

Testing the proposed expression for P-controller design for systems(a)(b)(c), given by Eq.(15), and comparing the results with First Ziegler-Nichols method, will result in response curves shown in Figure 3. The numerical values and response measures are shown in Table 2. Analyzing resulted response curves and data show, a response with less offset error can be achieve by soft tuning of parameter α (tuning K_p), also response curves show that for processes with small DC gain and/or small time constant, the proposed expression given by Eq.(18) result in more acceptable response than Ziegler-Nichols, that may result in unsatisfactory response.

$$K_P = \frac{T \cdot L}{K} \quad \Leftrightarrow \quad K_P = \frac{\alpha \cdot T \cdot L}{K} \qquad (17)$$

$$K_P = \frac{10 \cdot T \cdot L}{K} \quad \Leftrightarrow \quad K_P = \frac{10 \cdot \alpha \cdot T \cdot L}{K} \qquad (18)$$

Table 2: P-Controller for systems(a)(b)(c) applying proposed and Z-N tuning method.

P-Controller		α	T	K	L	K_p	Mp	5T	DC gain
System(a)	Proposed method	1	1	1	0.3	0.3	-	4.4	2.3
		6				1.8	0.65	2.4	6.43
		20					See Figure 3(a)		
Ziegler-Nichols		-				3.3333	3.51	4.55	7.69
System(b)	Proposed method	1	10	5	0.5	1	-	6.7	8.3
		3.5				3.5	3.54	7	9.46
		7				7	See Figure 3(b)		
Ziegler- Nichols		-				20	See Figure 3(b)		
System(c) (small DC gain) Eq.(18)	Pro. method	**1**	0.2	0.005	0.9	90	4	10	8.42
		2.5				900	2.11	10	8.18
		5				180	18	70	9.1
Ziegler- Nichols		-				0.2222	See Fig. 3(c)		2.1

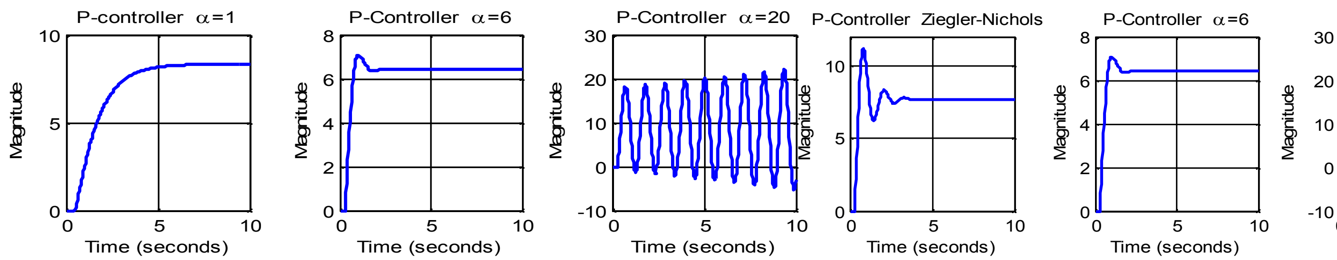


Figure 3(a) P-controller for system(a) applying proposed and Z-N methods

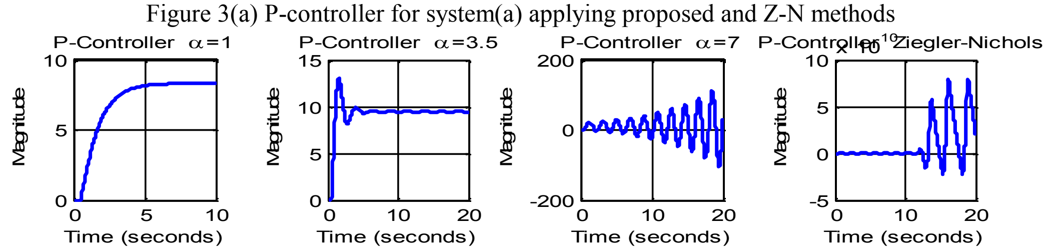


Figure 3(b) P-controller for system(b) applying proposed and Z-N methods

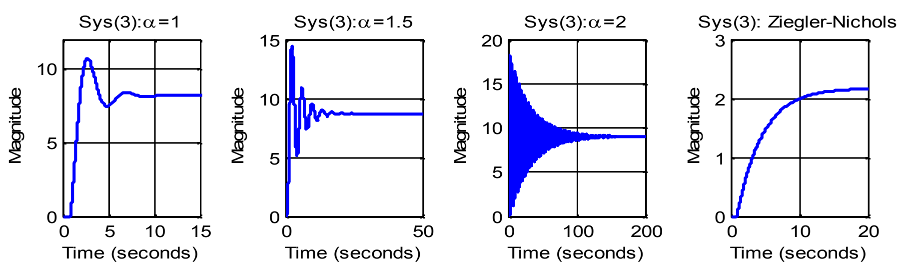


Figure 3(c) P-controller for system(c) applying proposed and Z-N methods

4. P-Controller for Second order plus delay time (SOPDT) process.

Based on SOPDT process's L, K, ζ and ω_n the expressions given by Eq.(19) are proposed to calculate P-controller gain (K_P), a long with tuning parameter α. For processes with small DC gain and/or small time constant, the expression given by Eq.(20) can be applied directly. The expressions are applied first assigning parameter α=1, and then parameter α can be tuned to soften resulted response. Also expression given by Eq.(20b) can be proposed.
Testing proposed expressions given by Eq.(20)for processes(1)(2)(3) given by Eq.(15), will result in response curves shown in Figure 4. Numerical data and response measures are shown in table 3. Analyzing data and responses show increasing α, (increases K_p), will result in increasing overshoot and reducing steady state error, and finally, assigning α big values will result in undamped response up to unstable response.

$$K_P = \frac{\alpha \cdot T}{K} \quad (20)$$

$$K_P = \frac{\alpha \cdot T \cdot L \cdot R \cdot \omega_n}{\xi \cdot K} \quad (19b)$$

Table 3: P-Controller for systems(1)(2)(3) applying proposed and Z-N tuning method.

