Comparison of ARMA and ARMAX stochastic models for Karoon river time series generation and forecasting

Sima Safarkhani, Javad Karimi Parchian

Abstract— Time series modeling and forecasting has fundamental importance to various practical domains. The main aim of time series modeling is to carefully collect and rigorously study the past observations of a time series to develop an appropriate model which describes the inherent structure of the series. This model is then used to generate future values for the series, i.e. to make forecasts. Time series forecasting thus can be termed as the act of predicting the future by understanding the past [5, 6]. Due to the fact that just using the historical datasets, suppress the chance of considering different scenarios in water resource studies, hydrological parameters forecasting such as precipitation and discharge plays a big role in hydrologic studies as considering different scenarios will increase the liability of the final forecasting.

Plenty of mathematical methods has been introduced for forecasting, however stochastic models are one of the most common. In this research, data generation and forecasting of ARMA and ARMAX stochastic techniques by using a numerical model on Karoon River (Iran) historical discharge time series has been compared. The final results represents how combined model of ARMA and an external variable X will grow fitness of the model in ARMAX stochastic modeling.

 ${\it Index\ Terms} {-\!\!\!\!--} ARMA, ARMAX, Stochastic\ modeling, Time Series$

I. INTRODUCTION

A time series is a sequential set of data points, measured typically over successive times. It is mathematically defined as a set of vectors x(t), t = 0,1,2,... where t represents the time elapsed [3,4,6].

In general models for time series data can have many forms and represent different stochastic processes. There are two widely used linear time series models in literature. Autoregressive (AR) [1, 2, 4] and Moving Average (MA) [1,4] models. Combining these two, the Autoregressive Moving Average (ARMA) [1, 2, 3, 4] and Autoregressive Moving Average Models with Exogenous Variables (ARMAX), Generalized.

In an AR (p) model the future value of a variable is assumed to be a linear combination of p past observations and a random error together with a constant term [2,4] while the MA(q) model uses past errors as the explanatory variables [2,3,4]. Fitting an MA model to a time series is more complicated than fitting an AR model because in the former one the random error terms are not fore-seeable.

Autoregressive (AR) and moving average (MA) models can be effectively combined together to form a general and useful class of time series models, known as the ARMA models. Mathematically an ARMA (p, q) model is represented as [2,3,4]:

$$y_t = c + \varepsilon_t + \sum_{i=1}^{p} \varphi_i y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j}$$
 (1)

Here \mathcal{Y}_t and \mathcal{E}_t are respectively the actual value and random error (or random shock) at time period t. $\varphi(i=1,2,...,p)$, $\theta(j=1,2,...,q)$ are model parameters, c is the constant and p,q refer to p autoregressive and q moving average model orders [2,3,4].

While in ARMAX model there is another term representing Xt time series which has a good correlation with the yt time series.

$$\gamma_{\mathbf{r}} = c + s_{\mathbf{r}} + \sum_{i=1}^{\mathbf{g}} \varphi_i \gamma_{\mathbf{r}-i} + \sum_{j=1}^{\mathbf{q}} \theta_j s_{\mathbf{r}-j} + \sum_{k=1}^{N_{\mathbf{g}}} \beta_k X_{(\mathbf{r},k)}$$
(2)

II. MATERIALS AND METHODS

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Historical time series in this research is 47 years recorded discharge of Karoon River at Armand station (Fig1-a) which has been standardized via using Logarithmic transformation in a program called SAMS (Fig 1-b).

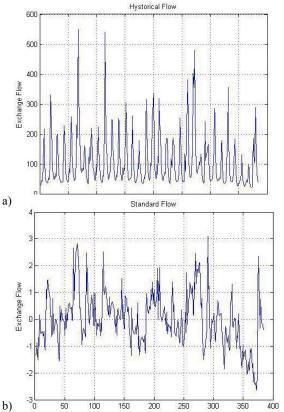


Fig 1- a) monthly historical discharge (x) of Karoon river at Armand station (cms) b) standard monthly historical discharge (z) of Karoon river at Armand station

The primary investigation on the historical data based on autocorrelation and partial auto correlation respectively in Fig 2, Fig 3 represents that after 2 lags in partial autocorrelation chart Φk is finite in extent while it's infinite in extent before 15 lag in autocorrelation chart. For that reason, AR (2) model (with $\Phi 1$ and $\Phi 2$ equals to 0.684 and 0.116) is the primary selected model for data generation.

