

# Design of a Delta-Sigma Modulator Structured in MASH 2-2-1 with Higher Order Shaped Dither

Seyed Ali Sadat Noori , Seyed Saeed Sadat Noorieh

**Abstract**— This paper presents a novel Delta Sigma Modulator (DSM) in form of MASH 2-2-1, which can suppress the spurious tones in spectrum for all constant inputs and initial conditions. We can increase the order of noise shaping of the applied dither to that of the modulator without any spurious tones appearing in the spectrum. Theoretical results prove that the quantization noise is asymptotically white and uncorrelated with the input; this is corroborated by behavioral simulations.

**Index Terms**— digital delta sigma modulator (DDSM), dither, fractional division, multistage noise shaping (MASH)

## I. INTRODUCTION

Frequency synthesizers based on fractional PLL (phase locked loop) are widely used in the field of Radar, wireless communication systems, and electronic measurements. To suppress the fractional spurious tones [1], the fractional-N frequency synthesizer DSM based was usually proposed. High-order DDSMs can be implemented using single-loop or MASH architectures. Single loop architectures allow for a large degree of design flexibility, but care must be taken to ensure stability of the loop. MASH DDSMs, on the other hand, are unconditionally stable and offer full coverage of the input fractional range. As a finite-state machine, featuring in a non-linear system, the DSM output is a periodic sequence and the spectrum of output is discrete while a constant input injected. As for a fractional PLL application, the output spectrum of VCO will be remarkably discrete in the vicinity of the desired carrier frequency [2]. Two classes of techniques have been developed to whiten the quantization noise: stochastic and deterministic. Stochastic techniques include the use of LSB dithering [3]-[5] and time varying noise transfer functions. Their goal is to make the quantization noise asymptotically white and independent of the modulator's input, thereby eliminating spurious tones in the output spectrum. Deterministic techniques include the setting of predefined initial conditions [9], using prime modulus quantizers [8] and architectural modification [6], [7]. The goal of these methods is to maximize the cycle length of the quantization error signal, thereby causing the quantization power per tone to be minimized. In one realization of the stochastic approach, noise-shaped dither is added to the input as shown in Fig. 2 to break up periodic

cycles and thereby to increase the effective cycle length. Although this approach results in smooth noise-shaped spectra, the noise floor is also raised. To compensate for this effect, the order of the noise shaping filter can be increased to lower the noise floor, but a spur-free spectrum cannot be guaranteed if the filter order is higher than 1, where  $1 < R-2$  and R is the order of the DDSM [4]. In this brief, we describe a MASH DDSM architecture that remains spur-free even when higher order filtering is used and build upon previous work by [10] providing a rigorous theoretical proof verifying the performance of the architecture.

This work is organized as follows. Section 2 describes the conventional MASH 1-1-1 DDSM architecture and summarizes the problem of spurious tones. Section 3 describes our modified MASH DDSM architecture. Finally, in Section 4, the conclusions of this work are given.

## II. CONVENTIONAL MASH 1-1-1 DDSM ARCHITECTURE

Before we explore our new design, we first review a conventional MASH 1-1-1 DDSM architecture comprising Error Feedback Modulators (EFMs). The EFM is realized as a digital accumulator, the model of which is shown in Figure. 1. The input to the modulator is a digital word with N bits. When  $v[n]$  is greater than M ( $2^N$ ), the quantizer overflows and the output signal  $y[n]$  will be 1. On the other hand, when  $v[n]$  is less than M, the quantizer does not overflow and  $y[n]$  will be 0.

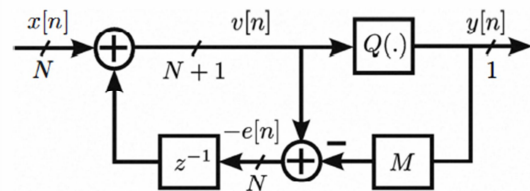


Figure 1. Block diagram of a first-order error feedback modulator (EFM1).

Mathematically, we write:

$$y[n] = \begin{cases} 0 & v[n] < M \\ 1 & v[n] \geq M \end{cases} \quad (1)$$

Figure.2 shows a block diagram of an 3th order MASH DDSM comprising a cascade of N-bit EFM1 blocks and a noise cancellation network. In this structure, the negative of the quantization error from each stage ( $-e_i[n]$ ) is fed to the next stage and the output of each stage ( $y_i[n]$ ) is fed to the noise cancellation network, which eliminates the intermediate quantization noise terms [12]. The output of the 3th order MASH DDSM can be expressed in the Z-domain as:

$$Y(z) = \frac{1}{M} \cdot X(z) - \frac{1}{M} (1 - z^{-1})^3 \cdot E_3(z) \quad (2)$$

where  $E_3$  is the Z-transform of the quantization error  $e_3$  in the third stage.