P-Controller		α	ζ	ω_n	K	L	K_p	Mp	5T	DC gain
System(1)	Proposed method	1	0.7071	7.0711	1	0.3	0.2	0.34	2.2	1.66
		3					0.6	2.09	4.4	3.75
		4					0.8	3.3	**6**	4.4
Ziegler-Nichols		-					0.6667	2.47	4.2	4
System(2)	Proposed method	1	0.5	1	1	0.3	2	6.2	100	6.6
		0.5					1	2.8	21	5
		0.3					0.6	1.6	16	3.7
Ziegler- Nichols							4	See Figure 4(b)		
System(3) (small DC gain) Eq.(20)	Pro. method	1	3.1820	0.7071	0.05	0.5	8.89	-	28	3
		15					177.78	1.89	9.5	9
		33					293.3	5.3	17	9.3
							0.89	-	42	0.425

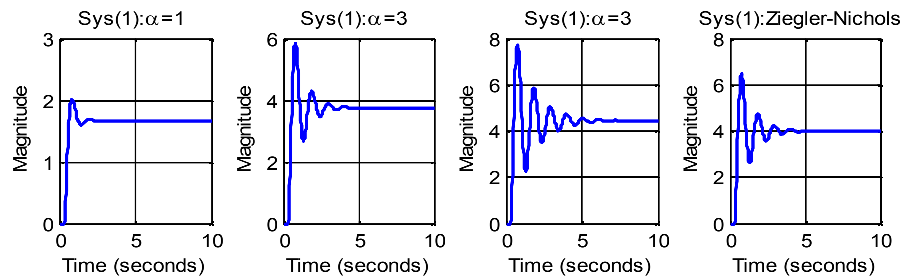


Figure 4(a) P-controller for system(1) applying proposed and Z-N methods

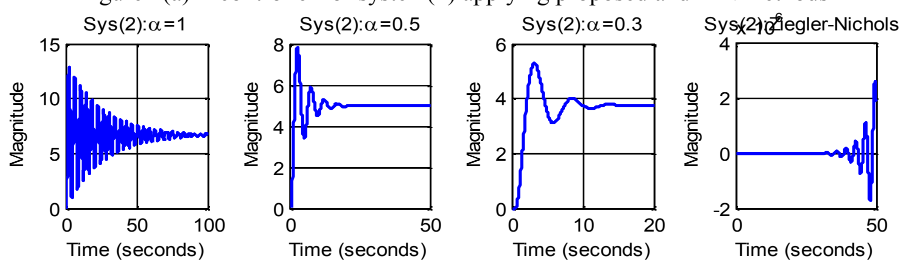


Figure 4(b) P-controller for system(2) applying proposed and Z-N methods

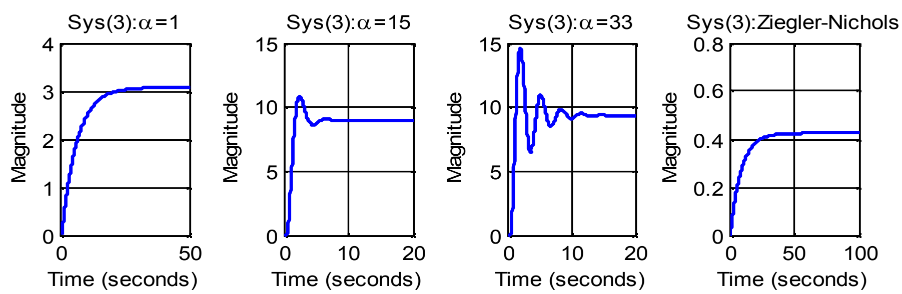


Figure 4(c) P-controller for system(3) applying proposed and Z-N methods

4. PD Controller for first order plus delay time (FOPDT) process

Based on FOPDT process's *L, T,* and *K*, expressions given by Eq.(21) are proposed to calculate PD-controller gains, a long with tuning parameter α. The expressions are applied first assigning parameter α=1, and then α can be tuned to soften resulted response. For processes with *small* DC gain *and/or small* time constant, to speed up response and to minimize time of selection and design process, the expression given by Eq.(22) can be applied directly.

By relating process's L to controller's derivative time T_D, more direct expression can be proposed and given by Eq.(22b), this expression can be applied for both types of systems. Testing the proposed expressions given by Eq.(21) for systems(a)(b)(c), will result in response curves shown in Figure 5, and data shown in Table 4, curves and date show that applying proposed expressions , a start point for designing PD controller can be achieved, then a tuning parameter can be adjusted to achieve suitable or desired response.

