HSU-STRUCTURE ON H-STRUCTURE MANIFOLDS II

Virendra Nath Pathak, Geeta Verma

***Abstract*— In this paper we have studied Hsu-structure & H-structure manifolds and obtained with constant holomorphic sectional curvature. On this communication we prove that every 2m dimensional connected NK-manifold of pointwise constant holomorphic sectional curvature is an Einstein manifold**

***Index Terms*— H-structure manifold , Hsu-structure ,Curvature tensor almost complex , almost product and almost tangent structures, NK-manifold.**
Mathematical Classification : 53C15, 53C25

I. INTRODUCTION

Let F be a (1,1) tensor field on C^{∞} n-dimensional differentiable manifold M, such that

$$F^2(X)=a^r X, \qquad (1)$$

where a is a real or a purely imaginary number and X an arbitrary vector field on M, Clearly, F is an endomorphism of the tangent space $T_p(M)$, for every point $p \varepsilon M$. F gives a differentiable structure on M called Hsu-structure defined by (1).

Let there be defined on V_n, a vector valued linear function F of class C such that

$$F^2 = a^r I_n \qquad 0 \le r \le n$$

where r is an integer and a is real or imaginry number. Then F is called Hsu – structure and V_n is called the **Hsu – structure manifold**.

If $a^{r/2} = 0$, we have an almost tangent structure.

If $a^{r/2} \neq 0$, we have a π – structure which is known from G. Legrand in [4], (1956), Especially if $a^r = 1$, we have an almost product structure, if $a^r = -1$, we have the well known almost complex structure, $(J^2 X = -X)$.

If the above mentioned Hsu-structure F is equipped with a Hermitian metric g such that

$$g(FX,FY)+a^r g(X,Y)=0.$$

for any vector fields X,Y on M, then (g,F) is called H-structure and M is said to be H-structure manifold. Many authors studied H-structure manifolds: K.L. Duggal in [1] and [2] was of the first ones.

If the structure tensor F is parallel $(i.e.(\nabla_X F)Y = 0,$ where ∇ is the Riemannian connection), then M is called K-manifold.

If the structure tensor F satisfies the condition $(\nabla_X F)Y = 0,$ for arbitrary vector field X on M, then M is called nearly K-manifold (briefly NK-manifold).

In the present communication we deal with some 2m-dimensional H-structure manifolds. In the second section we shall give some useful preliminaries. In the last section we shall give the main result of the present communication, which are referred in [1].

II. PRELIMINARIES

Let ϕ be (0,2)-tensor on a 2m-dimensional H-structure manifold M such that

$$\phi(X,Y) = g(FX,Y) = -g(X,FY). \qquad (2)$$

Using (2) we can prove:

$$\phi(X,Y)+\phi(Y,X)=0, \qquad (3)$$
$$\phi(FX,FY)+a^r\phi(X,Y)=0, \qquad (4)$$
$$(\nabla_X\phi)(Y,Z)+(\nabla_X\phi)(Z,Y)=0, \qquad (5)$$
$$(\nabla_X\phi)(FY,FZ)=a^r(\nabla_X\phi)(Y,Z). \qquad (6)$$

Denoting by $(W,X,Y,Z)=g((\nabla_W F)X,(\nabla_Y F)Z)$, the properties (3) and (4) give:

$$(W,X,Y,Z)=(Y,Z,W,X),$$
$$(W,FX,Y,FZ)=-a^r(W,X,Y,Z), \qquad (7)$$
$$(W,FX,Y,Z)=-(W,X,Y,FZ),$$

It is well known that the curvature tensor R is defined by:

$$R_{XY}Z=\nabla_{[X,Y]}Z-[\nabla_X,\nabla_Y]Z.$$

We denote by

$$R(W,X,Y,Z)=g(R_{WX}Y,Z),$$

for arbitrary vector fields W,X,Y and Z on M.

Manuscript received May 15, 2015
Virendra Nath Pathak, Department of Mathematics,Shri Ramswaroop Memorial Group of Professional Colleges,Tewariganj, Faizabaad Road, Lucknow
Geeta Verma, Department of Mathematics, ShriRamswaroop Memorial Group of Professional Colleges, Tewariganj, Faizabaad Road, Lucknow

It is also known that the holomorphic sectional curvature $H(x)$ is defined by

$$H(x) = R(x, Fx, x, Fx)/g(x,x)g(Fx,Fx),$$

for $x\varepsilon Tp(M), (p\varepsilon M)$.

