Modeling and Sensitivity Analysis on Cost, Volume and Profit in a Production Venture

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***Abstract*— This paper aims at modeling a single alternative production venture and a two alternative production venture using mathematical and analog models simultaneously. Sensitivity analysis was used to confirm the authenticity of the models by analyzing data collected from similar production ventures. The break even point for the production ventures were determined using both the mathematical and analog models, and this helped in taking decisions for a single alternative and two alternative production ventures which were considered in the study.**

***Index Terms*— modeling, break-even point, sensitivity analysis**

I. INTRODUCTION

A model is an idealized representation of a real life situation. Modeling helps us to examine the interrelationship existing between variables in real life situation. Mathematical models replace components or variables in the real-life system with symbols and the systems are generally related mathematically. Analog model establishes a relationship between variables in a system, examples are graphs. These two types of models are to be employed in this study to examine the interrelationship between cost, volume and profit.

The study of the interrelationship between cost, volume and profit at different levels of production process is called the cost-volume-profit analysis. The C-V-P is one of the cost planning and control techniques employed by management. By determining the level of activity at which the business makes neither profit not loss (the break-even point), management is able to determine what volume of output is required to meet a given profit target. It also helps management to take decision between two production alternatives as to which one is better.

Sensitivity analysis is the process where by one or more system input variables are changed and corresponding changes in the system output, or figure of merit, are observed if the decision is changed as the input is varied over a reasonable range of possible values, then the decision is said to be sensitive to that input; otherwise it is insensitive.

This paper examines Analog and mathematical model for single Alternative production and two alternatives production with reference to breaker-even and sensitivity analysis.

The single alternative model determines a minimum production volume (sales) where costs and income are equal (break-even point). While the tow alternative model determines which one of two (or more) production alternatives is better than the other.

II. LITERATURE

Iynama and Osuagwu (1999) applied mathematical and analog models, to model the rate of an epidemic spread in a given community, they concluded that modeling is an instrument for dissecting and reducing a complex life situation to simple and easy to comprehend situation. Earl and James (2005) Examined break-Even Analysis which he saw as an aid to management in decision making process where alternative course of actions are present. Herbert (2010) in his paper "Break-even analysis: A Re-evaluation proposed a modified approach to analysis of C-V-P problems. He presented a mathematical model with which he obtained a break-even point. Hopeman (2007) critically examined in his paper how sensitivity analysis and break-even point Analysis are two concepts that can not be easily separated for a better understanding in modeling an economic situation.

III. METHODOLOGY

Single alternative production ventures.

Analog model for single alternative production venture: This model determines a minimum production volume (sales) where costs and income are equal.

Let R=Total Revenue line, C_T=Total cost line, C_V=Variable cost line and C_F=Fixed cost line.

Procedure

- ❖ We plot on the Y-axis production volume (sales) and cost elements in Naira, and on the X-axis we scale off output level in output units.
- ❖ We plot the fixed cost line, which is fixed for various activity levels.
- ❖ We now draw the variable cost line which relates volume of output to cost of production. This line will cut the Y-axis at the value of the fixed cost.
- ❖ We derive the sales revenue (income) line, which relates sales (production volume) in units of output and sales in monetary outlay.

The procedures listed above are illustrated below

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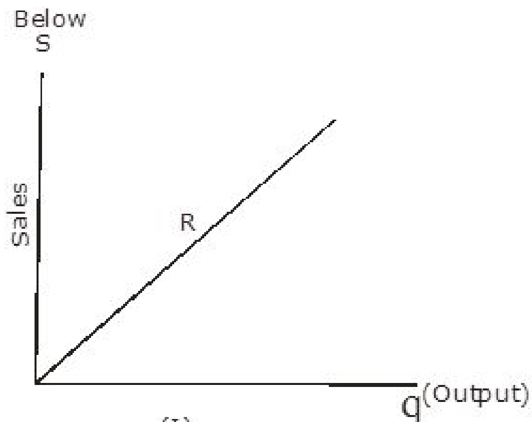
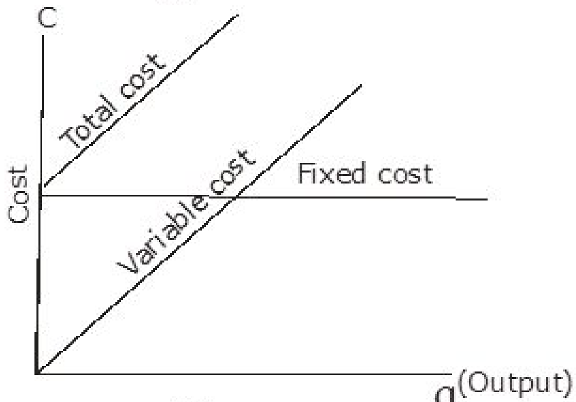


