Hsu-Structure motivate the mathematical Space

Virendra Nath Pathak, Geeta Verma

***Abstract*— The purpose of the present paper, we have studied Hsu-structure motivate the mathematical space of circle in one dimensional manifolds. A manifold is a mathematical space in which every point has a neighborhood which resembles with Euclidean space, but in which the global structure may be more complicated. In the present paper, we have discussed the manifolds, the idea of dimension is important. For example lines are one dimensional , and planes two dimensional. In a one dimensional manifold , every point has a neighborhood that looks like a segment of a line. Examples of one dimensional manifold include a line, a circle and two separate circles. In a two dimensional manifold, every point has a neighborhood that looks like a disk. Examples include a plane , the surface of a sphere, and the surface of tours. The trivial example of an n-dimensional manifold is M^n. It is assumed that , in section one which contains a brief introduction to Hsu-structure of mathematical manifold and modeling of Hsu-structure manifold, while in section two, defines the special quadratic F-structure and proves some theorems. In section three , we have defined the mathematical modeling in one or more dimensional manifold. In section four, we have discussed the motivational examples of manifold and construct the figures. In section five, we have obtained the geometrical projection and define the slope of the geometrical equations with point (1,0) and (-1,0). In section six, we have calculated the Nijenhuis tensor with Hsu- structure and proved some theorems. In the end; we have discussed the important role of mathematical space.**

***Index Terms*— Hsu-structure, Nijenhuis tensor, Modeling of manifold, Projection, Mathematical space, F-structure**

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I. INTRODUCTION

Definition Let there be defined on V_n, a vector valued linear function F of class C such that

$$F^2 = a^r I_n \qquad 0 \le r \le n$$

where r is an integer and a is real or imaginary number. Then F is called Hsu – structure and V_n is called the **Hsu – structure manifold**.

Let us consider n- dimensional differentiable manifold M^n of class C^∞ in which there exists a vector valued linear function F of differentiability of class C^∞ satisfying

(1.1) $F^2 X = a^r X$

(1.2) $\overline{X} = FX$, for an arbitrary value of X.

Where 'a' is a complex number , then the manifold M^n is said to be Hsu- Structure manifolds . Hsu- structure arise the cases as

(i) If $a^{r/2} = \pm i$, then it is almost complex structure.

(ii) If $a^{r/2} = \pm 1$, then it is almost product structure.

(iii) If $a^{r/2} = 0$, then it is almost tangent.

(iv) If $a^{r/2} \ne 0$, then it is π -structure.

If the Hsu- structure is endowed with Hermite metric tensor g, such that

(1.3) $g(\overline{X},\overline{Y}) = a^r g(X,Y),$

Then $[F,g]$ gives M^n, a Hermite structure or H-structure subordinate to Hsu-structure. Let a tensor $`F$ of the type (0,2) in M^n equipped with H-structure defined as

(1.4)
$$`F(X,Y) \overset{def}{=} g(\overline{X},Y)$$
$$`F(X,Y) \overset{def}{=} -g(X,\overline{Y})$$

It is easy to vary the following results:

(1.5) $`F(\overline{X},\overline{Y}) = -a^r F(X,Y)$

II. SPECIAL QUADRATIC F- STRUCTURE

The manifold M^n is defined the special quadratic F-structure and prove some theorems.

Theorem 2.1

The rank of F in the special quadratic F-structure is equal to have dimension of the manifold, we have rank (F) = n.

Proof : Assuming $\overline{X} = 0 \;\Rightarrow\; F^2 X = 0$

$\Rightarrow X = 0$

So from equation (1.1) it follows that $a^r X = 0 \;\Rightarrow\; X = 0.$ Hence $\overline{X} = 0$ has trivial solution $X = 0.$ Consequently $rank(F) = n$. And If V denotes the nullity of F , V = 0. If ρ be the rank of F, then from a well known theorem of linear algebra

(2.1) $\rho + V = n$

Since $\rho = 0$, hence $F\rho = n.$
This proves the theorems.

Theorem 2.2

The dimension of the manifold M^n equipped with the special quadratic F-structure for $1 < 4(a^r + I)$ is even.