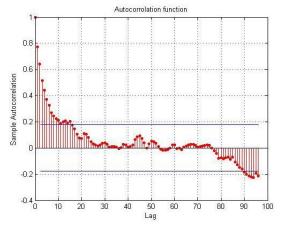


Fig2- Autocorrelation coefficient

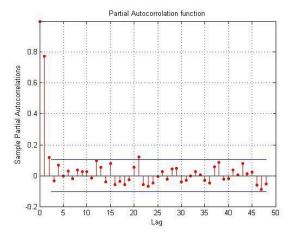


Fig 3- Partial autocorrelation coefficient

A. Data Generation

Unlimited discharge time series has been generated with using *GARCHSIM* instruction in *GARCH* toolbox of MATLAB programming as follows:

[e, s, y]=garchsim (coeff, NumSamples, NumPaths, PreInnovations, PreSigmas, PreSeries)

The final generated series consist of three terms which respectively represent: fitting error (e), the standard deviation of the fitted model (s) and generated data (y).

Here in the garchsim instruction coeff represents the fitted model coefficients (Φ_1 and Φ_2), NumSample is the number of historical data, NumPaths is the number of generated series, PreInnovation is the amount of ϵ , PreSigmas is the amount of σ and PreSeries is the standardized historical time series. Also the success of the fitted model has been investigated by the Porte Manteau Lack test on the $\boldsymbol{\mathcal{E}_{t}}$ time series with using lbqtest instruction in MATLAB

B. Data Forecasting

Data estimation for future (Lead time) is called forecasting. The target is estimating \mathbf{Z}_{t+L} data for $L \geq 1$. This has been fulfilled with using garchpred instruction in MATLAB environment as follows:

[SigmaForecast,MeanForecast] = garchpred (Spec,Series,NumPeriods)

Which Numperiods is the number of L and in this research was considered equal to 20.

III. RESULTS

A. Data Generation

The number of generated series (NumPaths) is unlimited and one of the outlets has been depicted in the form of three charts is represented in Fig 4.

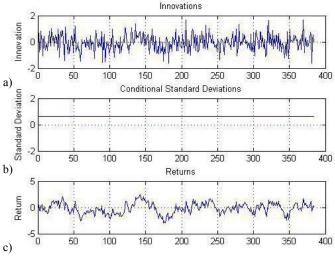


Fig 4- One sample result of the generated data from historical time series based on AR (2) model a)Generated time series b) standard deviation of the fitted model c) fitting errors

B. Data Forecasting

Forecasted data for the time lags of 1 to 20 is illustrated in Fig5.

In this research the liability of the fitted model has been investigated by forecasting each data of the historical time series based on the data one step behind then both predicted and historical time series has been depicted in one chart which shows a very strong fitness (Fig6).

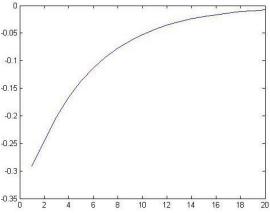


Fig 5- Forecasted data for lag times of 1 to 20

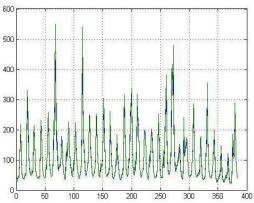


Fig 6- Historical and forecasted time series

IV. DISCUSSION

The best fitted model selection is a complicated process needing a numerical model which has been written in MATLAB environment enclosed to this article. For better selection each model has been compared based on following parameters (Table 1).

