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Quotient Intuitionistic Fuzzy Topology

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Abstract— **In this paper intuitionistic fuzzy quotient topology and intuitionistic fuzzy quotient map are introduced. It reveals the relation among intuitionistic fuzzy quotient map, intuitionistic fuzzy continuous function, and intuitionistic fuzzy homeomorphism and some of their properties are discussed.**

Index Terms— **Intuitionistic fuzzy quotient topology, intuitionistic fuzzy quotient map**

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I. INTRODUCTION

L. A. ZADEH [12] in 1965 introduced the notion of fuzzy set to describe vagueness mathematically in its very abstractness and tried to solve such problems by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set. This grade corresponds to the degree to the fuzzy set. After the introduction of the concept of fuzzy sets by L. A. ZADEH [12] several researches were conducted on the generalizations of the notion of fuzzy set. The idea of "intuitionistic fuzzy set" was first published by ATANASSOV [3]. A study of local compactness and the quotient fuzzy topology was started in [10, 11]. In [10, 11] WONG introduced the quotient fuzzy topology on the set of classes of an equivalence relation on a fuzzy topological space. F. T. CHRISTOPH generalized WONG's quotient fuzzy topology in [6]. The purpose of this paper is to investigate the quotient intuitionistic fuzzy topology.

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1. PRELIMINARIES

In this section, some basic definitions like intuitionistic fuzzy set, intuitionistic fuzzy point, intuitionistic fuzzy continuous and intuitionistic fuzzy homeomorphism have been recalled.

DEFINITION 1.1 [8]

Any surjective map f satisfying the condition U is open in $Y \Leftrightarrow f^{-1}(U)$ is open in X, is called a **quotient map**.

DEFINITION 1.2 [5]

Let X be a non-empty set and I be the unit interval. A fuzzy set in X is an element of the set I^{X} of all functions from X to I.

DEFINITION 1.3 [4]

Let X be a non- empty fixed set. An intuitionistic fuzzy set (IFS for short) A is an object having the form $A = \{\langle x, \mu_A(x), \gamma_A(x) : x \in X \rangle\}$ where the function $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for each $x \in X$.

DEFINITION 1.4 [4]

Let X be a non-empty set and the IFSs A and B be in the form $A = \{\langle x, \mu_A(x), \gamma_A(x) : x \in X \rangle\}$, $B = \{\langle x, \mu_B(x), \gamma_B(x) : x \in X \rangle\}$. Then

$A \subseteq B$ iff $\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$ for all $x \in X$;

A = B iff $A \subseteq B$ and $B \subseteq A$;

$\overline{A} = \{\langle x, \gamma_A(x), \mu_A(x) : x \in X \rangle\}$;

$A \quad B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x) : x \in X \rangle\}$;

$A \quad B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x) : x \in X \rangle\}$;

$[\ ]A = \{\langle x, \mu_A(x), 1 - \mu_A(x) : x \in X \rangle\}$;

$\langle\ \rangle A = \{\langle x, 1 - \gamma_A(x), \gamma_A(x) : x \in X \rangle\}$.

DEFINITION 1.5 [7]

Let X be a non-empty fixed set. Then $0_\sim = \{\langle x, 0, 1 \rangle : x \in X\}$ and $1_\sim = \{\langle x, 1, 0 \rangle : x \in X\}$.

DEFINITION 1.6 [7]

Let X and Y be two non-empty fixed sets and $f : (X, T) \to (Y, S)$ be a function. Then

(a) if $B = \{\langle x, \mu_B(y), \gamma_B(y) : y \in Y \rangle\}$ is an IFS in Y, then the pre image of B under f, denoted by $f^{-1}(B)$, is the intuitionistic fuzzy set in X defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x) : x \in X \rangle\}$.

(b) if $A = \{\langle x, \lambda_A(x), \gamma_A(x) : x \in X \rangle\}$ is an IFS in X, then the image of A under f, denoted by $f(A)$, is the intuitionistic fuzzy set in Y defined by $f(A) = \{\langle y, f(\lambda_A)(y), (1 - f(1 - \gamma_A))(y) : y \in Y \rangle\}$ where,

$$f(\lambda_A)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \lambda_A(x) & \text{if } f^{-1}(y) \neq 0 \\ 0, & \text{otherwise.} \end{cases}$$

$$(1 - f(1 - \gamma_A))(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \gamma_A(x) & \text{if } f^{-1}(y) \neq 0 \\ 1, & \text{otherwise.} \end{cases}$$

for the IFS $A = \{\langle x, \mu_A(x), \gamma_A(x) : x \in X \rangle\}$.