Proof: Let θ be the eigen value of F and V the corresponding Eigen vector. Then

$$\overline{V} = \theta V$$

This yield

$$\overline{\overline{V}} = \theta^2 V \text{ or } F^2 V = \theta^2 V$$

Substituting these values of $\overline{V}$ & $\overline{\overline{V}}$ in (1.1) & (1.2), we have

$$\theta^2 V = a^r V \text{ and } \theta V = I$$

Which gives

(2.2) $\theta^2 = a^r$ and $\theta = I$

Adding , we have

$$\theta^2 + \theta - (a^r + I) = 0$$

The roots of the above quadratic equation are given by

$$\theta = \frac{-1 \pm \sqrt{1 - 4.1.(a^r + I)}}{2}$$

(2.3) $\theta = \dfrac{-1 \pm \sqrt{1 - 4(a^r + I)}}{2}$

If $1 < 4(a^r + I)$, the Eigen value of F one of the form

$$\Rightarrow \quad F^2(\mu(x)) = a^r X, Y, `B(X, Y, Z) \overset{def}{=} g(B(X, Y), Z)$$

Where $\alpha = \dfrac{-1}{2}$ & $\beta = \dfrac{\sqrt{1 - 4(a^r + I)}}{2}$

Since the complex Eigen values occur in pairs, therefore, the dimension n of the manifold must be even.

Theorem 2.3

In an equation with real coefficients, complex roots occur in conjugate pair.

Proof : Let $F_n(X) = 0$ be the equation with real coefficients and let $\alpha + i\beta$ be complex roots of this equation, where α & β are real quantities and $b \neq 0.$

Now, we are to prove that $\alpha - i\beta$ is also a root of the equation $F_n(X) = 0,$

Let the polynomial $F_n(X)$ be divided by

$$[(X - \alpha)^2 + \beta^2] \text{ i.e.}$$

$$[(X - \alpha)^2 - i^2\beta^2] \text{ i.e.}$$

$$[(X - \alpha + i\beta)(X - \alpha - i\beta)]$$

Let θ be the quotient & (R X +R) be the remainder , if any , Then

$$F_n(X) = \{(X - \alpha)^2 + \beta^2\}\theta + (RX + R') \text{ or}$$

(2.4) $F_n(X) = \{(X - \alpha + i\beta)(X - \alpha - i\beta)\}\theta + (RX + R')$

Putting $X - \alpha + i\beta$ we find that $X - \alpha - i\beta$ vanishes and also F-structure vanishes as $\alpha + i\beta$ is root of the equation $F(X) = 0$.

From (2.4), we have

$$R(\alpha + i\beta) + R' = 0 \quad or \quad (R\alpha + R') + iR\beta = 0$$

Equating real and imaginary parts on both sides of this relation

We have $R + \alpha R' = 0 \quad \& \quad R\beta = 0$

$\beta = 0$, so we have $R = 0 \quad \& \quad \therefore\ R' = 0$

$\therefore$ From (2.4) , we have

$$F_n(X) = (X - \alpha + i\beta)(X - \alpha - i\beta)\theta$$

F-structure vanishes when $X = \alpha - i\beta$ i.e. $\alpha - i\beta$ is a root of the equation.

$$F_n(X) = 0.$$

Theorem 2.4

The special quadratic F-structure is not unique.

Proof : Let us put

(2.5) $\mu(F'(X)) = F(\mu(X))$

Where F' is a tensor field of type (1,1) and m is non singular vector function on M^n ,Then

(2.6) $\mu(F'^2(X)) = \mu F'(F'(X)) = F(\mu(F'(X))) = F(F\mu(X)) = F^2(\mu(X)) = a^r \mu(X)$

Thus we have

$$\mu\{F'^2(X)\} = a^r X$$

$$\Rightarrow \quad F^2(\mu(X)) = a^r X$$

By virtue of the equation (1.1), we obtain,

$$F'^2 = a^r$$

Where μ is non singular. Hence F' gives the special quadratic F- structure on the manifold M^n

III. Mathematical Modeling

If ϕ is a map of M into N and ψ is a map of P into T, then $\psi \quad \phi$ will denote their combination , that is, $\psi \quad \phi$ is ϕ followed by ψ . Here M, N, P,T are any sets and we understand that the domain of $\psi \quad \phi$ is $\phi^{-1}(P) \cap M$. The domain is the largest meaningful set, will be used in the formation of sums , products, and other combinations of map.

