Bit-Rate controlled Image Compression based on Discrete Wavelet Transform

Abhirup Sinha, Rashmi pandey

Abstract— In this paper, an image compression technique is presented based on wavelet transform. The proposed technique has been compressing the image using wavelet transform based multiresolution analysis with Huffman encoding technique. It is tested over the different gray scale images and performance evaluated in term of compression ratio (CR), peak signal to noise ratio (PSNR) and mean square error (MSE). Here, compression obtained with different compression rate using iterates the bit-per-pixel. Overall, analysis has shown the effect of controlled bit rate over the quality of image for compression application.

Index Terms— Image Compression, Wavelet Transform, quantization, Thresholding.

I. INTRODUCTION

An image is essentially a 2-D signal that processed by the human visual system. The signals representing images are usually in analog form (1). However, for processing, storage and transmission by computer applications, they are converted from analog to digital form. A digital image is basically a 2-Dimensional array of pixels. An image is an array, or a matrix, of square pixels (picture elements) arranged in columns and rows. An image as defined in the “real world” is considered to be a function of two real variables, for example, a(x, y) with a as the amplitude (e.g. brightness) of the image at the real coordinate position (x, y). Digitization of the spatial coordinates (x, y) is called image sampling. Amplitude digitization is called gray-level quantization. Currently, images are very common multimedia content that shown the information very quickly, unfortunately its consuming lots of memory of devices as well as transmission system. Therefore, compression is essential for multimedia content to save memory as well as transmission bandwidth.

Basically, Image compression is minimizing the size in bytes of a graphics file without degrading the quality of the image to an unacceptable level for problem of reducing the amount of data required to represent a image. Image compression reduces the number of bits required to represent the image, therefore the amount of memory required to store the data set is reduced. It also reduces the amount of time required to transmit a data set over a communication link at a given rate [2-5].

Image Compression addresses the problem of reducing the amount of data required to represent the digital image. Compression is achieved by the removal of one or more of three basic data redundancies:
(1) Coding redundancy, which is present when less than optimal (i.e. the smallest length) code words are used;
(2) Inter pixel redundancy, which results from correlations between the pixels of an image, &/or
(3) psycho visual redundancy which is due to data that is ignored by the human visual system (i.e. visually nonessential information). Huffman codes contain the smallest possible number of code symbols (e.g., bits) per source symbol (e.g., grey level value) subject to the constraint that the source symbols are coded one at a time. So, Huffman coding when combined with technique of reducing the image redundancies using discrete wavelet Transform (DWT) helps in compressing the image data to a very good extent.

![Fig.1Basic block structure of compression of image](image_url)

The earlier techniques when are based on transformation and encoding techniques gave better performance in term of good compression and minimum mean square error for compressed data [2]. Bit rate reduction was further improved in block truncation coding. This technique uses one bit nonparametric quantize over local region of image [3,4]. In the earlier time analog signals are more common, here using sampling approach possible to convert analog to digital signal. A digital encoded data is suitable to further encoding schemes such as run length encoding (RLE) or entropy encoding [5]. Image compression basically started with quantization (scalar and vector), which has rejected spatial redundancy of digital image data [6, 7]. Further introduces several algorithms based on prediction encoding, block segmentation of image for data compression. Here, prediction encoding schemes adopt different method such as LMS algorithms, which are uses adaptive prediction filter for image source encoding [8]. There are spatial and optimal prediction compression schemes are used, when apply predictive compression on image data there is implicit assumption that the image is scanned in particular order of image blocks [9,10,11]. Where

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block size segmentation provide high quality variable-rate image is achieved by segmenting an image into different block size, this is perform as a lossless compression [12, 13, 14]. Recently in area of image data compression DCT and DWT based compression area more popular, both are gave good compression using different encoding schemes [15].