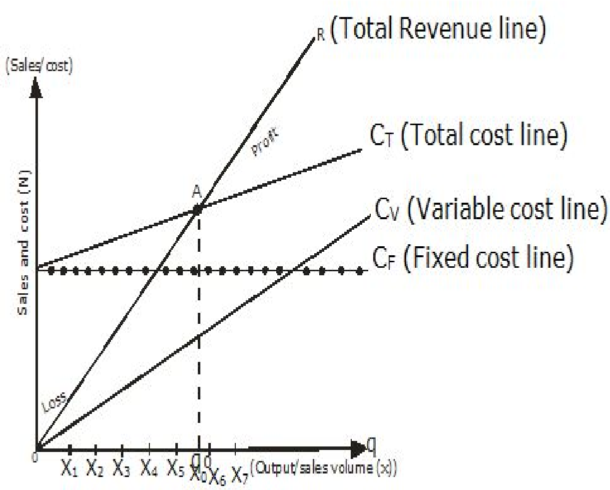
(I)



(II)

If we combine (I) and (II) we have the Analog model as presented below

(Figure1) The Analog Model



q_0 = output/sales unit at break-even point (x_0).

The four major lines required on the analog model are: Revenue (income) line (R), fixed cost line (C_F), variable cost line (C_V) and total cost line (C_T).

The Analog model in figures I shows that the firms total costs and total revenue (income) are equal at point A, this point is known as the break-even point. Therefore a net profit is indicated at any Revenue volume beyond point A, on the Total revenue line. At point A, the firms total cost equals its total revenue. In such case, profit is zero (no gain and no loss).

In a simple alternative production venture an analog model as presented above will guide management to consider the desirability of a venture by knowing its costs and basing its decision on predicted production volume (sales), then such as if predicted production volume (sales) exceed the break-even point (point A) in terms of volume, then a profit from the venture can be anticipated.

Mathematical model for single Alternative Production Venture:

Definition of variables.

Let F_C = Fixed cost, a constant
TVC = Total variable cost,(unit variable cost) x (number of units)
V = Unit variable cost
TC = Fixed cost + TVC
X = Production volume in units
R = Total Revenue (income)
S = Selling price per unit

Thus at break-even point,
Total Revenue = Fixed Cost + Total variable cost or $R = F_C + TVC$, Thuesen and Frabrycky (2012)
But TVC = (Unit cost). (production volume) = V.X
and Revenue (R) = (sales price). (production volume) or R = S.X
Therefore S.X = Fc + V.X=Revenue or Income

$$X = \frac{FC}{S-V}\ ;\ \text{(The breakeven point)}$$

One can readily see the logic in this equation by a moment of thought. That part of the equation S-V is in reality unit profit when no consideration is given to fixed cost. But, obviously, fixed cost must be absorbed or allocated over the production volume since S-V is the net profit after considering variable unit cost, this net profit divided into total fixed cost will yield the break-even point in terms of units sold or production volume.

IV. Sensitivity/Data Analysis

The data below is for the production of fish piles and meat piles from a fast foods production firm.