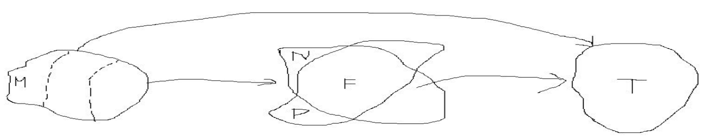

Fig.-(3.1)- model of manifold

A manifold is a mathematical space which is not fixed shape and size and in other words we can say that extended to n-dimensional space in a covering area.

Let us consider a topological manifold is defined the top half of the unit circle, $X^2 + Y^2 = 1$, where Y-ordinate is positive and X-coordinate is negative. So projection onto the first coordinate is a continuous and invertible to the open interval (-1, 1).

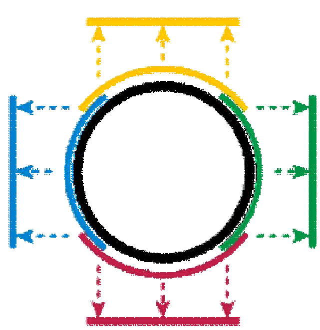

Fig. (3.2) circle to an open interval

(3.1)(a) $F_{top}(X,Y) = X,$

(b) $F_{bottom}(X,Y) = Y,$

Then the cover of the whole circle of the map and mathematical modeling of a circle of 1-dimensional manifold.

PROBLEM (3.1) A modeling of 1-dimensional manifold based on slope define a function is a covering all but one point of a circle in transition map.

SOLUTION: The top and right maps overlap: their intersection lies in the quarter of the circle where both the X- coordinates and Y- coordinates are positive . The two map F_{top} and F_{right} structure each map of this part into the interval (0, 1). Thus the function T from (0, 1) to itself can be constructed, which first uses the inverse of the top map to reach the circle and then follows the right map back to the interval. Let a be any number in (0, 1), then

(3.2) $T(a^{r/2}) = F_{right}(F_{top}^{-1}(a^{r/2})) = F_{right}(a^{r/2},\ \sqrt{1-a^r}) = \sqrt{1-a^r}$

Such a mathematical modeling of a function is called a transition map. Hence the mathematical modeling of manifold can do extended to one or more dimension.

IV. MOTIVATIONAL EXAMPLE OF MATHEMATICAL SPACE

Hsu- structure defines a motivational example in a mathematical modeling of a circle manifold based on slope , covering all but one point of the circle.

PROBLEM (4.1)

Let the circle is the simplest example of topological manifold after a line.

SOLUTION: Let the unit circle,

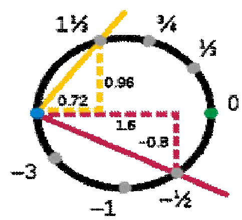

Fig.(4.1)- unit circle in one dimensional manifold

There are map for bottom , left, top and right part of the circle . The F_{top} and F_{right} exacts map this part into the interval (0,1) and using the equation (3.1) and (3.2), then such a function is called a transition map. Thus top, bottom, left and right show that the circle is one – dimensional manifold.

PROBLEM (4.2)

Show that a C^∞ map is necessarily continuous .

SOLUTION : A covering map $\phi : M \to F$ is continuous map such that for every $n \in F$ there is a neighborhood U of n such that $\phi^{-1}(U)$ is a disjoint union of neighborhoods of points of $\phi^{-1}(n)$ such that ϕ is a homomorphism on each such neighbor hood ϕ is said to evenly cover U and U is said to be a distinguished neighborhood of ϕ when M and N are C^∞ manifolds, then M is said to be a C^∞ covering of N if ϕ is a C^∞ maps.

PROBLEM (4.3)

Prove that if $\phi : M \to F$ is a covering map and F has a C^∞ structure , then there is a unique C^∞ structure on M such that M is a C^∞ covering of N.

PROBLEM (4.4)

Prove that if $\phi : M \to F$ is a covering map and M has a C^∞ structure such that for every $U_{i,}\ \ U_j$ open set in M on which ϕ is a homeomorphism and $\phi(U_i) = \phi(U_j)$ we have $(\phi / U_j)^{-1}\ \ \phi$ is a C^∞ map on U_i, then N has a unique C^∞ structure such that M is a C^∞ covering of F.

PROBLEM (4.5)

Prove that if N is a connected C^∞ manifold, then there exists an essentially unique simply connected C^∞ covering of F.