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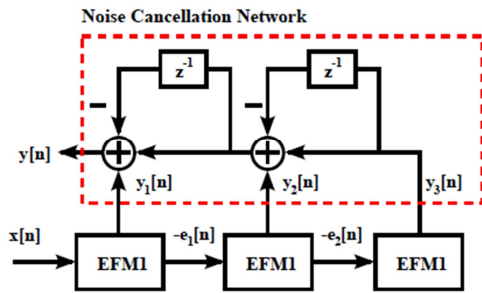


Figure 2. Block diagram of a MASH 1-1-1 DDSM

Figure.3 highlights the fundamental problem associated with a conventional MASH 1-1-1 DDSM. Because the input is constant and the modulator is a deterministic Finite State Machine (FSM), the quantization error is periodic; for this input value, it produces a short cycle. Applying a constant input to the model in Figure. 2 and plotting the Power Spectral Density (PSD) gives an insight into the spectral behavior of the model. The idealized PSD of the quantization error is  $\frac{1}{12}$ . From this, the smooth curve for shaped white quantization noise, shown in Figure. 3, is given by

$$P(f) = \frac{1}{12} \cdot |2\sin(\frac{\pi f}{f_s})|^6 \tag{3}$$

Where  $P(f)$  is the PSD of the shaped quantization noise for the MASH 1-1-1 in this case.

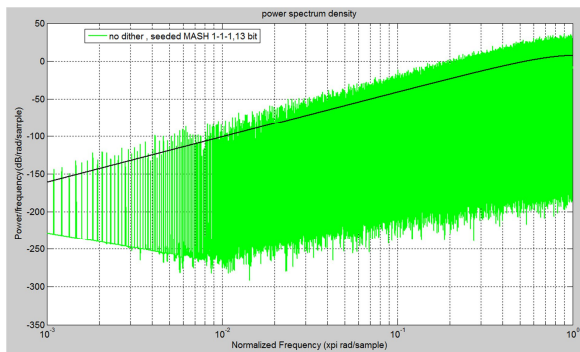


Figure 3. Output spectra of a 13-bit MASH 1-1-1 DDSM for a constant input X = 13. The solid curve represents the ideal shaped white quantization noise.

### III. PROPOSED MASH DDSM ARCHITECTURE

Figure 4 shows our proposed second order EFM structure. Compared with EFM1 shown in Figure. 2, the delay in the feedback loop has been increased from one cycle to two. In same way, the output,  $Y(z)$  can be written as:

$$Y(z) = \frac{1}{M} \cdot X(z) - \frac{1}{M} (1 - z^{-2}) \cdot E(z) \tag{4}$$

The function of quantization noise has been changed as  $\frac{1}{M} (1 - z^{-2})$ . Combining two structures, EFM1 and EFM2, we can construct a higher order MASH DSM 2-2-1, illustrated in Figure. 5. EFM stages due to an increased delay in the EFM2 feedback loop, the filter in the noise cancellation network has to be modified accordingly to remove the effects of  $e_1$  and  $e_2$ .

From Figure. 5, we can obtain

$$Y_1(z) = \frac{1}{M} \cdot X(z) - \frac{1}{M} (1 - z^{-2}) E_1(z) \tag{5}$$

$$Y_2(z) = \frac{1}{M} \cdot E_1(z) - \frac{1}{M} (1 - z^{-2}) E_2(z) \tag{6}$$

$$Y_3(z) = \frac{1}{M} \cdot E_2(z) - \frac{1}{M} (1 - z^{-1}) E_3(z) \tag{7}$$

From Figure. 5, we can conclude

$$Y(z) = Y_1(z) + (1 - z^{-2}) \cdot Y_2(z) + (1 - z^{-2})^2 \cdot Y_3(z) \tag{8}$$

Replacing with (3), we can obtain:

$$Y(z) = \frac{1}{M} \cdot X(z) - \frac{1}{M} (1 - z^{-2})^2 (1 - z^{-1}) E_3(z) \tag{9}$$