Testing the proposed expressions given by Eq.(22b) for systems(a)(b)(c), will result in response curves shown in Figure 5(d). Due to the high frequency gain of the derivative term, the closed-loop performance of processes with a large *L*/T ratio applying controllers with D-term may not give significant achievement (Rames C. Panda, et all, 2004).

$$K_P = 3.5\alpha \frac{T \cdot L}{K}$$
$$K_D = 0.9\frac{T \cdot L}{K} \qquad (21)$$
$$T_D = \frac{K_D}{K_P} = \frac{0.2571}{\alpha}$$

$$K_P = \alpha \frac{T \cdot L}{10K}$$
$$K_D = 0.2\frac{T \cdot L}{K} \qquad (22)$$
$$T_D = \frac{0.02}{\alpha}$$

$$K_P = \alpha \frac{T}{K \cdot L}$$
$$T_D = 0.5 \cdot L \qquad (22b)$$
$$K_D = K_P * T_D = 0.5\frac{T}{K}$$

Table 4: PD-Controller for systems(a)(b)(c) applying proposed method.

P-Controller	α	T	K	L	K_p	Mp	5T	DC gain
System(a) Eq.(21)	1	1	1	0.3	1.05	-	4.2	5.12
	5				5.25	6.4	6.7	8.4
	6				6.3	See Figure 4(a)		
System(b) Eq.(21)	1	10	5	0.5	3.5	0.34	4.4	9.46
	1.2				4.2	2.05	5	9.55
	2				7	See Figure 4(b)		
System(c) (small DC gain) Eq.(22)	1	0.2	0.005	0.9	9	-	44.4	3.1
	8				72	0.79	6.7	7.82
	15				135	4.55	12	8.71
	20				See Figure 4(c)			

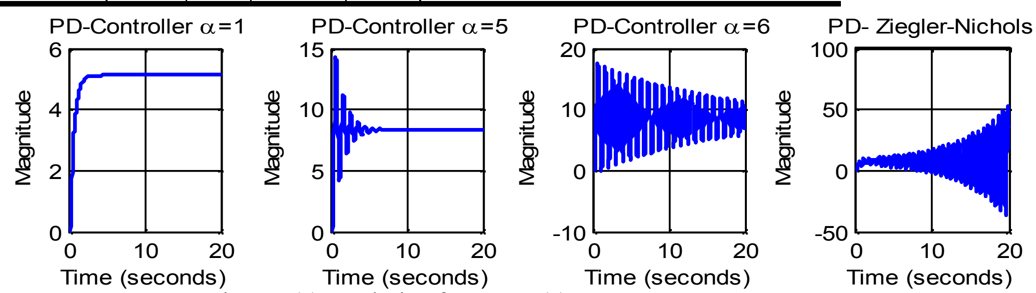


Figure 5(a) PD design for system(a)

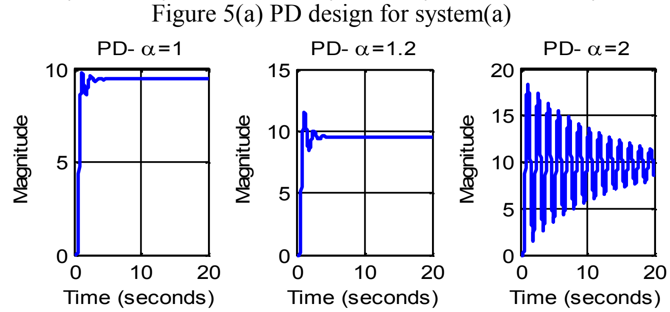


Figure 5(b) PD design for system(b)

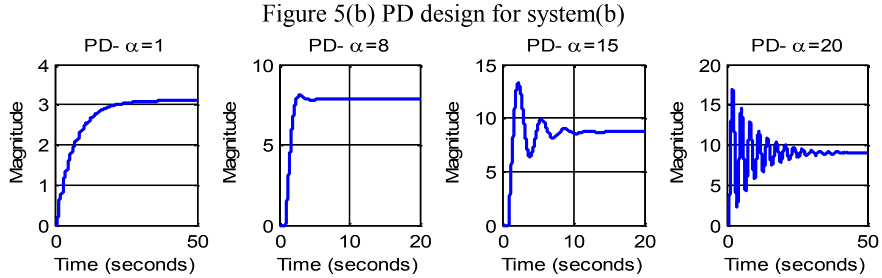


Figure 5(c) PD design for system(c)

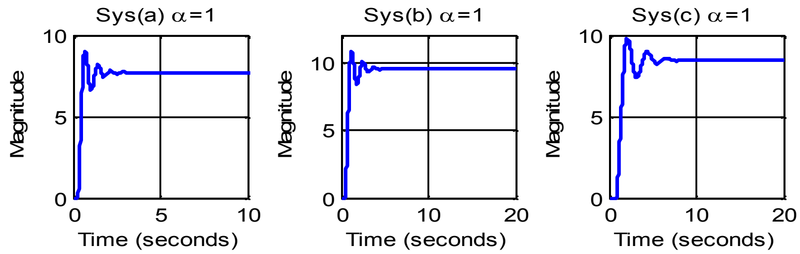


Figure 5(c) PD design for system(a)(b)(c) applying Eq.(22b)

4. PI-Controller for first order plus delay time (FOPDT) process

In order to design PI controllers for FOPDT process, various methods have been suggested during the past sixty years(Saeed Tavakoli, et all,2003). The second method of Ziegler and Nichols known as the "*process reaction curve" method(*J. G. Ziegler, et all, 1943*)* and that of Cohen and Coon (G. H. Cohen, et all, 1953) are the most prominent methods mentioned in most control textbooks. Similar to the Ziegler and Nichols methods, Cohen and Coon technique sometimes brings about oscillatory responses, because it was designed to provide closed loop responses with a damping ratio of 25% (M. Zlokarnik, 1991). However, Ziegler and Nichols methods are still widely used, either in their original form or with some modifications (Katsuhiko Ogata, 1997).