V. GEOMETRICAL PROJECTION

Let ordinary sphere $S^d = \{X \in R^{d+1} \middle| \sum u_i^2(X) = 1\}$ and define

$\phi : S^d - \{0,0.......,0,1)\} \to R^d$,$\psi : S^d - \{0,0.......,0,-1)\} \to R^d$,

$\psi : S^d - \{0,0.......,0,-1)\} \to R^d$, by stereographic projection from (0, ……….,0, 1) , (0,……,0, -1) respectively.

Then $F = \{\phi, \psi\}$ is a basis for a C^∞ structure on S^d . The projection $\phi(X)$ is the point where the straight line from (0,…….,0,1) through X intersects $u_{d=1}^{-1}(0) = R^d$.

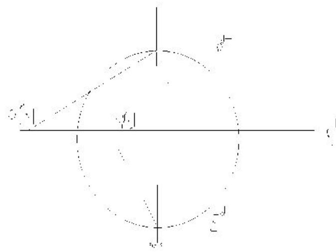

Fig.(5.1)-projection in d-space

Let P^d be a real projective d-space, that is, the collection of straight lines through the origin in $R^{d=1}$

The natural covering map $\phi : S^d \to P,$ which takes X into the line through X , induces a C^∞ structure on P^d ,that is , there is a unique C^∞ structure on P^d such that ϕ is a C^∞ map with local C^∞ inverses. Let us consider the projection of the slope

(5.1) $F_{\min us}(X,Y) = s$,

(5.2) $F_{plus}(X,Y) = t$

where

(5.3)(a) $s = \dfrac{Y}{1+n}$

(b) $t = \dfrac{Y}{1-n}$

Here s is the slope of the line through the point at coordinate (X, Y) and the fixed pivot point (-1, 0); t is the mirror image, with pivot point (+1,0). The inverse mapping from s to (X,Y) is given by

(5.4)(a) $X = \dfrac{1-s^2}{1+s^2}$

(b) $Y = \dfrac{2s}{1+s^2}$

It can easily be confirmed that $X^2 + Y^2 = 1$ for all values of the slope S. These two maps provide a second atlas for the circle , with

(5.5) $t = \dfrac{1}{s}$

Each map omits a single point , either (-1, 0) for S or (+1, 0) for t , so neither map alone is sufficient to cover the whole circle. It is clear that it is not possible to cover the full circle with single point. For example although it is possible to construct a circle from a single line interval by overlapping and 'glueing' ends, this does not produce a map. A portion of the circle will be mapped to both ends once , losing invertibility.

VI. MATHEMATICALLY CALCULATE NIJENHUIS TENSOR WITH HSU-STRUCTURE

In what follows that we shall study some theorems of Nijenhuis tensor with Hsu-structure. Mathematically calculate of Nijenhuis tensor with respect to F is a vector valued bilinear function N and B given by

(6.1) $N(X,Y)=\left[\overline{X},\overline{Y}\right]+\left[\overline{\overline{X,Y}}\right]-\left[\overline{\overline{\overline{X}},Y}\right]-\left[\overline{X,\overline{\overline{Y}}}\right]$

Or equivalently

(6.2) $N(X,Y)=\left[\overline{X},\overline{Y}\right]+a^{r}[X,Y]-[\overline{\overline{\overline{X}},Y}]-\left[\overline{X,\overline{\overline{Y}}}\right]$

If arbitrary vector field X, Y will be complex then ,

(6.3) $N(X,Y)=N(X,Y)$

Theorem (6.1)

We have

(6.4) $N(X,\overline{Y})=N(\overline{X},Y)$

(6.5) $N(\overline{X},\overline{Y})=a^{2r}\left\{[\overline{X},Y]+[X,\overline{Y}]-[X,Y]\right\}+a^{r}[\overline{X},\overline{Y}]$

(6.6) $N(\overline{\overline{X}},Y)=a^{r}\left[\overline{X},\overline{Y}\right]+a^{2r}[X,Y]-a^{2r}\left[X,\overline{Y}\right]+a^{2r}\left[\overline{X},Y\right]$

(6.7) $N(X,\overline{\overline{Y}})=-a^{r}[\overline{X},\overline{Y}]-a^{2r}[X,Y]-a^{2r}\left[X,\overline{Y}\right]+a^{2r}\left[\overline{X},Y\right]$