RMSEX = Root Mean Square x data

RMSEZ = Root Mean Square z data- standardized

AIC = Akaike Information Criterion

LLF = Likelihood function

Based on this comparison the best fitted models are GARCH (1,0,0,1), GARCH (1,0,1,1), GARCH (2,0,0,1) and GARCH (2,1,0,1).

As Discussed before, there are some newly developed stochastic models called ARMAX (Autoregressive Moving Average Models with Exogenous Variables) which uses other related parameters time series and historical time series correlation for better fitness and forecasting of the model.

In this research, different X time series (Precipitation, Temperature, Downstream station recorded discharge and Large-scale climate signal El-Nino with using southern Oscillation Index (SOI)) has been considered. In the numerical model (enclosed to this article), first there is a correlation process between these parameters and historical discharge of Karoon River and then all the process of data generation and forecasting has been followed.

Based on correlation factors, precipitation and SOI time series are the two most fitted parameters to the Karoon River discharge. The fitness investigation of the ARMAX model for the same ARMA model order results has been reviled in table 2 and table 3 respectively with using precipitation data and SOI data.

V. CONCLUSION

It is a common belief that in every aspect of life, having better knowledge of past, will raise the likelihood of forecasting the future. In this research it also has been proved even in hydrologic data forecasting. That is, using other related climatic parameters such as precipitation (32 years recorded data) and large scale climatic signal (SOI), has slightly improved the fitness of the forecasted Karoon river discharge.

Comparison of ARMA and ARMAX stochastic models for Karoon river time series generation and forecasting

Table1- Summarized results of the stochastic ARMA model fitting process

	30.57	Parameters			510	100	Result						
Num.	model	R	M	P	Q	X	RMESz	RMESx	LLF	Q-test	AIC	Sellected	
1	AR(1)	1	0	0	0	0	0.2224	15.5918	-363.2294	ok	-357.2855		
2	100.000	1	0	0	1	0	0.1820	13.0330	-350.1360	ok	-366.5992	1	
3		1	0	1	1	0	0.2002	14.1585	-348.8530	ok	-365.6224	1	
4		1	0	0	2	0	0.2318	15.5000	-347.6897	ok	-363.3785		
5		1	0	2	1	0	0.2108	14.1697	-348.8530	ok	-363.6059		
6		1	0	1	2	0	0.2318	15.4998	-347.6897	ok	-361.3917	- 0	
7		1	0	2	2	0	0.2318	15.5000	-347.6897	ok	-359.3853		
8	AR(2)	2	0	0	0	0	0.2427	17.0166	-360.3867	ok	-358.9714		
9		2	0	0	1	0	0.2151	15.2803	-348.1599	ok	-370.1535	1	
10		2	0	1	1	0	0.2247	15.8410	-347.1745	ok	-368.2982		
11		2	0	0	2	0	0.2375	16.5316	-346.4483	ok	-365.6152		
12		2	0	2	1	0	0.2247	15.8410	-347.1745	ok	-366.2919		
13		2	0	1	2	0	0.2375	16.5327	-346.4483	ok	-363.6169		
14		2	0	2	2	0	0.2375	16.5326	-346.4483	ok	-361.6072		
15	AR(3)	3	0	0	0	0	0.2519	17.0160	-360.6123	ok	-360.5203		
16	ANG	3	0	0	1	0	0.2272	15.3862	-347.8360	ok	-369.0207		
17		3	0	1	1	0	0.2360	15.8994	-346.8729	ok	-367.0973		
18		3	0	0	2	0	0.2471	16.5130	-346.1876	ok	-364.5436	- 1	
19	t	3	0	2	1	ō	0.2360	15.8983	-346.8729	ok	-365.0871		
20	t	3	0	1	2	0	0.2471	16.5151	-346.1876	ok	-362.5071		
21	1	3	0	2	2	0	0.2471	16.5150	-346.1876	ok	-360.5022	- 6	
22	ARMA(1,1)	1	1	0	0	0	0.2316	16.8832	-360.8183	ok	-360.1077		
23		1	1	0	1	0	0.1996	14.9231	-348.4038	ok	-369.4530		
24		1	1	1	1	0	0.2113	15.5871	-347.3720	ok	-367.7444		
25		1	1	0	2	0	0.2362	16.4110	-346.5859	ok	-365.0586		
26		1	1	2	1	0	0.2215	15.5973	-347.3720	ok	-365,7293		
27		1	1	1	2	0	0.2362	16.4106	-346.5859	ok	-363.0533		
28		1	1	2	2	0	0.2362	16.4106	-346.5859	ok	-361.0524		
29	ARMA(2,1)	2	1	0	0	0	0.2465	17.2124	-359.7235	ok	-360.3010		
30	ONWO(2,1)	2	1	0	1	0	0.2183	15.4582	-347.4398	ok	-369.9161	1	
31		2	1	1	1	Ō	0.2271	15.9305	-346.1892	ok	-368.2254		
32		2	1	0	2	0	0.2183	15.4582	-347.4398	ok	-369.9161		
33		2	1	2	1	0	0.2272	15.9683	-346.4750	ok	-366.1124		
34		2	1	1	2	0	0.2377	16.5328	-345.8009	ok	-363.8259		
35		2	1	2	2	0	0.2377	16.5314	-345.8009	ok	-361.8241		
36	ARMA(1,2)	1	2	0	0	0	0.2417	16.8444	-360.6154	ok	-358.5150		
37	ARIVIA(1,2)	1	2	0	1	0	0.2142	15.0595	-348.1077	ok	-368.4111	Ī	
38	t	1	2	1	1	0	0.2246	15.6530	-347.1037	ok	-366.5517		
39	+	1	2	0	2	0	0.2378	16.3814	-346.3618	ok	-363.8299	- i	
40	+	1	2	2	1	0	0.2246	15.6526	-347.1037	ok	-364.5369	Ē	
41	+	1	2	1	2	0	0.2378	16.3805	-346.3618	ok	-361.8436		
42	t t	1	2	2	2	0	0.2378	16.3796	-346.3618	ok	-359.8483		