Expressions given by Eq.(23a) are proposed to calculate PI-controller gains, a long with tuning parameter α. The expressions are applied first assigning parameter α=1, and then parameter α can be tuned to soften resulted response. For processes with *small* DC gain *and/or small* time constant, the expression given by Eq.(24) can be applied directly.

By relating controller's integral time constant T_I to process's Time constant T, more soft expression can be proposed and given by Eq.(24b), these expressions can be applied for most types of FOPDT processes, where decreasing α, will reduce overshoot and slow response, meanwhile decreasing β, will increase overshoot and slow response, these expressions are applied first assigning parameters α= β =1, then if needed soft tuning may be required

Testing the proposed expressions given by Eq.(23a) for systems(a)(b)(c), and applying Ziegler Nichols tuning method, will result in response curves shown in Figure 6, and data shown in Table 5(a)(b)(c). The response curves and data show that, increasing α will reduce error and may increase overshoot, an acceptable response can be achieved applying proposed expressions.

Testing the proposed expressions given by Eq.(24b) for systems(a)(b)(c), will result in response curves shown in Figure 5(d)(e), analyzing these response curves show almost identical response curve with 50% overshoot. Reducing α (less than unity) ,will reduce overshoot up to removing it, based on this, and analysis of testing similar systems, Eq.(24c) are proposed for PI controller design.

$$K_P = \alpha \cdot \frac{1.5K}{T \cdot L}$$
$$K_I = \frac{1.5L}{T} \qquad (23a)$$
$$T_I = \frac{K_P}{K_I}$$

$$K_P = \alpha \cdot \frac{15K}{T \cdot L}$$
$$K_I = \beta \frac{K_P \cdot K}{L} \qquad (24)$$
$$T_I = \frac{K_P}{K_I}$$

$$K_P = \alpha \frac{T}{K \cdot L}$$
$$T_I = \beta T \qquad (24b)$$
$$K_I = K_P / T_I$$

$$K_P = 0.3 \frac{T}{K \cdot L}$$
$$T_I = T \qquad (24c)$$
$$K_I = K_P / T_I$$

Table 5 PI-Controller for systems(a)(b)(c) applying proposed and Z-N tuning method.

P-Controller		α	β	T	K	L	K_p	K_I	T_I	Mp	5T	DC gain
System(1)	Proposed method Eq.(24)	1	-	1	1	0.3	5	0.45	11.11	5.8	44	10
		0.1	-				0.5	0.45	1.11	-	11	10
		2	-				10	0.45	22.22	**4**	**4**	**10**
Ziegler-Nichols		-					3	3	1	1.47	16	10
System(2)	Proposed method	1	-	10	5	0.5	1.5	0.075	20	-	38	9.9
		1.5					2.25	0.075	30	0.4	45	9.9
		3	-				4.5	0.075	60	5.72	13.5	9.84
Ziegler- Nichols		-					18.8	1.667	See Figure 6(b)			
System(3)	Pro.	1	1	5	0.05	0.9	1.67	0.463	3.6	-	200	10

(*small DC*) Eq.(24)	method	1	3				1.67	1.389	1.2	-	71	10
		1	10				1.67	4.63	0.36	2.48	58	10
Ziegler- Nichols		-	-				5	3	1.67	0.7	40	10

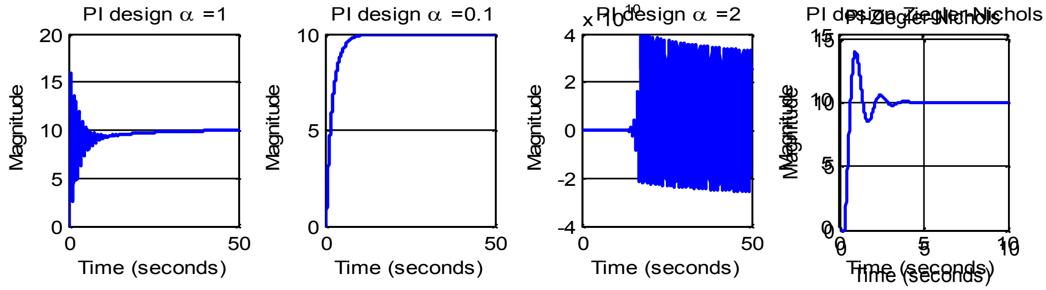


Figure 6(a) PI-Controller design for system(a) .

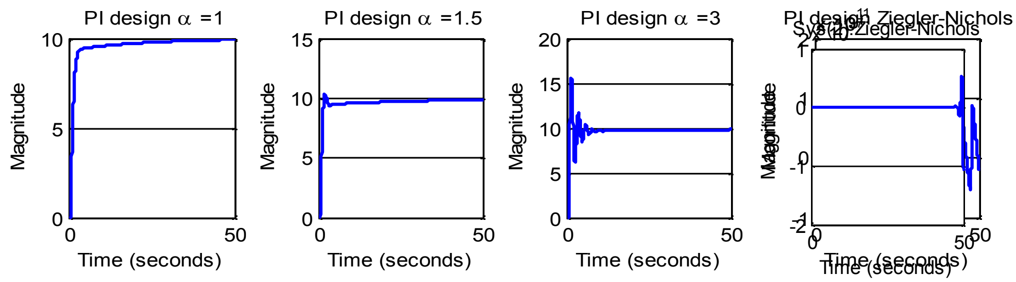


Figure 6(b) PI-Controller design for system(b)