There are several techniques where are used in image compression such as: Karhuen-loeve Transform (KLT), Hadmard Transform (HT), Fourier Transform (FT), Discrete Linear Transform (DLT), Block Truncation Coding (BTC) algorithm, Arithmetic coding, Non-uniform sampling, Adaptive vector quantization, Least mean square (LMS) adaptive algorithm, Discrete Cosine Transform (DCT), variable block size segmentation, spatial prediction, optimal prediction, adaptive lifting, Discrete Wavelet Transform (DWT). In last two decade, there is compression standards are introduced such as: Joint Picture Experts Group (JPEG) and Joint Picture Experts Group-2000 (JPEG-2000), which are extremely used for image compression.

II. TRANSFORM TECHNIQUES

In the compression application, transform technique such as discrete cosine transform (DCT) and discrete wavelet transform (DWT) are used to change the pixels in the original image into frequency domain coefficients (called transform coefficients) as well as different frequency sub-bands. These coefficients have several desirable properties. One is the energy compaction property that results in most of the energy of the original data being concentrated in only a few of the significant transform coefficients. This is the basis of achieving the compression. Only those few significant coefficients are selected and the remaining is discarded. The selected coefficients are considered for further quantization and entropy encoding. DCT coding has been the most common approach to transform coding.

2.1 Overview of DCT

The discrete cosine transform (DCT) is the most recent know transform in the image compression field because of its excellent properties of energy compaction [1]. The process is as follows: first, the image is broken into 8x8 blocks. Second, DCT is applied on each block from the left to the right and from the top to the bottom. Then, quantization is applied for compression process, and data are stored following a specific process to reduce the information in the memory. And to reconstruct the compressed image we apply IDCT transform. DCT method is widely applied for the compression because it is insensitive energy of coefficient concentrated in a few transform coefficients helped us to improve the compression.

Discrete Cosine Transform (DCT) definition for a 1-D sequence \( b[n] \) of length \( N \) is

\[
B[m] = \left( \frac{2}{N} \right)^{\frac{1}{2}} \sum_{n=0}^{N-1} b[n] \cos \left( \frac{(2n+1)m\pi}{2N} \right) \quad (7)
\]

For \( M=0,1,\ldots,N-1 \). Similarly, the inverse transformation IDCT is defined as

\[
b[n] = \left( \frac{2}{N} \right)^{\frac{1}{2}} \sum_{m=0}^{N-1} c_m B[m] \cos \left( \frac{(2n+1)m\pi}{2N} \right) \quad (8)
\]

In both equation (1) and (2) \( c_m \) is defined as

\[
c_m = \begin{cases} 
\left( \frac{1}{2} \right)^{\frac{m}{2}} & \text{for } m=0 \\
1 & \text{for } m \neq 0 
\end{cases}
\]

The coefficient \( B[0] \), which is directly related to the average value of the time-domain block, is often called the DC coefficient, and the remaining coefficients are called AC coefficients.

2.2 Overview of DWT

Wavelet function is defined over a finite interval and having an average value zero. The basic idea of the wavelet is to represent any arbitrary function \( f \) as a superposition of a set of such wavelet of basic functions [17-23]. These basis functions or baby wavelets are obtained from a single prototype wavelet called the mother wavelet, by dilations or contractions (scaling) and translations (shifts).

\[
\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi \left( \frac{t-b}{a} \right) \quad (9)
\]

Where ‘a’ is the scaling parameter and ‘b’ is the shifting parameter.

Wavelets Transform is a method to analysis a image in time and frequency domain, it is effective for the analysis of image. Wavelet transform give the multi resolution decomposition of image [18]. There is the basic concept of multi resolution: (i) sub-band coding; (ii) vector space and (iii) pyramid structure coding. DWT decompose a image at several \( n \) levels in different frequency bands. Each level decomposes a image into approximation coefficients (low frequency band of processing) and detail coefficients (high frequency band of processing), and the result is down sampled by 2 shown in Fig. 2[19, 20].

Fig 2 Filter bank representation of DWT decomposition:

At each step of DWT decomposition, there are two outputs: scaling coefficients \( x^{l+1}(n) \) and the wavelet coefficients \( y^{l+1}(n) \). These coefficients are:

\[
x^{l+1}(n) = \sum_{i=0}^{2^n} h(2n-i)x^l(n) \quad (10)
\]

And \( y^{l+1}(n) = \sum_{i=0}^{2^n} g(2n-i)x^l(n) \quad (11)\)
Where, the original image is represented by $x^j(n)$ and $j$ shows the scaling number. Here $g(n)$ and $h(n)$ represent the low pass and high pass filter, respectively. The output of scaling function is input of next level of decomposition, known as approximation coefficients.