Consequently,

(6.8) $N(\overline{\overline{X}},Y)+N(X,\overline{\overline{Y}})=+2a^{2r}\left\{-[X,\overline{Y}]+[\overline{X},Y]\right\}$

(6.9) $N(\overline{\overline{X}},Y)-N(X,\overline{\overline{Y}})=2a^{r}\left\{[\overline{X},\overline{Y}]+a^{r}[X,Y]\right\}$

(6.10)
$$N(\overline{X},\overline{Y})+N(X,Y)=\left[\overline{X},\overline{Y}\right](a^{r}+1)+(a^{r}+1)a^{r}[\overline{X},Y]+a^{r}(a^{r}-1)[X,\overline{Y}]$$
$$+a^{r}(1-a^{r})[X,Y]$$

(6.11)
$$N(\overline{X},\overline{Y})-N(X,Y)=(a^{r}-1)[\overline{X},\overline{Y}]-a^{r}(a^{r}+1)[X,Y]+a^{r}(a^{r}-1)[\overline{X},Y]$$
$$+a^{r}(a^{r}+1)[X,\overline{Y}]$$

(6.12) $\overline{N(\overline{X}.Y)}+\overline{N(X,Y)}=-a^{2r}\left\{[\overline{X},Y]+2[X,Y]\right\}+a^{r}\left\{[X,Y]-[\overline{X},Y]-[X,\overline{Y}]\right\}$

(6.13)
$$\overline{N(\overline{X},\overline{Y})}+\overline{N(X,Y)}=a^{r}\left\{[\overline{X},\overline{Y}]-[\overline{X},Y]-[X,\overline{Y}]\right\}-a^{2r}\left\{[\overline{X},Y]+[X,Y]\right\}$$
$$-a^{3r}[X,Y]$$

Proof : Barring X and Y in equation (6.2) separately and using (1.1), we get (6.3). Barring X and Y both in (6.2) and using (1.1), we obtain (6.4). Barring X and Y in equation(6.2) separately in two times and using equation (1.1), we get (6.6) and (6.7) respectively. On adding and subtracting one by one equation (6.6) and (6.7) separately we get (6.8) and (6.9) respectively. On adding and subtracting one by one equation (6.3) and (6.5) separately we get (6.10) and (6.11) respectively. And barring (6.4) and (6.3) , then adding we get (6.10). Barring (6.5) and using (1.1), we get (6.13).

Theorem (6.2)

If we put

(6.14)
$$B(X,Y)=[\overline{X},\overline{Y}]+[\overline{\overline{X,Y}}]$$
$$Or$$
$$B(X,Y)\overset{def}{=}[\overline{X},\overline{Y}]+a^{r}[X,Y]$$

Then

(6.15)
$$B(\overline{X},Y)=[\overline{\overline{X}},\overline{Y}]+a^{r}[\overline{X},Y]=[a^{r}X,\overline{Y}]+a^{r}[\overline{X},Y]=a^{r}[X,\overline{Y}]+a^{r}[\overline{X},Y]$$
$$=a^{r}\left\{[X,\overline{Y}]+[X,Y]\right\}$$

(6.16) $B(X,\overline{Y})=-a^{r}[\overline{X},Y]+a^{r}[X,\overline{Y}]$

(6.17) $B(\overline{X},\overline{Y})=-a^{2r}[X,Y]+a^{r}[\overline{X},\overline{Y}]$

(6.18) $\overline{B(X,Y)} = B(\overline{X},\overline{Y})$

(6.19) $\overline{B(\overline{X},\overline{Y})} = -a^{2r}\left\{[X,Y] - a^{3r}[\overline{X},\overline{Y}]\right\}$

(6.20) $B(\overline{\overline{X}},Y) = -a^{r}\left\{[\overline{X},\overline{Y}]\right\} - a^{2r}\left\{[X,Y]\right\}$

(6.21) $B(X,\overline{\overline{Y}}) = -a^{r}\left\{[\overline{X},\overline{Y}]\right\} - a^{2r}\left\{[X,Y]\right\}$

(6.22) $B(\overline{\overline{X}},\overline{\overline{Y}}) = -a^{3r}[X,Y] - a^{2r}[\overline{X},\overline{Y}]$