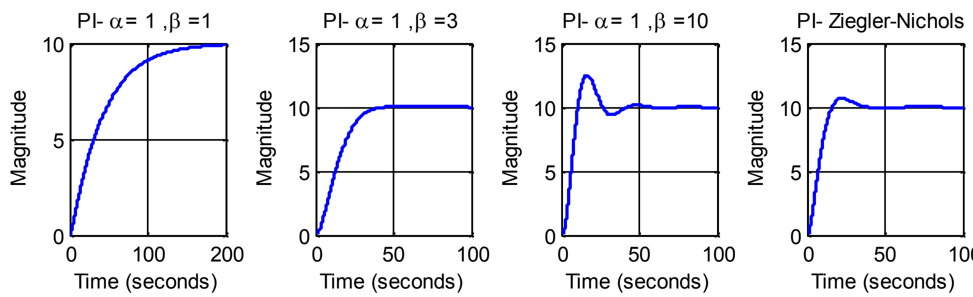


Figure 6(c) PI-Controller design for system(c)

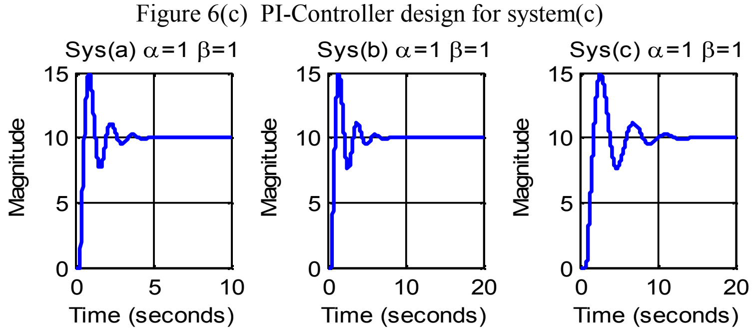


Figure 6(c) PI-Controller design for system(a)(b)(c) applying Eq.(24b)

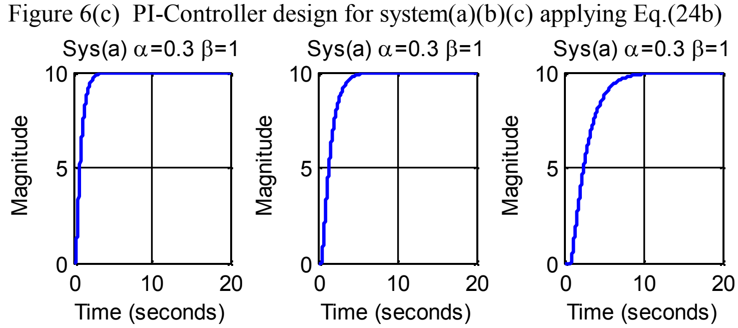


Figure 6(d) PI- design for system(a)(b)(c) applying Eq.(24b), with α=0.3 and β=1

4. PI-Controller for second order plus delay time (SOPDT) process

Expressions given by Eq.(25) are proposed to calculate PI-controller gains, a long with tuning parameter α. The expressions are applied first assigning parameter α=1, and then parameter α can be tuned (increased or decreased) to soften resulted response. For processes with small DC gain and/or small time constant, α can be assigned a small decimal value less than one (e.g. 0.1, 0.01), with limits for increasing α, since it may lead to instability.
Applying similar approach as with PI for SOPDT processes, by relating controller's integral time constant T_I to process's Time constant T, more simple expressions given by Eq.(25b) considering the effect of processes damping ratio ζ and relating it to, gives Eq. (25c), these expressions can be applied for most types of FOPDT processes,
Testing the proposed expressions given by Eq.(25),for systems(1)(2)(3), and applying Ziegler Nichols tuning method, with will result in response curves shown in Figure 7, and data shown in Table 6. Analyzing responses and data, shows that assigning α small values (e.g. 0.1:3), can result in more acceptable response, meanwhile increasing α may lead in instability.

Testing the proposed expressions given by Eq.(25b) for systems(a)(b)(c), will result in response curves shown in Figure 7(d), and testing the proposed expressions given by Eq.(25c) for systems(a)(b)(c), will result in response curves shown in Figure 7(e), analyzing and comparing these response curves show that an acceptable design compromise and starting point can be achieved applying proposed expressions given by Eq.(25c).

$$K_I = \alpha \frac{(\xi + \omega_n)}{(\xi \cdot \omega_n)} \qquad K_P = \alpha \frac{0.01}{K_I \cdot \xi \cdot T \cdot L} \tag{25}$$

$$K_P = 0.3 \frac{T}{K \cdot L} \qquad K_I = T \tag{25b}$$

$$K_P = 0.3 \frac{T\,\xi}{K \cdot L} \qquad K_I = T \tag{25b}$$

Table 6: PI-Controller for systems(1)(2)(3) applying proposed and Z-N tuning method.