Table2- Summarized results of the stochastic ARMAX model fitting process with the precipitation as the X time series

Num.	model	Parameters R	M	Р	Q	x	Result RMESz	RMESx	LLF	Q-test	aic
1	ARMAX(1,0,1)	1	0	0	0	1	0.2162	14.3474	-304.6449	ok	-472.4551
2		1	0	1	1	1	0.1886	12.8104	-289.7069	not	-478.6843
2		1	0	0	1	1	0.1852	12.6075	-289.8213	not	-479.8321
4	ARMAX(2,0,1)	2	0	0	0	1	0.2360	15.7729	-301.4020	ok	-476.9401
5		2	0	0	1	1	0.1999	13.0761	-289.6562	ok	-480.0796
6	ARMAX(3,0,1)	3	0	0	0	1	0.2451	15.7785	-301.2001	ok	-475.3447
7	ARMAX(1,1,1)	1	1	0	0	1	0.2859	20.4664	-294.6113	ok	-490.5224
8	ARMAX(1,2,1)	1	2	0	0	1	0.2975	20.9233	-294.5313	ok	-488.6823
9	ARMAX(2,1,1)	2	1	0	0	1	0.3068	21.8804	-293.0400	ok	-491.6704
10		2	1	0	1	1	0.2738	19.4562	-283.1388	ok	-495.9657

Table3- Summarized results of the stochastic ARMAX model fitting process with the SOI as the X time series