DEFINITION 1. 7 [7]

An IFT on a non-empty set X is a family T of IFSs in X satisfying the following axioms:

(T_1) $0_\sim, 1_\sim \in T$,

(T_2) $G_1 \quad G_2 \in T$ for any $G_1, G_2 \in T$,

(T_3) $\quad G_i \in T$, for any arbitrary family $\{G_i : i \in J\} \subseteq T$.

In this case the pair (X, T) is called an **intuitionistic fuzzy topological space** (IFTS for short) and any intuitionistic fuzzy set in T is known as an **intuitionistic fuzzy open set** (IFOS for short) in X. The complement of an IFOS is called an **intuitionistic fuzzy closed set** (IFCS for short) in X.

DEFINITION 1.8 [7]

Let (X, T) and (Y, S) be two IFTSs and let $f:(X,T)\to(Y,S)$ be a function. Then f is said to be an **intuitionistic fuzzy continuous** iff the pre image of each IFS in S is an IFS in T.

DEFINITION 1.9 [7]

Let (X, T) and (Y, S) be two IFTSs and let $f:(X,T)\to(Y,S)$ be a function. Then f is said to be an **intuitionistic fuzzy open** iff the image of each IFS in T is an IFS in S.

The complement of an intuitionistic fuzzy open set is said to be an intuitionistic fuzzy closed.

DEFINITION 1.10 [9]

Let f be a bijection mapping from an IFTS (X, τ) into an IFTS (X, σ). Then f is said to be **intuitionistic fuzzy homeomorphism** (IF homeomorphism in short) if f and f^{-1} are intuitionistic fuzzy continuous mappings.

DEFINITION 1.11 [2]

Let (X, T) be an intuitionistic fuzzy topological space and A be an intuitionistic fuzzt set in (X, T). Then, if a family $\left\{\left\langle x,\mu_{G_i},\gamma_{G_i}\right\rangle : i\in J\right\}$ of an intuitionistic fuzzy open sets in (X, T) satisfies the condition $A\subseteq \left\{\left\langle x,\mu_{G_i},\gamma_{G_i}\right\rangle : i\in J\right\}$, then it is called an **intuitionistic fuzzy open cover** of A. A finite subfamily of an intuitionistic fuzzy open cover $\left\{\left\langle x,\mu_{G_i},\gamma_{G_i}\right\rangle : i\in J\right\}$ of A, which is also an intuitionistic fuzzy open cover of A is called a finite subcover of $\left\{\left\langle x,\mu_{G_i},\gamma_{G_i}\right\rangle : i\in J\right\}$.

an intutionistic fuzzy set $A=\left\langle x,\mu_A,\gamma_A\right\rangle$ in an intuitionistic fuzzy topological space (X, T) is called an **intuitionistic fuzzy compact** iff every intuitionistic fuzzy open cover of A has finite subcover.

DEFINITION 1.12 [1]

An IFTS (X, τ) is called **Hausdorff** if for all pair of distinct intuitionistic fuzzy points $x_{(\alpha,\beta)}$, $y_{(\gamma,\delta)}$ in X , $\exists\, U, V\in\tau$ such that $x_{(\alpha,\beta)}\in U$, $y_{(\gamma,\delta)}\in V$ and $U \quad V = 0_{\sim}$.

2. QUOTIENT INTUITIONISTIC FUZZY TOPOLOGY

In this chapter, intuitionistic fuzzy quotient topology and intuitionistic fuzzy quotient map are introduced and some of their properties are discussed.

DEFINITION 2.1

Let (X, T) be an intuitionistic fuzzy topological space, Y a set, and $f: X\to Y$ a surjection. Then IF-quotient topology for Y is the intuitionistic fuzzy topology whose intuitionistic fuzzy open sets are $\left\{B : f^{-1}[B]\in T\right\}$. With this IF-quotient topology Y is called an IF-quotient topological space.