P-Controller		α	ζ	ωn	K	L	Kp	KI	Mp	5T	DC gain
System(1)	Proposed method	1	1.0607	7.0711	50	0.3	0.1515	1.56	1.64	4.5	10
		1.5					0.1515	2.3335	5.3	8.9	10
		2					0.6	1	See Figure 6(a)		
Ziegler-Nichols		-					0.6	.6	-	11	1-
System(2)	Proposed method	1	3.182	0.7071	0.05	0.5	0.0082	1.7285	3.64	8.5	10
		1.5					0.0082	2.5927	7.7	19	10
		0.1					0.0082	1.3828	2	5.5	10
Ziegler- Nichols		-					0.6	.6	-	11	1-
System(3)	Pro. method	1	0.5	1	1	0.5	0.0067	3	9.6	57	10
		0.1					0.0067	0.3	-	20	10
		2					0.0067	6	See Figure 6(b)		
Ziegler- Nichols		-					5	1.67	2.8	8.5	10

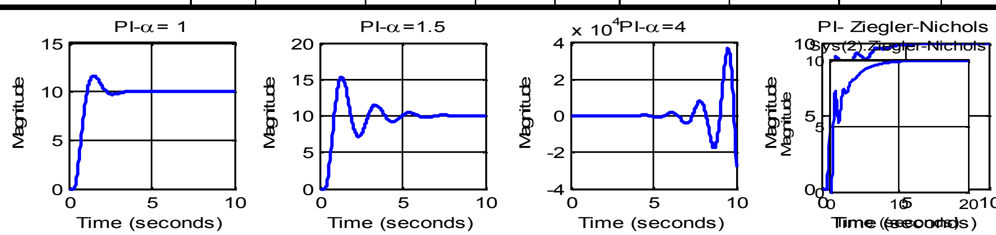


Figure 7(a) PI-Controller design for system(a)

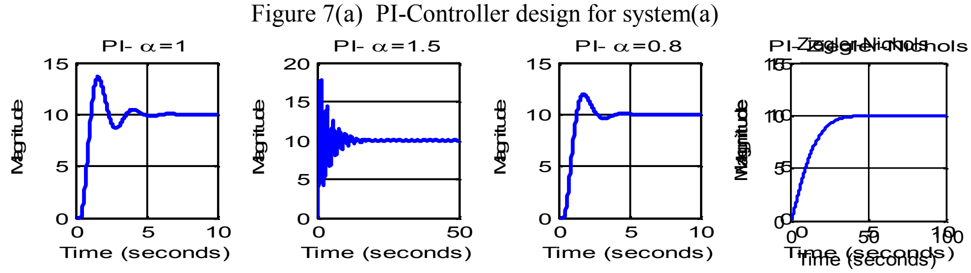


Figure 7(b) PI-Controller design for system(b)

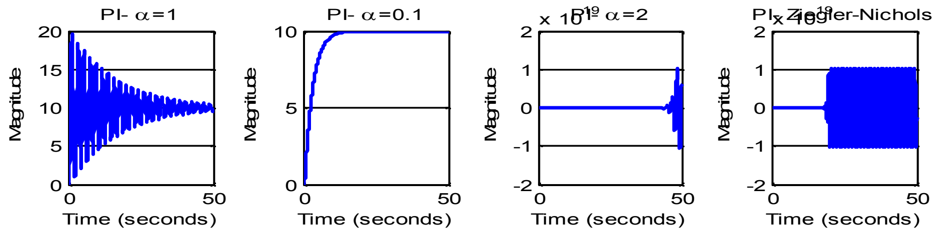


Figure 7(c) PI-Controller design for system(c)

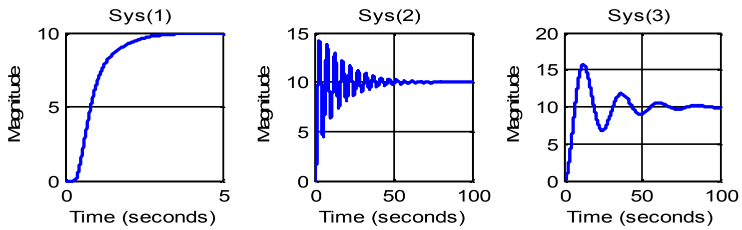


Figure 7(d) PI-Controller design for system(c) applying Eq.(25b)

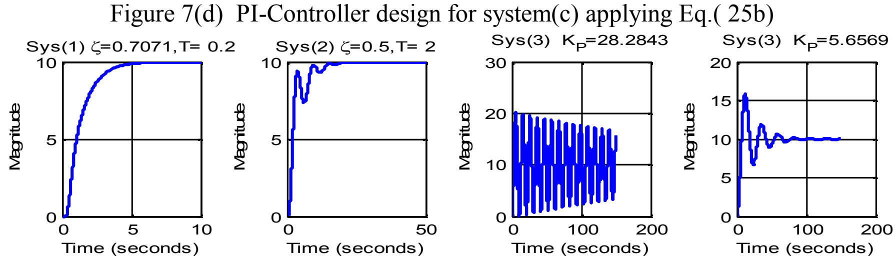


Figure 7(e) PI-Controller design for system(c) applying Eq.(25b)

4. PID Controller for first order plus delay time (FOPDT) process

Due to the high frequency gain of the derivative term, the closed-loop performance of processes with a large *L*/T ratio applying PID controllers may not give significant achievement over the same with PI (Rames C. Panda, et all, 2004). Based on process's parameters L, T, *K*. The expressions shown in Table 7(a), are proposed to calculate PID gains, along with tuning parameters (α, β). The expressions are applied first assigning parameter $\alpha=\beta=1$, and then parameters α and β can be tuned where, increasing β will speed up response, and increase overshoot, meanwhile, increasing α will increase overshoot and slow response, the divisor *N* is chosen in the range 2 to 20.