Selling price Per unit	**Variable cost Per unit**	**Fixed Cost**
N300	N150.00	N300,000

Table 1 Source: Mr Biggs Aba

Sensitivity analysis on the mathematical model:
Contribution (unit profit) = (selling price–Variable cost)
ie S-V = N300- ₦150 = ₦150
Quantity at break-even point (X) $= \frac{FC}{S-C}$

$$X = \frac{300,000}{150} = 2000$$

Therefore quantity at break-even point = 2000 units
Sensitivity analysis on the analog Model

Table 2: Table of values

Output units (x)	Fixed '₦' cost (FC)	Variable cost V '₦'	Total cost TC ₦	Sales '₦'	Profit '₦'
0	300,000	-	300,000	-	-300,000
1000	300,000	150,000	450,000	300,000	-150,000
2000	300,000	300,000	600,000	600,000	0
3000	300,000	450,000	750,000	900,000	150,000
4000	300,000	600,000	900,000	1200,000	3000,000
5000	300,000	750,000	1050,000	1500,000	450,000

We note that at the output/quality unit (X) = 2000, profit = 0 (Break-even point), and at this point total cost (TC) = sales = 600,000

Sales here stand for income (Revenue)

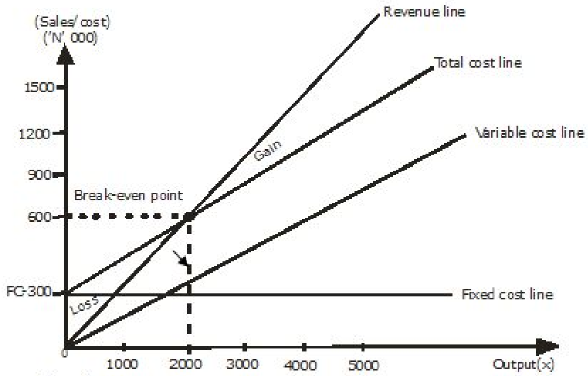


Figure 2

From the above, at Break-even point (BEP) output (x) = 2000 units, and total cost = ₦ 600,000

This conform the results from the mathematical model.

Two or more alternative production Venture

The previous discussion is related to a single venture and information presented was an aid in deciding whether or not to undertake that venture. In this case the decision to be made is not whether to attempt the venture but rather which one of the two (or more) alternatives is better. Consider a situation where a work area to be lighted can be done satisfactorily by either one of two methods. Using Lamp A, the initial cost is low but the operating costs are relatively high. For Lamp B, the initial cost is high but substantially has a lower operating cost. In addition, the number of hours of operation must be considered since this result in a variable cost common to both alternatives. This can be illustrated in analog model as follows.

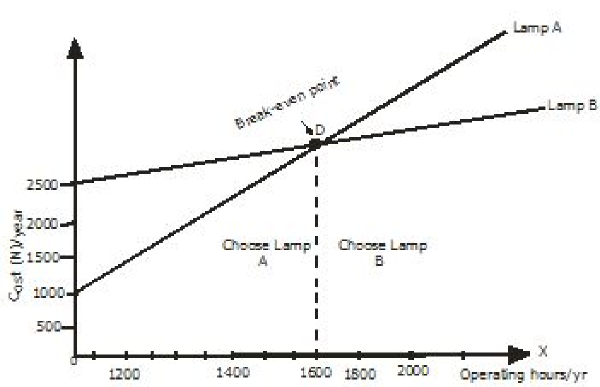


Figure 3

The analog model indicates that at point D costs are equal for the two ventures and that if the system is to operate X =1600 hours per year it is a matter of indifference which lamp is used. But for operating hours less than X =1600 hours, lamp A venture is preferable because of lower cost of production,

and for more than X= 1600 hours, lamp B venture is the better alternative. We note again that at point D, lamb A and lamp B have equal costs, and equal operating hours 1600hours and

non is preferable more than the other, but going towards the right from the point D, lamp B is considered better, while

towards the left from point D lamp A is considered, and all objectives for consideration based on lower cost of production.

Mathematical model for two Alternatives production ventures.

A mathematical model can be developed, based on the fact that at break-even point D, costs are equal. This knowledge permits us to state that at this point:

Cost A = cost B

Now let C_A = Total Annual cost of using Lamp A.