In order to reconstruct the original image, at each level of reconstruction, approximation components and the detailed components are up by 2 and the detailed components are up sampled by 2, and then convolved which is shown in Fig.3.

![Fig.3. Reconstruction using inverse wavelet filter representation](image)

### III. METHODOLOGY

The image compression is achieved using wavelet transformation and its process is based on uniform quantization and entropy encoding. The basic methodology for image compression is shown in Fig. 4.

![Fig. 4.DWT based bit-rate controlled compression methodology for image compression](image)

The algorithm of image compression is performing in three stages: (i) DWT decomposition, (ii) Threshold & Quantization, (iii) Entropy encoding. After DWT decomposition of the image, their wavelet coefficients are selected on the basis of energy packing efficiency of each sub-band. Then, apply a thresholding (level or global), which suggested that a fixed percentage of wavelet coefficients should be zero. Further, uniform quantizer is applied in these coefficients. The actual compression is achieved at this stage and then compression achieved based on the entropy encoding techniques (Huffman). Finally compressor system gives the compressed data value of image.

### A. THRESHOLDING

The next step after decomposition of image is thresholding, after decomposing of the image a threshold is applied to coefficients for each level from 1 to $N[5,18,21]$. So many of the wavelet coefficients are zero or near to zero so due to thresholding near to zero coefficients are equal to zero. By aping a hard thresholding the coefficient below the level is zero so produce a many consecutive zero’s which can stored in much less space and transmission speed is up, and in the case of global thresholding the value are set manually, this value are chosen from the range $(0, \ldots, C_{max})$ where $C_{max}$ maximum coefficient in the decomposition.

### B. Quantization

Next step after the thresholding stage is uniform quantization; aim of this step is to decreases the information which found in the wavelet coefficients in such a way so no error is formed. We quantize the wavelet coefficients using uniform quantization, the computation of step size depend on three parameters $[3, 5, 17]$ are:

1. **Maximum value, $M_{max}$ in the matrix**
2. **Minimum value, $M_{min}$ in the matrix**
3. **Number of quantization level, $L$**

Once these parameters are found, then step size,

$$\Delta = \left( \frac{M_{max} - M_{min}}{L} \right)$$  \hspace{3cm} (12)

Then, the input is divided in to $L+1$ level with equal interval size ranging from $M_{min}$ to $M_{max}$ quantization step. When quantization step is done, then quantization value is fed to the next stage of compression. Three parameters defined above are stored in the file because to create the quantization table during reconstruction step for de-quantization [1].

### C. Huffman Encoding

In quantization process, the quantized data contains some un useful data, means repeated data, it is wastage of space. To overcome this problem, Huffman encoding $[5,1]$ is exploited. In Huffman encoding, the probabilities of occurrence of the symbols in a image are computed. These symbols indices in the quantization table. Then these symbols are arranged according to the probabilities of occurrence in descending order and build a binary tree and codeword table.

Over all compression algorithm summarized in following steps:

The image $f(x,y)$ is subdivided into non-overlapping $M \times M$ blocks. The Transformed is applied to each block of image (for DWT the image is treated as one block).

The transformation coefficients values less than a given threshold are set to zero. The threshold takes a percentage of the minimum and maximum coefficient values throughout the whole block coefficients.

After that, uniform quantization exploited to compute new coefficient in different bit rate on quantization scale. Apply an entropy coding technique.

The reconstructed image $F(x,y)$ is compared to the original image to get the compressed image.

### IV. RESULT

In this paper, wavelet transform based image compression technique has been presented using the uniform quantization and Huffman based entropy coding. Where, quantization process controls the bit-rate of compressed coefficients of transformed image data. Technique has been tested with three different gray scale images obtained from USC-SIPI image database $[28]$ and also compare with discrete wavelet
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transform based compression scheme in term of considered fidelity parameters. Here, three different well known signal processing fidelity metrics exploited to evaluate performance of presented technique such as compression ratio (CR), peak signal to noise ratio (PSNR), mean square error (MSE).