Num.	model	Parameters				Ĥ	Result				
		R	M	Р	Q	X	RMESz	RMESx	LLF	Q-test	aic
1	ARMAX(1,0,1)	1	0	0	0	1	0.2364	16.4916	-361.2118	ok	-359.3209
2		1	0	1	1	1	0.2257	15.5808	-346.3332	ok	-367.4722
2		1	0	0	1	1	0.2029	14.2349	-347.7870	ok	-369.5663
4	ARMAX(2,0,1)	2	0	0	0	1	0.2560	17.9103	-358.4972	ok	-362.7500
5		2	0	0	1	1	0.2317	16.2637	-345.9357	ok	-372.7652
6	ARMAX(3,0,1)	3	0	0	0	1	0.2649	17.9110	-358.2462	ok	-361.2512
7	ARMAX(1,1,1)	1	1	0	0	1	0.2463	17.6307	-358.6762	ok	-362.3919
8	ARMAX(1,2,1)	1	2	0	0	1	0.2556	17.6961	-358.4258	ok	-360.8936
9	ARMAX(2,1,1)	2	1	0	0	1	0.2562	18.0854	-358.3725	ok	-360.9999
10		2	1	0	1	1	0.2314	16.4434	-345.5949	ok	-371.0545

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Fitted ARMAX numerical model compression based on RMSEx, RMSEz, AIC, LLF

```
clear all
iR=input('Please input parameter R:');
iM=input('Input Parameter M:');
ip=input('Input parameter p :');
iq=input('Input parameter q :');
ix=input('Input Number of parameter(X)(1-13):');
ij=input('Input Number of parameter(X)(0-12):');
Nsim=input('Input Number of data generating:');
LeadN=input('Input Leadtime Number to forecast:');
imfd=xlsread('Strem Flow Normal Data.xls'
skimfd=skewness(imfd);
%======transform to column format==========
cif=imfd':
cimfd=[cif(:,1);cif(:,2);cif(:,3);cif(:,4);cif(:,5);cif(:,
6);cif(:,7);cif(:,8);cif(:,9);...
cif(:,10);cif(:,11);cif(:,12);cif(:,13);cif(:,14);cif(:,15
);cif(:,16);cif(:,17);..
cif(:,18);cif(:,19);cif(:,20);cif(:,21);cif(:,22);cif(:,23
);cif(:,24);cif(:,25);.
cif(:,26);cif(:,27);cif(:,28);cif(:,29);cif(:,30);cif(:,31
nmfd \hspace{-0.08cm}=\hspace{-0.08cm} \textit{nmfd} \hspace{-0.08cm}=\hspace{-0.08cm} \textit{(imfd); \$normalized monthly flow data}
sknmfd=skewness(nmfd);
       ===standardlized monthly flow data==========
mef=mean(nmfd);
sdf=std(nmfd);
smfd=[(nmfd(:,1)-
                              mef(1))./sdf(1)
                                                         (nmfd(:,2)-
(nmfd(:,10) - mef(10))./sdf(10) (nmf
mef(11))./sdf(11) (nmfd(:,12) - mef(12))./sdf(12)];
                                                          (nmfd(:,11)-
        transform to column format===
cmf=smfd':
csmfd=[cmf(:,1);cmf(:,2);cmf(:,3);cmf(:,4);cmf(:,5);cmf(:,
6);cmf(:,7);cmf(:,8);cmf(:,9);...
cmf(:,10);cmf(:,11);cmf(:,12);cmf(:,13);cmf(:,14);cmf(:,15
cmf(:,16);cmf(:,17);...
cmf(:,18);cmf(:,19);cmf(:,20);cmf(:,21);cmf(:,22);cmf(:,23)
);cmf(:,24);cmf(:,25);...
cmf(:,26);cmf(:,27);cmf(:,28);cmf(:,29);cmf(:,30);cmf(:,31
);cmf(:,32)1;
impd=xlsread('Precipitation Data.xls');% initial onmthly
precipitation data of armand station
skimpd=skewness(impd);
 %====transform to column format==
