Sensitivity Analysis of 3- PRR Planar Manipulator

Shaik Himam Saheb, G.Satish Babu

Abstract— **This paper deals with the sensitivity analysis of 3-PRR planar parallel manipulators (PPMs). First, the sensitivity coefficients of the pose of the manipulator moving platform to variations in the geometric parameters and in the actuated variables are expressed algebraically. Moreover, two aggregate sensitivity indices are determined, one related to the orientation of the manipulator moving platform and another one related to its position. Then, a methodology is proposed to compare 3-PRR PPMs with regard to their dexterity, workspace size and sensitivity. Finally, the sensitivity of a 3-PRR PPM is analyzed in detail**

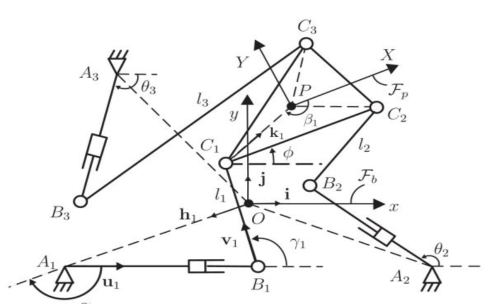
I. Introduction

Variations in the geometric parameters of parallel kinematics machines PKMs can be either compensated or amplified. For that reason, it is important to analyze the sensitivity of the mechanism performance to variations in its geometric parameters. For instance, Wang and Masory 1 studied the effect of manufacturing tolerances on the accuracy of a Stewart platform. Kim and Choi 2 used a forward error bound analysis to find the error bound of the end-effector of a Stewart platform when the error bounds of the joints are given, and an inverse error bound analysis to determine those of the joints for the given error bound of the end-effectors. Kim and Tsai 3 studied the effect of misalignment of linear actuators of a three-degree of freedom 3DOF translational parallel manipulator on the motion of its moving platform. Caro et al. 4 developed a tolerance synthesis method for mechanisms based on a robust design approach. Caro et al. 5 proposed two indices to evaluate the sensitivity of the end-effector pose position+ orientation of Ortho glide three-axis, a 3DOF translational PKM, to variations in its design parameters. Besides, they noticed that the better the dexterity, the higher the accuracy of the manipulator. However, Yu et al. 6 claimed that the accuracy of a3DOF planar parallel manipulator PPM is not necessarily related to its dexterity. Meng et al. 7 proposed a method to analyze the accuracy of parallel manipulators with joint clearance and obtained a standard convex optimization problem to evaluate the maximal pose error in a prescribed work space. This paper deals with the sensitivity analysis of 3DOF PPMs to variations in their geometric parameters and actuated joints. Without loss of generality, we focus on the sensitivity analysis of the3-RPR manipulator within the framework of this paper. The singularities of this manipulator were analyzed. Here, we used a methodology to derive the sensitivity coefficients of the moving platform pose to variations in the geometric parameters in algebraic form. The underlying methodology was applied to derive the sensitivity coefficients of other PPMs such as 3-RPR,3-RRR, 3-RRR, and 3-PRR PPMs. First, the architecture of the manipulator is described. Then, the sensitivity coefficients of the moving platform pose to variations in the geometric parameters and in the prismatic actuated variables are expressed algebraically. Moreover, two aggregate sensitivity indices are determined, one related to the orientation of the manipulator moving platform and another one related to its position. Then, a methodology is proposed to compare 3- PRR PPMs with regard to their dexterity, workspace size, and sensitivity. Finally, the sensitivity of an arbitrary 3- PRR PPMs is analyzed in detail and four 3- PRR PPMs are compared as illustrative example

Shaik Himam Saheb, Department of Mechanical Engineering, GNIT, Hyderabad
G.Satish Babu, Department of Mechanical Engineering, JNTUCEH, Hyderabad

II. Manipulator Architecture

Here and throughout this paper, R, P, and P denote revolute, prismatic, and actuated prismatic joints, respectively. Figure 1 illustrates the architecture of the manipulator under study. It is composed of a base and a moving platform MP connected by means of three legs. Points *A*1, *A*2, and *A*3, *C*1, *C*2, and *C*3, respectively lie at the corners of a triangle, of which point *O* point *P*, respectively is the circum center. Each leg is composed of a R, a P, and a R joint in sequence. The three P joints are actuated. Accordingly, the manipulator is named 3-RPR manipulator. *Fb* and *Fp* are the base and the moving platform frames of the manipulator. In the scope of this paper, *Fb* and *Fp* are supposed to be orthogonal. *Fb* is defined with the orthogonal dihedron ***O****x*,***O****y*, point *O* being its center and ***O****x* parallel to segment *A* 1*A*2. Likewise, *Fp* is defined with the orthogonal dihedron ***PX***,***PY***, point *C* being its center and ***PX*** parallel to segment *C*1*C*2. The manipulator MP pose, i.e., its position and its orientation, is determined by means of the Cartesian coordinates vector **p**=*px*, *py T* of operation point *P* expressed in frame *Fb* and angle , namely, the angle between frames *Fb* and *Fp*. Finally, the passive joints do not have any stop.