R_A = Annual cost of using Lamp A (maintenance, depreciation)

O_A = Hourly operating cost for Lamp A

C_B = Total Annual Cost of using Lamp B

R_B = Annual cost using Lamp B (maintenance, depreciation)

O_B = Hourly operating cost for Lamp B

X = Annual hours of operating the system

Thus $C_A = C_B$

But $C_A = R_A + O_AX$

And $C_B = R_B + O_BX$

Therefore, $R_A + O_AX = R_B + O_BX$

Solving for X, results in $X = \frac{R_B - R_A}{O_A - O_B}$

Where X represents the annual hours of operating the two systems (Lamp A or Lamp B) (ie the Break-even point)

Sensitivity/Data Analysis

Sensitivity analysis on the mathematical model:

The application of a theory of procedure to a real problem is helpful in visualizing other problems that can be solved in a similar manner.

The problem considered here is that of deciding between two alternative methods of production.
The data below shows the cost breakdown of producing two different types of Plastic Chairs, Armed Plastic Chairs and Not Armed Plastic Chairs.
Table 3: Data on production cost of Armed and Not Armed Plastic Chairs.

a. **Annual cost of production breakdown (Armed chairs) per Annum:**

Depreciation cost	=	N3082.00
Maintenance cost	=	N5485.00
Variable cost	=	N0.1136 per Armed chair
Fixed cost	=	N982.00

b. **Annual cost of production breakdown (Armed chairs) per Annum:**

Depreciation cost	=	N2,017.00
Maintenance cost	=	N7,921.00
Variable cost	=	N0.0081 per Armed chair
Fixed cost	=	N358.00

Source: Stanpole plastics company

Derivation of the mathematical model

ARMED CHAIRS (A):

Let: A = Total cost of production
D_a = Depreciation cost
M_a = Maintenance cost
V_a = Variable cost
FC_a = Fixed cost
N = Number of chairs produced per year

Then $A = FC_a+D_a+M_a+V_aN$ = Total cost of production (Armed chairs)

NOT ARMED CHAIRS (B):

Let: B = Total cost of production
D_b = Depreciation cost
M_b = Maintenance cost
V_b = Variable cost
FC_b = Fixed cost
N = Number of chairs produced per year

Then $B= FC_b+D_b+M_b+V_bN$ = Total cost of production (Not Armed chairs)

If all values except N are known, there is a value for N, the common variable, for which the costs of the armed chairs and the Not armed chairs are equal. This value of N may be found as follows:

Let: A = B

Then $FC_a+D_a+M_a+V_aN = FC_b+D_b+M_b+V_bN$

$$N = \frac{FC_b-FC_a+D_a-D_a+M_a-M_a}{V_a-V_b}$$

Substituting from Table 3:

$$N= \frac{358-982+2017-3082+7921-5485}{0.1136-.0081}$$

$$= 229{,}000 \text{ chairs per year or } 628 \text{ chairs per day.}$$

Explanation

Thus, at a daily production of 628 chairs or 229,000 chairs per year, the costs of production are equal for producing either the armed chairs or the Not-armed chairs. For daily production volumes greater than 628 chairs the Not-armed chairs would be more economical to produce than the equivalent armed chairs. Verification of this statement can be made by substituting into the mathematical model or by using analog model method as previously explained. Although this condition would seem paradoxical, the reason for it can be seen by comparing variable costs for each production alternative. Also, as volumes increase other cost factors could arise and affect the final decision, while the mathematical models tend to appear somewhat complex, but in actuality it is quite easy to use and greatly simplify this important activity of determining the preferable production alternative.

CONCLUSION

This paper has succeeded in combining mathematics and analog modeling to examine the interrelationship between cost, profit and Revenue in a production Venture. The Analog and mathematical models showcased the break-even point and how it could simply be used to determine minimum production sales (volume) where costs (expenditure) and income (revenue) are equal. The mathematical and analog models were also applied to establish decisions when two alternative course of action are being considered in production.
This paper has been able to x-ray the relationship between break-even point and sensitivity analysis as inseparable and indispensable tools in management decision making.
Although, the mathematical models developed appear to be somewhat complex, but the symbols become familiar through frequent usage.
The models are in actuality quite easy to use and greatly simplify the important activity of determining preferable alternatives in a decision making situation involving production ventures.

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