Table 1 listed the performance of wavelet analysis as well as discrete cosine transform for three different images. These results clearly shown the efficiency of DWT is better than the DCT in term of CR, PSNR and MSE. The average performance of presented technique is 10.0, 4.52, $2.44\times10^4$ and $12.54, 24.08, 2.59\times10^4$ for DCT and DWT respectively in term of CR, PSNR and MSE.

Table 1 Performance of DCT and DWT based Image compression technique

<table>
<thead>
<tr>
<th>Image</th>
<th>DCT</th>
<th>DWT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CR</td>
<td>PSNR</td>
</tr>
<tr>
<td>Aerial</td>
<td>10.0</td>
<td>2.82</td>
</tr>
<tr>
<td>Girl</td>
<td>10.0</td>
<td>5.00</td>
</tr>
<tr>
<td>Boat</td>
<td>10.0</td>
<td>5.73</td>
</tr>
<tr>
<td>Average</td>
<td>10.0</td>
<td>4.52</td>
</tr>
</tbody>
</table>

The presented technique gives efficient image compression with DWT technique as results illustrate in Table 1, here, quantization process also play key role to truncate different compression efficiency. Table II contain the performance of DWT based technique using different quantization bit-rate, it illustrate the lower bit-rate (Q) can give lower compression rate and increase with increment of quantization bit rate. Where, found maximum compression at 2 bit/pe-pixel is 25.17, 26.35 and 26.07 for three different images. Overall, analysis has illustrated the efficiency of DWT based technique for image compression. Fig. 5 had shown the effect of compression on output images as compare to original signal.

CONCLUSION

This paper presents image compression techniques based on DWT technique with bit rate controlled compression. There are basically two types of compression techniques. One is Lossless Compression and other is Lossy Compression Technique. Comparing the performance of compression technique is difficult unless identical data sets and performance measures are used. Some of these techniques are obtained good for certain applications like security technologies. Some techniques perform well for certain classes of data and poorly for others. Analysis has clearly illustrate the presented technique is widely suitable for image compression.

REFERENCES


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Table II Performance analysis of different quantization bit-rate for image compression

<table>
<thead>
<tr>
<th>Bit-Rate (Q)</th>
<th>Aerial</th>
<th>CR</th>
<th>PSNR</th>
<th>MSE</th>
<th>Girl</th>
<th>CR</th>
<th>PSNR</th>
<th>MSE</th>
<th>Boat</th>
<th>CR</th>
<th>PSNR</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>5.99</td>
<td>25.86</td>
<td>1.68×10^2</td>
<td>6.71</td>
<td>26.82</td>
<td>1.35×10^2</td>
<td>5.43</td>
<td>28.63</td>
<td>0.89×10^2</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>12.49</td>
<td>23.71</td>
<td>2.76×10^2</td>
<td>13.18</td>
<td>23.14</td>
<td>3.15×10^2</td>
<td>11.94</td>
<td>25.38</td>
<td>1.88×10^2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>18.31</td>
<td>22.08</td>
<td>4.02×10^2</td>
<td>19.70</td>
<td>20.70</td>
<td>5.53×10^2</td>
<td>19.10</td>
<td>22.71</td>
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<tr>
<td>2.0</td>
<td>24.26</td>
<td>16.86</td>
<td>1.33×10^3</td>
<td>24.04</td>
<td>16.04</td>
<td>1.50×10^3</td>
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<td>17.95</td>
<td>1.04×10^3</td>
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<tr>
<td>2.5</td>
<td>25.17</td>
<td>18.24</td>
<td>9.73×10^2</td>
<td>26.35</td>
<td>17.50</td>
<td>1.15×10^3</td>
<td>26.07</td>
<td>19.16</td>
<td>7.88×10^2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 5 Performance analysis of Presented Compression Technique: Original Image (A) Ariel, (B) Girl, (C) Boat; Compressed Image using DCT (D), (E) and (F); and Compressed Image using DWT (G), (H), and (I)