(6.23) $\overline{B(\overline{\overline{X}},\overline{\overline{Y}})} = -a^{3r}[\overline{X},\overline{Y}] + a^{4r}[X,Y]$

Consequently,

(6.24) $B(\overline{X},Y) + B(X,\overline{Y}) = 2a^{r}[X,\overline{Y}]$

(6.25) $B(\overline{X},Y) - B(X,\overline{Y}) = 2a^{r}[\overline{X},Y]$

(6.26) $\overline{B(\overline{X},\overline{Y})} - B(\overline{X},\overline{Y}) = -a^{r}(a^{2r}+1)[\overline{X},\overline{Y}]$

(6.27) $B(\overline{\overline{X}},Y) + B(X,\overline{\overline{Y}}) = -2a^{r}[X,Y]$

(6.28) $B(\overline{\overline{X}},Y) - B(X,\overline{\overline{Y}}) = -2a^{r}[\overline{X},\overline{Y}]$

(6.29) $\overline{B(\overline{\overline{X}},\overline{\overline{Y}})} + B(X,\overline{\overline{Y}}) = a^{2r}(a^{2r}-1)[X,Y] - a^{r}(a^{2r}+1)[\overline{X},\overline{Y}]$

(6.30) $\overline{B(\overline{\overline{X}},\overline{\overline{Y}})} - B(\overline{\overline{X}},Y) = a^{r}(a^{2r}+1)\left\{a^{r}[X,Y] - [\overline{X},\overline{Y}]\right\}$

(6.31) $\overline{B(\overline{\overline{X}},\overline{\overline{Y}})} + B(X,\overline{\overline{Y}}) = \overline{B(\overline{\overline{X}},\overline{\overline{Y}})} - B(\overline{\overline{X}},Y)$

Proof : Barring X and Y in (6.14) separately and using (1.1), we get (6.15) and (6.16) respectively. Barring X and Y both in (6.14) and using (1.1), we obtain (6.17). Barring (6.17) and using (1.1), we get (6.19). Barring X and Y in(3.2) and (3.3) separately and using (1.1), we get (3.7) and (3.8) respectively. Barring (6.17) and using (1.1), we get (6.22). Barring (6.22)and using (1.1) we get (6.23). Adding & subtracting (6.15) and (6.16) separately, we get (6.24) and (3.12) respectively. Again adding equation (6.19) and (6.17), adding & subtracting (6.20) and (6.21), adding (6.23) and (6.21), subtracting (6.23) and (6.20) and using (6.29) and (6.30) separately, we get (6.26), (6.27), (6.28), (6.29), (6.30) and (6.31) respectively.

Theorem (6.3)

(6.32) $N(X,Y) = -\frac{1}{a^{r}}\overline{B(\overline{X},Y)} + B(X,Y)$

(6.33) $N(\overline{X},Y) = B(\overline{X},Y) - \overline{B(X,Y)}$

(6.34) $N(X,\overline{Y}) = B(X,\overline{Y}) - \overline{B(X,Y)}$

(6.35) $N(\overline{X},\overline{Y}) = B(\overline{X},\overline{Y}) - \overline{B(\overline{X},Y)}$

Proof: Barring (6.14) , adding the resulting equation obtains in (6.15) and using (6.2), we get (6.32). Barring (6.14) and subtracting the resulting equation from (6.16) and (6.18) separately , we get (6.33). Barring (6.14) and subtracting the resulting equation from (6.16) and (6.18) separately, we get (6.34). Barring X and Y both in (2.6) and subtracting the resulting equation from (6.17) and barring (6.15), we get (6.35).

Theorem (6.4)

If we put

(6.36) $W(X,Y) \overset{def}{=} \overline{[\overline{X},Y]} + \overline{[X,\overline{Y}]}$,

Then

$$N(X,Y) = B(X,Y) - W(X,Y)$$

(6.37) $N(X,Y) = W(X,Y) - [\overline{X},\overline{Y}] - a^{r}[X,Y]$

(6.38) $W(\overline{X},\overline{Y}) = a^{r}W(X,Y)$

(6.39) $W(X,Y)+W(\overline{X},\overline{Y})=(a^r+1)W(X,Y)$

(6.40) $W(\overline{X},Y)-W(X,\overline{Y})=0$

Consequently,

(6.41) $N(X,Y)=B(X,Y)-W(X,Y)$

(6.42) $N(X,Y)=\frac{1}{a^r}\overline{W(\overline{\overline{X}},Y)}-W(X,Y)$

(6.43) $N(\overline{X},\overline{Y})=-W(X,Y)-\overline{W(X,\overline{\overline{Y}})}$