Testing proposed expressions given in Table 7(a) for systems(a)(b)(c), will result in response curves shown in Figure 6, the calculated gains and response measures are shown in table 8.

Table 7(a) Proposed expressions for PID design for first (FOPDT) system,

Plant	PID parameters			
	K_P	K_I	K_D	N
T,K,L	$K_P = 0.5\alpha TL$	$K_I = 0.5\beta LT$	$K_D = 0.01/L$	2 ÷ 20

Table 8 PID-Controller for systems(a)(b)(c) applying proposed and Z-N tuning method.

P-Controller		α	β	T	K	L	K_p	K_I	K_D	T_I	T_D	Mp	5T	DC gain
System(1)	Proposed method Eq.(24)	1	1	1	1	0.3	0.15	0.15	0.0333	1	0.2222	-	34	10
		1	5				0.15	0.75	0.0333	0.2	0.2222	1.4	15	10
		5	8				0.75	1.2	0.0333	0.625	0.0444	0.9	5.3	10
Ziegler-Nichols		-	-				4	6.67	0.6	0.6	0.15	5	6	10
System(2)	Proposed method	1	1	10	5	0.3	2.5	1.25	0.02	2	0.008	4.6	5	10
		2	0.5				5	0.63	0.02	8	0.004	6.3	13	10
		0.5	0.1				1.25	0.13	0.02	10	0.016	-	3.6	10
Ziegler- Nichols		-	-				24	24	6	1	0.25	See figure6(b)		
System(3) (*small DC*) Eq.(24)	Pro. method	1	1	5	0.05	0.9	2.25	0.02	0.0111	100	0.0049	2.6	75	10
		1	200				2.25	4.5	0.0111	0.5	0.0049	1.4	42	10
		1	150				2.25	2.25	0.0111	1	0.0049	1.4	50	9.8
Ziegler- Nichols		-	-				6.67	3.70	3	1.8	0.45	0.96	45	10

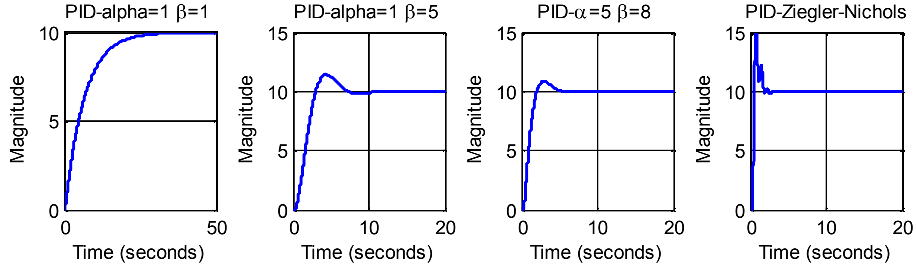


Figure 6(a) PID design for system(a) applying expressions in table 7(a)

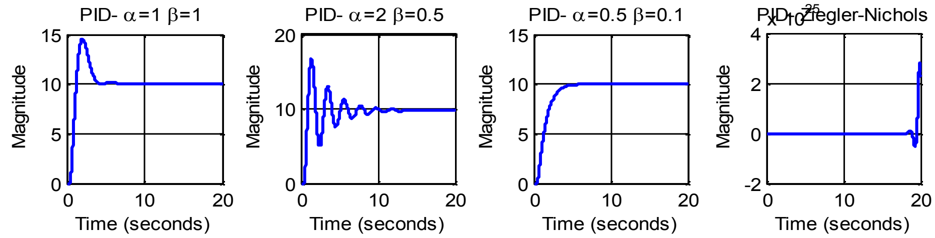


Figure 6(b) PID design for system (b) applying expressions in table 7(a)

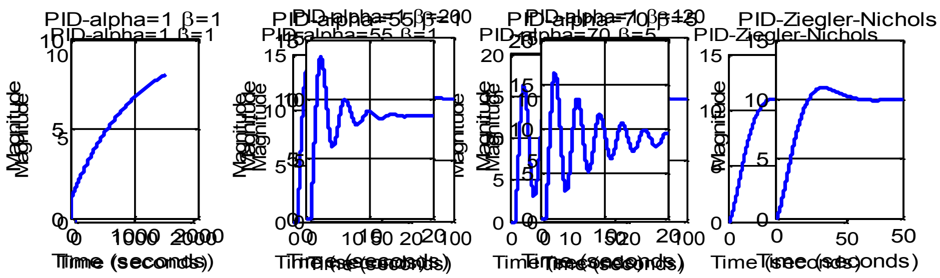


Figure 6(c) PID design for system (c) applying expressions in table 7(a)

4. PID Controller for Second order plus delay time (SOPDT) process

Chen and Seborg (Chen, D, et all, 2002) and Lee (Lee, Y, et all, 1998) have presented PID tuning rules for SOPDT systems. Huang *(*Huang, H.P, et all, 2000*)* presented inversed based design methods with a modified PID controller for different kinds of model structures.

The expressions shown in Table 9 are proposed to calculate PID-controller gains, along with tuning parameters (α and β). The expressions are applied first assigning parameter α=β=1, and then parameters can be tuned to soften resulted response. Where increasing alpha will reduce overshoot and speeding up response. Testing proposed expressions given in Table 9 for systems(1)(2)(3), will result in response curves shown in Figure 7, the calculated gains and response measures are shown in table 8.