Let $\{E_1 ....., E_m, E_{m+1} ....., E_{2m}\}$ be an orthonormal frame field such that:

$$E_{m+i} = \sqrt{-1}E_i / a^{r/2}, (i = 1,...., m).$$

We denote by k *and* k^* the Ricci tensor and the Ricci *tensor on M, respectively. The Ricci *tensor k^* is defined by

$$k^*(x, y) = trace\ of\ (z \to R(Fz, x)Fy),$$

for $x, y, z\varepsilon T_p(M), p\varepsilon M$.

3. Main results

In the present section we shall state the main results of the present communication.

Theorem 1

Let M be on H-structure manifold of pointwise constant holomorphic sectional curvature c(p). Then

$$\begin{aligned}
&4a^r c(p)[-2\phi(x,y)Fw - \phi(x,w)Fy + \phi(y,w)Fx + a^r g(x,w)y - a^r g(y,w)x] = \\
&= -3a^{2r}R(w,x)y + 3FR(Fw,Fx)Fy + a^r R(Fw,Fx)y - a^r FR(w,x)Fy - \\
&a^r R(Fw,x)Fy - 3a^r FR(Fw,x)y + 3a^r R(w,Fx)Fy + a^r FR(w,Fx)y + \\
&3a^r R(w,y)x - 3FR(Fw,Fy)Fx - a^r R(Fw,Fy)x + a^r FR(w,y)Fx + \\
&a^r R(Fw,y)Fx + 3a^r FR(Fw,y)x - 3a^r R(w,Fy)Fx - a^r FR(w,Fy)x.
\end{aligned}$$

We now state some lemmas.

Lemma 1

If M is an H-structure manifold and $\{E_i\}$ is an orthonormal frame field, for every vector field X we have:

$$\sum_{i=1}^{2m} [R(X, FE_i)]FE_i + a^r R(X, E_i)E_i] = 0,$$

$$\sum_{i=1}^{2m} [R(X, E_i)]FE_i + a^r R(X, FE_i)E_i] = 0,$$

Lemma 2

Let M be a H-structure manifold. Then for arbitrary vector fields X,Y on M we have:

$$\begin{aligned}
&k(X,Y) = k(Y,X) \\
&k^*(FX,FY) = -a^r k^*(Y,X) \\
&k(FX,Y) = -k^2(FY,X).
\end{aligned}$$

If s and s^* are the scalar and the *scalar curvatures of M respectively, then we have:

Proposition 1

If M be a 2m-dimensional H-structure manifold of pointwise constant holomorphic sectional curvature c(p), then for arbitrary vector fields X,Y on M, we have:

$$\begin{aligned}
a^r k(X,Y) &= k(FX,FY) - 3[k^*(X,Y) + k^*(Y,X)] \\
&= 4(m+1)c(p)a^r g(X,Y), \\
a^r s - 3s &= 4m(m+1)a^r c(p).
\end{aligned}$$

The results of the theorem 1 and the proposition 1 have been obtained by G.B. Rizza for $a^{r/2} = -1$,([5]).

For every NK-manifold we have:

$$k(X,Y) = (a^r - 3)^{-1} \sum_{im1}^{2m} (X, E_{i,} Y, E_i),$$

$$k(FX, FY) = -a^r k(X,Y),$$
$$k^*(X,Y) = k^*(Y,X),$$
$$k^*(X,Y) = \left(a^r - 3\right)^{-1} a^r k(X,Y).$$

Using the above relations we can obtain the following:

Theorem 2

If M is a 2m-dimensional connected NK-manifold of pointwise constant holomorphic sectional curvature, then M is an Einstein manifold.

REFERENCES

[1] Cb. Cristophoridou and Ph. Xenox, On some H-structure manifolds with constant holomorphics sectional curvature, Note Matem, XI, No.2, (1993).
[2] L, Duggal, On differentiable structures defined by algebraic equations, I. Nijenhuis tensor, Tensor, V.S. 22(1971), 238-242.
[3] L. Duggal, On differentiable structures defined by algebraic equations, II F- connections, Tensor, N.S. 22(1971), 255-260.
[4] C Legrand, Sur les varietes a structure de Presque produit complexe, C.R. Acad. Sci. Paris 242(1956), 335-337.
[5] Q.B. Rizza, On almost Hermitian manifolds with constant holomorphic curvature at a point, Tensor N.S. 50,(1991), 79-89.