cimpd=[cip(:,1);cip(:,2);cip(:,3);cip(:,4);cip(:,5);cip(:,
6);cip(:,7);cip(:,8);cip(:,9);...
cip(:,10);cip(:,11);cip(:,12);cip(:,13);cip(:,14);cip(:,15
);cip(:,16);cip(:,17);...
\verb|cip(:,18)|; \verb|cip(:,19)|; \verb|cip(:,20)|; \verb|cip(:,21)|; \verb|cip(:,22)|; \verb|cip(:,23)|
);cip(:,24);cip(:,25);.
cip(:,26);cip(:,27);cip(:,28);cip(:,29);cip(:,30);cip(:,31
);cip(:,32)];
 %====standardlized monthly flow data============
mep=mean(impd);
sdp=std(impd);
smpd=[(impd(:,1)-
                              mep(1))./sdp(1)
                                                          (impd(:,2)-
mep(2))./sdp(2) (impd(:,3) - mep(3))./sdp(3)...
(impd(:,4) - mep(4))./sdp(4) (impd(:,5) - mep(5))./sdp(5)
(impd(:,6) - mep(6))./sdp(6)...
(impd(:,7)- mep(7))./sdp(7) (impd(:,8)- mep(8))./sdp(8) (impd(:,9)- mep(9))./sdp(9)... (impd(:,10)- mep(10))./sdp(10) (impd(:,11)-
mep(11))./sdp(11) (impd(:,12)- mep(12))./sdp(12)];
          ======transform
                                                                 column
cmp=smpd';
 \begin{array}{l} {\rm csmpd=[cmp\,(:,1)\,;cmp\,(:,2)\,;cmp\,(:,3)\,;cmp\,(:,4)\,;cmp\,(:,5)\,;cmp\,(:,6)\,;cmp\,(:,7)\,;cmp\,(:,8)\,;cmp\,(:,9)\,;...} \end{array} 
cmp(:,10);cmp(:,11);cmp(:,12);cmp(:,13);cmp(:,14);cmp(:,15
);cmp(:,16);cmp(:,17);...
{\it cmp}\,(:,18)\,; {\it cmp}\,(:,19)\,; {\it cmp}\,(:,20)\,; {\it cmp}\,(:,21)\,; {\it cmp}\,(:,22)\,; {\it cmp}\,(:,23)\,
);cmp(:,24);cmp(:,25);...
cmp(:,26);cmp(:,27);cmp(:,28);cmp(:,29);cmp(:,30);cmp(:,31
ACFcsmfd=autocorr(csmfd,length(csmfd)/4,2);
% title('Autocorrolation function')
PACFcsmfd=parcorr(csmfd,length(csmfd)/8,2);
         \#\#\# select kind of moded & set the model \#\#
garchset('R',iR,'M',iM,'Regress',[1:ix],'p',ip,'q',iq)
```

```
BB1=[csmpd(length(csmpd));csmpd(1:length(csmpd)-1)];
BB2=[BB1 (length (csmpd)); BB1 (1:length (csmpd)-1)];
BB3=[BB2(length(csmpd));BB2(1:length(csmpd)-1)];
BB4=[BB3(length(csmpd));BB3(1:length(csmpd)-1)];
BB5=[BB4(length(csmpd));BB4(1:length(csmpd)-1)];
BB6=[BB5(length(csmpd));BB5(1:length(csmpd)-1)];
BB7 = [BB6(length(csmpd)); BB6(1:length(csmpd)-1)];
BB8=[BB7(length(csmpd));BB7(1:length(csmpd)-1)];
BB9=[BB8(length(csmpd));BB8(1:length(csmpd)-1)];
BB10=[BB9(length(csmpd));BB9(1:length(csmpd)-1)];
BB11 = [BB10 (length (csmpd)); BB10 (1:length (csmpd)-1)];
BB12=[BB11(length(csmpd));BB11(1:length(csmpd)-1)];
precipsc=[csmpd BB1 BB2 BB3 BB4 BB5 BB6 BB7 BB8 BB9 BB10 BB11
BB12];
%############ Parameter Estimation ######################
[coeff11000,error11000,LLF11000,innovations11000,sigmas110
00,summary11000]
 =garchfit(spec,csmfd,precipsc(:,ij+1:ij+ix));
 [coeff11000,error11000,LLF11000,innovations11000,sigmas110
00,summary110001