EXAMPLE 2.1

Let $X=\{a,b,c\}$ and let $A=\left\langle x,\left(\frac{a}{0.5},\frac{b}{0.5},\frac{c}{0.4}\right),\left(\frac{a}{0.2},\frac{b}{0.4},\frac{c}{0.4}\right)\right\rangle$

$B=\left\langle x,\left(\frac{a}{0.5},\frac{b}{0.6},\frac{c}{0.4}\right),\left(\frac{a}{0.2},\frac{b}{0.3},\frac{c}{0.3}\right)\right\rangle$ be intuitionistic fuzzy sets in X. Then the family $T=\{0_{\sim},1_{\sim},A,B\}$ is an intuitionistic fuzzy topology on X. Clearly, the ordered pair (X, T) is an intuitionistic fuzzy topological space. Let $Y=\{u,v,w\}$ be a set. Then the fami $S=\{0_{\sim},1_{\sim},C,D\}$ is an intuitionistic fuzzy topology on Y, where $C=\left\langle y,\left(\frac{u}{0.4},\frac{v}{0.5},\frac{w}{0.5}\right),\left(\frac{u}{0.4},\frac{v}{0.2},\frac{w}{0.4}\right)\right\rangle$

and $D=\left\langle y,\left(\frac{u}{0.4},\frac{v}{0.5},\frac{w}{0.6}\right),\left(\frac{u}{0.3},\frac{v}{0.2},\frac{w}{0.3}\right)\right\rangle$

Define a mapping $f : X \to Y$ by $f(a) = v, f(b) = w, f(c) = u$. Then clearly S is an IF-quotient topology for Y. Hence, (Y, S) is an IF-quotient topological space.

DEFINITION 2.2

If $f : X \to Y$ is an intuitionistic fuzzy continuous surjection of intuitionistic fuzzy topological space and Y has the IF-quotient topology, then f is called an IF-quotient map.

EXAMPLE 2.2

Let $X = \{a, b, c\}$, let $Y = \{u, v, w\}$ and let $A = \left\langle x, \left(\frac{a}{0.6}, \frac{b}{0.4}, \frac{c}{0.7}\right), \left(\frac{a}{0.4}, \frac{b}{0.6}, \frac{c}{0.3}\right)\right\rangle$,

$B = \left\langle x, \left(\frac{a}{0.3}, \frac{b}{0.2}, \frac{c}{0.6}\right), \left(\frac{a}{0.7}, \frac{b}{0.8}, \frac{c}{0.4}\right)\right\rangle$,

$C = \left\langle y, \left(\frac{u}{0.6}, \frac{v}{0.7}, \frac{w}{0.4}\right), \left(\frac{u}{0.4}, \frac{v}{0.3}, \frac{w}{0.6}\right)\right\rangle$ and

$D = \left\langle y, \left(\frac{u}{0.3}, \frac{v}{0.6}, \frac{w}{0.2}\right), \left(\frac{u}{0.7}, \frac{v}{0.4}, \frac{w}{0.8}\right)\right\rangle$

be intuitionistic fuzzy sets in X and Y respectively. Then the family $T = \{0_\sim, 1_\sim, A, B\}$ and $S = \{0_\sim, 1_\sim, C, D\}$ are intuitionistic fuzzy topologies on X and Y respectively. Define a mapping $f : X \to Y$ by $f(a) = u, f(b) = w, f(c) = v$. Then clearly S is an IF-quotient topology for Y and f is an intuitionistic fuzzy continuous surjective. Hence f is an IF-quotient map.

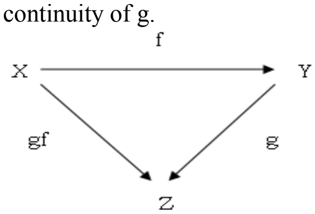
DEFINITION 2.3

Let X be an IFTS. A decomposition D of X is a collection of disjoint subsets of X whose union is $1_\sim$. We define an intuitionistic fuzzy topology on D by $S \subset D$ is intuitionistic fuzzy open $\Leftrightarrow$ ${}_{S \in S} S$ is intuitionistic fuzzy open in X. With this intuitionistic fuzzy topology D is called a decomposition space of X or IF-quotient spaces of X.

There is a natural surjective map $p : X \to D$ that takes a point $x \in X$ to the set $S \in D$ that contains x.

PROPOSITION 2.1

Let $f : X \to Y$ be an intuitionistic fuzzy continuous surjection of IFTS. Then f is an IF-quotient map if and only if for each IFTS Z and each function $g : Y \to Z$, the intuitionistic fuzzy continuity of the composition gf implies the intuitionistic fuzzy continuity of g.



Proof

$(\Rightarrow)$ Let $g : Y \to Z$ be a function of IFTS such that $gf : X \to Z$ is intuitionistic fuzzy continuous. If U is an intuitionistic fuzzy open set in Z, then $(gf)^{-1}[U] = f^{-1}\left[g^{-1}[U]\right]$. Thus $f^{-1}\left[g^{-1}[U]\right]$ is an intuitionistic fuzzy open in X and hence $g^{-1}[U]$ is an intuitionistic fuzzy open in Y since f is an IF-quotient map. Therefore g is an intuitionistic fuzzy continuous.