(c) 3-PRR PPM

Figure 1: PPMs under study

Sensitive indices:

The pose errors of the manipulators MP depends on variations in geometric parameters as well as on the manipulator configuration. In order to analyse the influence of manipulator configuration on those errors. The indices are formulated to assess the aggregate sensitivity of MP pose to variation in the geometric parameters

$$\begin{bmatrix} \delta\phi \\ \delta p \end{bmatrix} = \mathbf{J_s}[\delta a_i \ \ \delta p_i \ \ \delta c_i]^T$$

$\delta\phi$ is error in orientation

a_i, p_i, c_i = link lengths

δp is error in position

$\mathbf{J_s} = [\mathbf{J_a} \ \ \mathbf{J} \ \ \mathbf{J_c}]$

$\boldsymbol{\delta a_i} = [\ \delta a_{1x} \ \ \delta a_{1y} \ \ \delta a_{2x} \ \delta a_{2y} \ \ \delta a_{3x} \ \ \delta a_{3y}\ ]$

$\boldsymbol{\delta p_i} = [\delta p_1 \ \ \delta p_2 \ \ \delta p_3]$

$\boldsymbol{\delta c_i} = [\delta c_{1x} \ \ \delta c_{1y} \ \ \delta c_{2x} \ \delta c_{2y} \ \ \delta c_{3x} \ \ \delta c_{3y}\ ]$

Now **Js** is a 3 x 15 matrix which can be written as

$$\mathbf{Js} = \begin{bmatrix} j_{s_\phi} \\ J_{s_p} \end{bmatrix} \begin{bmatrix} j_{s_\phi} \\ J_{s_p} \end{bmatrix}$$

With

$$j_{s_\phi} = [\ \mathbf{j}_{A_1\phi} \ \mathbf{j}_{A2\phi} \ \mathbf{j}_{A_3\phi} \ j_{1\phi} \ \ j_{2\phi} \ \ j_{3\phi} \ \mathbf{j}_{C_1\phi} \ \mathbf{j}_{C_2\phi} \ \mathbf{j}_{C_3\phi}\ ]$$

$$J_{s_p} = [\ \mathbf{J}_{A_2p} \ \mathbf{J}_{A_3p} \ j_{1p} \ \ j_{2p} \ \ j_{3p} \ \ \mathbf{J}_{C_1p} \ \mathbf{J}_{C_2p} \ \mathbf{J}_{C_3p}\ ]$$

The aggregate sensitivity index v_ϕ of the orientation of the MP of the manipulator to variations in its geometric parameters and revolute actuated joint , namely

$$v_\phi = \frac{\left\| J_{s_\phi} \right\|_2}{n_v}$$

n_v = no.of variations that are considered.

Likewise form the above equations, an aggregate sensitivity index v_p of the position of the 3MP of th manipulator to the variations in its geometric parameters and revolute actuated joints.

$$v_p = \frac{\left\| J_{s_p} \right\|_2}{n_v}$$

For any given manipulator configuration , lower the v_ϕ ,the lower the overall sensitivity of the orientation its MP to variations in the geometric parameters. Similarly , lower the v_p the overall sensitivity of the MP positions to variations in the geometric parameters.

Sensitivity analysis of a general 3-RRR PPM:

The geometric parameters of 3-RRR PPM are below

$a_1 = a_2 = a_3 = R_1 = 0.60$

$c_1 = c_2 = c_3 = R_2 = 0.25$

$\{ \alpha_1, \alpha_2, \alpha_3\} = \{-2.40,-0.56,2.20\}$

$\{\beta_1, \beta_2, \beta_3\} = \{-2.85,-0.24,0.70\}$

α_i and β_i i= 1,2,3 being expressed in radiance and R_1 and R_2 are in ,meters

Orientation sensitivity indices:

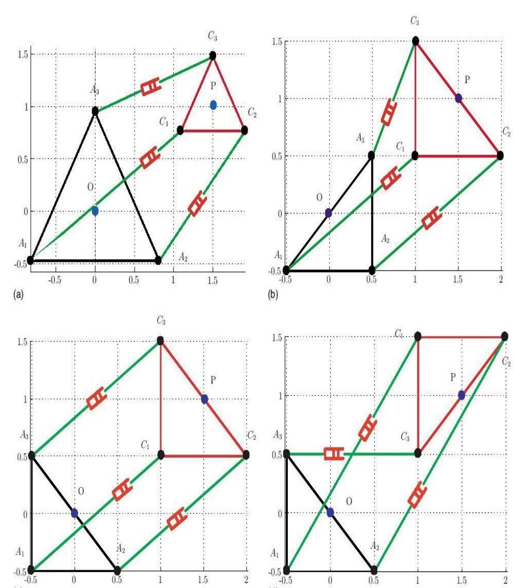
These are related to the variations in coordinates of A_i ,C_i and in $\boldsymbol{p}_i$ defined with equations $v_{\phi A_i} = \left\| \mathbf{j}_{A_i\phi} \right\|_2$, $v_{\phi C_i} = \left\| \mathbf{j}_{C_i\phi} \right\|_2$, $v_{\phi p_i} = \left| j_{i\phi} \right|$ are smaller than 3rad/m and 6rad/m ,respectively for the three bars above indices $v_{\phi q_i}$,i= 1,2,3. Are associated with the sensitivity of MP orientation ϕ to variations' in q_1 ,q_2 ,q_3 respectively for the first set of three bars , q_i stands for A_i for the second set of three bars , q_i stands for p_i. .For the first set of three bars , q_i stands for C_i it is apparent that higher the bar ,the smaller the sensitivity of MP orientation to variations in the corresponding geometric parameters or variables.