 garchfit(spec,csmfd,precipsc(:,ij+1:ij+ix),innovations110
00,sigmas11000,csmfd);
 garchdisp(coeff11000,error11000);
%########## Simulation ######################
[e11000,s11000,y11000] =
garchsim(coeff11000,384,Nsim,[],precipsc(:,ij+1:ij+ix),[],
innovations11000, sigmas11000, csmfd);
 [SigmaForecastA11000,MeanForecastA11000] =
garchpred(coeff11000,csmfd,LeadN,precipsc(:,ij+1:ij+ix));
nn=[iR iM ip iq ix];
for j=max(nn):length(csmfd)
[SigmaForecastB11000(j), MeanForecastB11000(j)] = garchpred(c
oeff11000,csmfd(1:j),1,precipsc(:,ij+1:ij+ix
hold on
end
 [H11000,pValue11000,Stat11000,CriticalValue11000]
=lbqtest(innovations11000,[10 30 50]',0.05);
[H11000 pValue11000 Stat11000 CriticalValue11000]
d=length(csmfd)-length(MeanForecastB11000);
Error=csmfd(1+d:end) -MeanForecastB11000';
SSE=sum(Error.^2);
LMSE=SSE/length(Error);
RMSEz=LMSE^0.5
a=MeanForecastB11000';
b=[a(1:12) a(13:24) a(25:36) a(37:48) a(49:60) a(61:72)
D-[a11.12] a(13.24) a(23.36) a(37.48) a(49.00) a(01.72) a(73.84) a(85:96)...
a(97:108) a(109:120) a(121:132) a(133:144) a(145:156) a(157:168) a(169:180)...
a(181:192) a(193:204) a(205:216) a(217:228) a(229:240)
a(241:252) a(253:264)...
a(265:276) a(277:288) a(289:300) a(301:312) a(313:324)
a(325:336) a(337:348)...
a(349:360) a(361:372) a(373:384)];
c=b';
t=[(c(:,1)*sdf(1))+mef(1)
(c(:,3)*sdf(3))+mef(3)...
                                        (c(:,2)*sdf(2))+mef(2)
(c(:,4)*sdf(4))+mef(4)
                                        (c(:,5)*sdf(5))+mef(5)
(c(:,6)*sdf(6))+mef(6)...
(c(:,7)*sdf(7))+mef(7)
                                        (c(:,8)*sdf(8))+mef(8)
(c(:,9) *sdf(9))+mef(9)...
(c(:,10) *sdf(10)) +mef(10)
                                     (c(:,11) *sdf(11)) +mef(11)
(c(:,12)*sdf(12))+mef(12)];
tt=10.^t;
ttt=sum((imfd-tt).^2);
tttt=sum(ttt);
\label{eq:loss_theory}    \texttt{ttttt=tttt/length\,(MeanForecastB11000)\,;}    \\    \texttt{RMSEx=ttttt}.5 
citt=tt';
ctt=[citt(:,1);citt(:,2);citt(:,3);citt(:,4);citt(:,5);cit
t(:,6);citt(:,7);citt(:,8);citt(:,9);.
citt(:,10);citt(:,11);citt(:,12);citt(:,13);citt(:,14);cit
t(:,15);citt(:,16);citt(:,17);.
citt(:,18);citt(:,19);citt(:,20);citt(:,21);citt(:,22);cit
t(:,23);citt(:,24);citt(:,25);.
citt(:,26);citt(:,27);citt(:,28);citt(:,29);citt(:,30);cit
t(:,31);citt(:,32)];
arid on
                     'DisplayName',
plot(ctt,
 YDataSource', 'MeanForecast');
hold all:
plot(cimfd, 'DisplayName', 'csmfd', 'YDataSource', 'csmfd');
hold off; figure (gcf)
[AIC11000.BIC110001
                                              aicbic(LLF11000,
garchcount(coeff11000),384);
LLF11000
AIC11000
BIC11000
aic11000=384*log(mean(sigmas11000)^2)+2*garchcount(coeff11
```