With the addition of filtered dither, the transfer function becomes as shown in Figure.6:

$$Y(z) = \frac{1}{M} \cdot X(z) - \frac{1}{M} (1 - z^{-2})^2 (1 - z^{-1}) E_3(z) + V(z) \cdot D(z) \tag{10}$$

Note that the spectral characteristics of the proposed MASH structure shown in Figure.7 are similar to those of the conventional MASH structure for (i) non-shaped and (ii) first order shaped dither. However, for second order shaped dither, the output spectrum (iii) is smoother. In this case, the solid lower curve in Figure.7 in the case of additive second order shaped dither is defined by:

$$P(f) = \frac{1}{12} \cdot \frac{1}{M^2} \cdot |2\sin(\frac{\pi f}{f_s})|^4 + \frac{1}{12} \cdot |2\sin(\frac{2\pi f}{f_s})|^4 \cdot |2\sin(\frac{\pi f}{f_s})|^2 \tag{11}$$

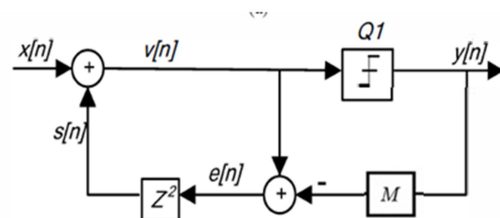


Figure 4. Block diagram of a second-order error feedback modulator (EFM2).

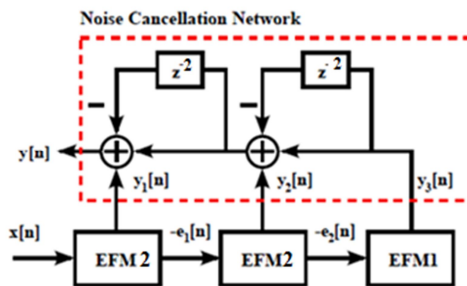


Figure 5. Block diagram of a MASH 2-2-1 DDSM

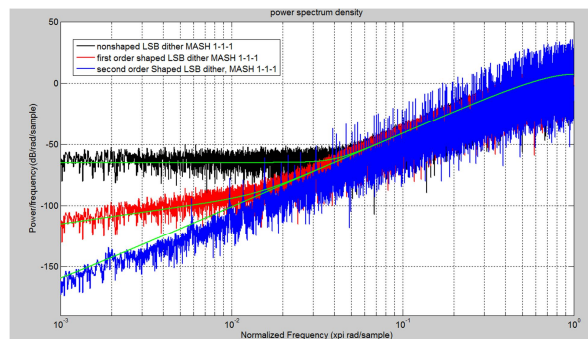


Figure.6: Output spectra of a MASH 111 DDSM for three cases with constant input X = 11: (i) non-shaped dither added, (ii) first order shaped dither added, (iii) second order shaped dither added.

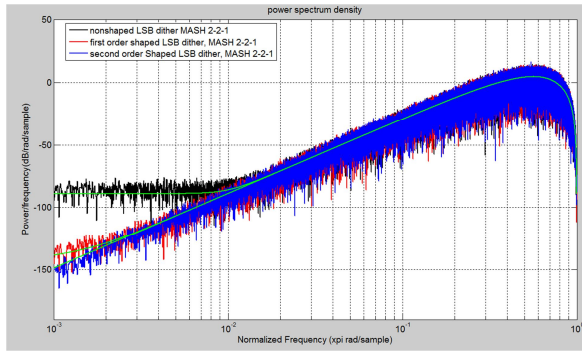


Figure.7: Output spectra of a MASH 221DDSM for three cases with constant input  $X = 11$ : (i) non-shaped dither added, (ii) first order shaped dither added, (iii) second order shaped dither added.

#### CONCLUSION

A new type of DSM was proposed by combining two commonly used methods to suppress spurious tones. The new structure can push spurious tones to higher frequencies effectively with a low level noise floor and can reduce the hardware costs and difficulty. Additive LSB dither can be used to break up cycles in an effort to eliminate the tonal behavior, but this raises the noise floor. An additive LSB dither signal can be noise shaped but the resulting correlations cause tonal behavior to reappear if  $l > R-2$  [7]. We have invented a new structure to overcome this problem; it incorporates modified second order EFM blocks as in the MASH architecture with a nonzero initial condition. We have shown that  $L$ th order filtering of the dither is achievable in the new architecture without producing spurs.

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