(6.44) $N(\overline{X},Y)=\overline{W(X,Y)}-W(\overline{X},Y)$

(6.45) $N(X,\overline{Y})=\overline{W(X,Y)}-W(X,\overline{Y})$

Proof : Barring in X and Y in (6.36) and using equation (1.1), we get (6.37) and solving Nijenhuis tensor, we get (6.2). Barring X and Y in (6.36), using (1.1) and then adding and subtracting the resulting equation in (6.36), we get (6.39) and (6.40) respectively. The relation (6.41) is the consequence of (6.14), (6.36) and (6.2). The equation (6.42), (6.43), (6.44) and (6.45) follow from the equations (6.2), (6.36), (6.39), (6.40) and (1.1).

Corollary (6.5)

We have

(6.46) $W(X,Y)+B(X,Y)=[\overline{X},\overline{Y}]+a^r\{[X,Y]+[X,\overline{Y}]-[\overline{X},Y]\}$

(6.47) $W(\overline{X},Y)-B(\overline{X},Y)=a^r[X,\overline{Y}]-a^{2r}[X,Y]$

(6.48) $W(\overline{X},Y)+\overline{B(X,Y)}=2a^r\overline{[X,Y]}+2[\overline{X},\overline{Y}]$

(6.49) $W(\overline{X},Y)-B(X,Y)=0$

(6.50) $W(\overline{X},Y)-\overline{B(\overline{\overline{X}},\overline{\overline{Y}})}=a^r(1+a^r)\overline{[X,Y]}+(-1+a^r)[X,Y]$

(6.51) $W(\overline{X},\overline{Y})-\overline{B(\overline{\overline{X}},Y)}=0$

(6.52) $\overline{W(X,Y)}+\overline{B(X,\overline{\overline{Y}})}=a^r(a^r+1)W(X,Y)$

(6.53) $\overline{W(X,Y)}-\overline{B(X,\overline{\overline{Y}})}=a^r(a^r-1)W(X,Y)$

(6.54) $\overline{W(\overline{\overline{X}},\overline{\overline{Y}})}+\overline{B(\overline{\overline{X}},Y)}=2a^rW(X,Y)$

PROOF: The above equation immediately follow from (6.14), (6.36) & (1.1) by simple manipulations.

Theorem (6.6)

Let we put

(6.55) $`N(X,Y,Z)\overset{def}{=}g(N(X,Y),Z)$

Then $`N(X,Y,Z)$ is skew –symmetric in X & Y i.e.

(6.56) $`N(X,Y,Z)=-`N(Y,X,Z)$

And

(6.57) $`N(\overline{X},Y,Z)=-a^r`N(X,\overline{Y},Z)$

(6.58) $`N(\overline{X},\overline{Y}Z)=a^r`N(X,Y,Z)$

Proof : Relation (6.55) obviously holds. From the theorem of [1], we have

(6.59) $N(\overline{X},Y)=-N(X,\overline{Y})$

(6.60) $N(\overline{X},\overline{Y})=a^rN(X,Y)$

Then using (6.55), (1.4), (1.5) and (1.3) in (6.59) and (6.60) respectively, we have (6.59). Relation (6.58) is obtained by using (6.55) , (1.4) and (1.5) in (6.60).

Remark : Let us put

$`B(X,Y,Z)\overset{def}{=}g(B(X,Y),Z)$

$$`W(X,Y,Z) \overset{def}{=} g(W(X,Y),Z)$$

Then

$$`N(X,Y,Z) = `B(X,Y,Z) - `W(X,Y,Z)$$

DISCUSSION

Manifold are important role of dealing the extended the n-dimensional space of modeling heavenly body because it is construct the higher dimensional space and they allow more complicated structures. We can easily calculate all structures and spaces of manifold form H s u -structure manifold and discuss the motivational examples of manifold. Examples of Mathematical space with additional structure include the differentiable manifold then the unit circle is the fixed point (1,0), (0,1), (-1,0) and (1,-1) respectively.

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