Table 9(a) Proposed expressions for PID design for (SOPDT) system,

Plant	**PID parameters**			
	K_P	**K_I**	**K_D**	**N**
T,K,L	α	$\beta \frac{L\omega_n}{2\xi}$	$\varepsilon \frac{L}{2\xi\omega_n}$	2 ÷ 20
	For systems with small DC gain and/or small time constant			
	α	$K_I = \beta L\omega_n\xi$	$\varepsilon \frac{L}{2\xi\omega_n}$	

Table 10: PID-Controller for systems(1)(2)(3) applying proposed and Z-N tuning method.

P-Controller		**α**	**β**	**ε**	**ζ**	**ω_n**	**K**	**L**	**K_p**	**K_I**	**K_D**	**Mp**	**5T**	DC gain
System(1)	Proposed method	1	1	1	1.0607	7.071	1	0.3	1	1.5	0.03	3.37	12	9.8
		0.5	1	1					0.5	1.5	0.03	-	3	10
		0.1	0.1	0.1					0.1	0.15	0.003	-	30	10
Ziegler-Nichols		-	-	-					0.8	1.333	0.12	0.8	6.2	10
System(2)	Proposed	1	1	1					1	0.5	0.5	0.7	12	10

	method	1	1	2.5	0.5	1	1	0.5	1	0.5	1.25	0.03	11.5	10
		1.1	1	2.8					1.1	0.5	1.4	-	7	10
Ziegler- Nichols		-	-	-					4.8	4.8	1.2	See figure 7(b)		
System(3)	Pro. method	1	1	1	3.182	0.707	0.05	0.5	1	0.056	0.111	4.6	42	9.9
		250	1	1					300	1.125	0.11	3.3	11	9.35
		0.1	250	400	-				250	13.89	27.78	2.6		
Ziegler- Nichols			-	-						1.067	1.067	0.267	0.5	100

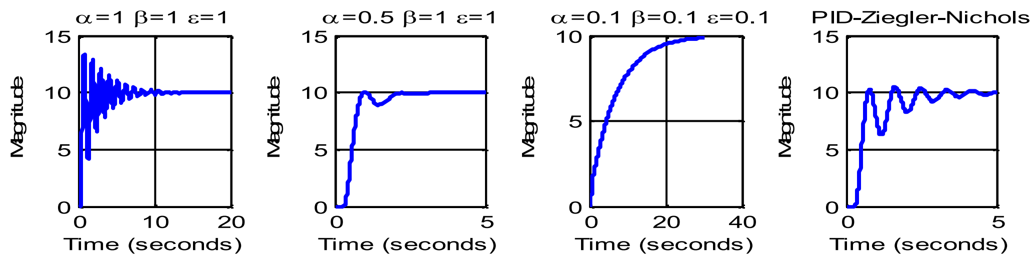


Figure 7(a) PID design for system (1) applying expressions in table 9

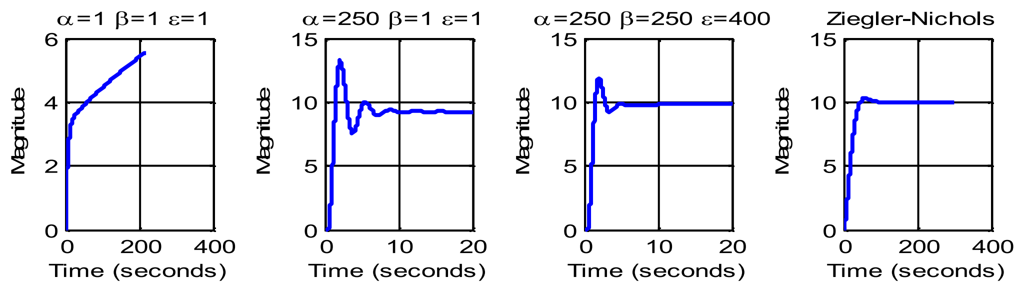


Figure 7(b) PID design for system (2) applying expressions in table 9

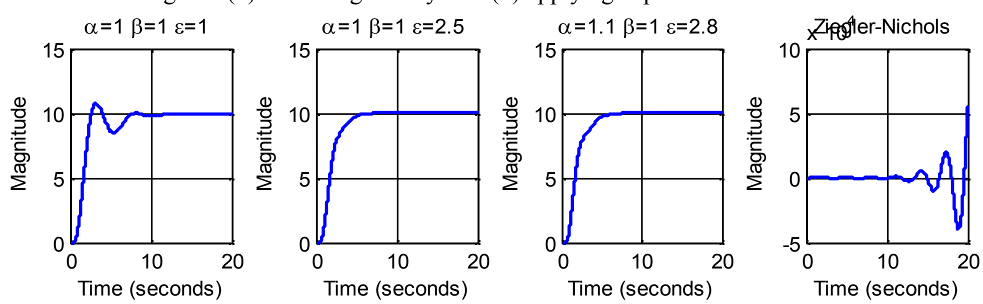


Figure 7(c) PID design for system (3) applying expressions in table 9

Conclusion

A new simple and efficient controllers design methods for FOPDT/SOPDT process, to achieve an important design compromise; acceptable stability, and medium fastness of response are proposed and tested, the proposed method is based on relating and selecting controllers' gains based on process's parameters. Expressions for calculating and soft tuning of controllers' gains, a long with soft tuning parameters are proposed, the proposed controllers design methods was tested using MATLAB/Simulink for different FOPDT/SOPDT process. The result obtained show applicability and simplicity of proposed design expressions

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