Position sensitivity indices:

These are related to the variations in coordinates of A_1 ,C_i and in $\boldsymbol{p}_i$ defined with equations $v_{pA_i} = \left\| \mathbf{J}_{A_ip} \right\|_2$, $v_{pC_i} = \left\| \mathbf{J}_{C_ip} \right\|_2$, $v_{pp_i} = \left\| \mathbf{J}_{ip} \right\|_2$ are smaller than 3rad/m and 6rad/m ,respectively for the three bars above indices v_{pq_i} ,i= 1,2,3. Are associated with the sensitivity of MP orientation ϕ to variations in q_1 ,q_2 ,q_3 respectively for the first set of three bars , q_i stands for A_i for the second set of three bars , q_i stands for p_i. .For the first set of three bars , q_i stands for C_i the higher the bar ,the smaller the sensitivity of MP position to variations in the corresponding geometric parameters or variables.

Manipulator under study:

If we consider different manipulators under study the below figure 2 illustrates 4 manipulators named as M_1,M_2,M_3 and M_4 respectively ,M_1 and M_2 are non degenerate, where as M_3 and M_4 are degenerate the base and moving platform of M_1 are equilateral . the base and moving platform of M_2 are identical but in a different geometric configuration for an orientation ϕ = 0



The four 3-RPR manipulators under study with ϕ=0 and p= [1, 1.5] T: (a) and (b) non degenerate manipulators and(c) and (d) degenerate manipulators

The Below isocontour figures of v_ϕ and v_p respectively illustrates the maximum value for a given orientation ϕ of MP throughout the RDW of M_1. M_1 has the least sensitive position of its MP to variations in geometric parameters. On the other side, M_1 has the most sensitive position of its MP to variations in geometric parameters.

Results: Following results are obtained from Matlab 2015a programe

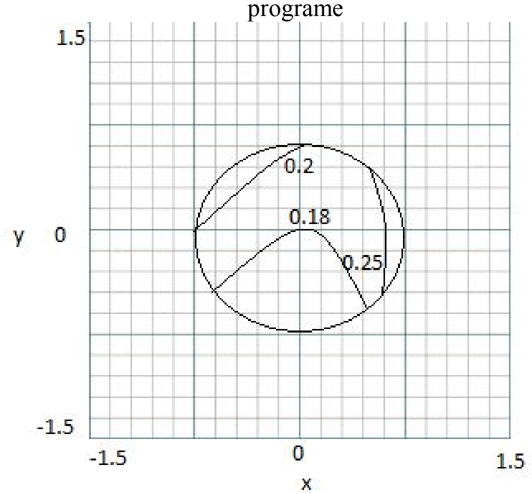


Figure v_p isocontour of Mechanism M_1 (non degenerate)

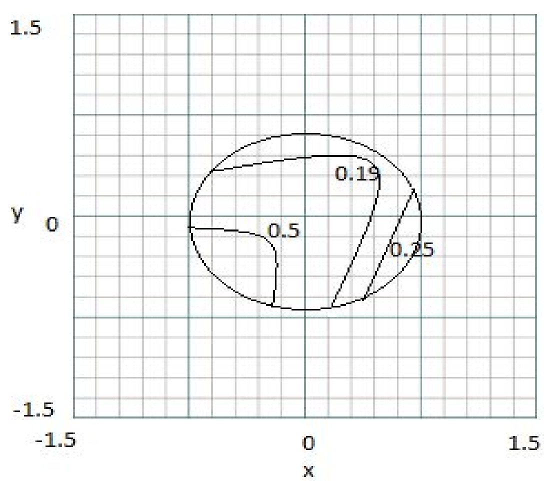


Figure v_ϕ isocontour of Mechanism M_1 (non degenerate)

CONCLUSIONS

Their sensitivity coefficients were derived and expressed algebraically. Moreover, two aggregate sensitivity indices were determined for each manipulator under study, one related to the orientation of the moving platforms of the manipulator and another one related to their position. Then, a methodology was proposed to compare planar parallel manipulators with regard to their workspace size and sensitivity. Finally, the sensitivity of 3- PRR Planar Parallel Manipulators, were compared as illustrative examples. this paper should help the designer of planar parallel manipulators at the conceptual design stage. Joint clearances and flexibilities also affect the positioning accuracy. The sensitivity to joint clearances and flexibilities in the revolute joints can be taken into account in the definition of the variations in the positions of the revolute joint centers. Prismatic joint clearances and link flexibilities will be studied in future work, considering also spatial manipulators.

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