$(\Leftarrow)$ Let Z be the IFTS whose set is Y with the IF-quotient topology determined by h = if, where $i : Y \to Z$ is the identity function. Then h is an intuitionistic fuzzy continuous and hence i is an intuitionistic fuzzy continuous. Also, $i^{-1}h = i^{-1}if = f$ is intuitionistic fuzzy continuous and thus i^{-1} is an intuitionistic fuzzy continuous by the first half of the proof. Therefore i is an IF homeomorphism and f is an IF-quotient map.

PROPOSITION 2.2

If $f : X \to Y$ is an intuitionistic fuzzy continuous and intuitionistic fuzzy open (intuitionistic fuzzy closed) surjection of IFTS (X, T_1), (X, T_2), then f is an IF-quotient map.

Proof

If $B \in T_2$, then $f^{-1}[B] \in T_1$ since f is an intuitionistic fuzzy continuous. Conversely, if U is an intuitionistic fuzzy set in Y such that $f^{-1}[U] \in T_1$, then $f\left[f^{-1}[U]\right] = U \in T_2$. Thus f is an IF-quotient map. The proof is similar if f is an intuitionistic fuzzy closed.

PROPOSITION 2.3

Let $f : X \to Y$ be an intuitionistic fuzzy continuous surjection of IFTS. If there exists an intuitionistic fuzzy continuous function $g : Y \to X$ such that fg is the identity on Y, then f is an IF-quotient map.

Proof

Let B be an intuitionistic fuzzy set in Y such that $f^{-1}[B]$ is an intuitionistic fuzzy open, then $B = (fg)^{-1}[B] = g^{-1}\left[f^{-1}[B]\right]$ which is an intuitionistic fuzzy open in Y. Thus f is an IF-quotient map.

PROPOSITION 2.4

Let $f : X \to Y$ be an IF-quotient map. A surjection $g : Y \to Z$ is an IF-quotient map if and only if $gf : X \to Z$ is an IF-quotient map.

Proof

$(\Rightarrow)$ The map $g\ \ f$ is surjective because f and g are surjective. A subset $U \subset Z$ is an intuitionistic fuzzy open if and only if $g^{-1}(U) \subset Y$ is an intuitionistic fuzzy open, which is an intuitionistic fuzzy open if and only if $(g\ \ f)^{-1}(U)$ is intuitionistic fuzzy open in X. Hence, $g\ \ f$ is an IF-quotient map.

$(\Leftarrow)$ By **Proposition 2.1**, $g : Y \to Z$ is an IF-quotient map

PROPOSITION 2.5

Let $q : X \to Q$ be an IF-quotient map of topological spaces. Let Y be an IFTS and $f : X \to Y$ such that $q(a) = q(b) \Rightarrow f(a) = f(b)$ holds for all $a, b \in X$. Then there is a unique map $\bar{f} : Q \to Y$ such that $f = \bar{f}\ \ q$.

Proof

Define $\bar{f} : Q \to Y$; $q(x)\ \ \ f(x)$. Then $\bar{f}$ is well-defined, since for all $x, y \in X$ with $q(y) = q(x)$, we have $f(y) = f(x)$. This $\bar{f}$ satisfies $f = \bar{f}\ \ q$. It is also an intuitionistic fuzzy continuous, because for any intuitionistic fuzzy open $U \subset Y$, $q^{-1}(\bar{f}^{-1}(U)) = f^{-1}(U)$ is an intuitionistic fuzzy open. By definition of the IF-quotient topology on Q, this means that $\bar{f}^{-1}(U)$ is an intuitionistic fuzzy open and therefore $\bar{f}$ is an intuitionistic fuzzy continuous. It is unique, since all $\bar{x}$ should be sent to $f(x)$ in order to get $f = \bar{f}\ \ q$.

Corollary 2.1

Let X, Y, Z be IFTS and $f : X \to Y$, $g : X \to Z$ are IF-quotient maps such that for all $a, b \in X$, $f(a) = f(b) \Leftrightarrow g(a) = g(b)$. Then there is a unique IF homeomorphism $h : Y \to Z$ such that $g = h\ \ f$. This h sends $f(x)$ to $g(x)$.

Proof

By **proposition 2.5**, there is unique $\bar{g} : Y \to Z$ such that $g = \bar{g}\ \ f$. This $\bar{g}$ sends $f(x)$ to $g(x)$. By **proposition 2.5**, there is also a map $\bar{f} : Z \to Y$, $g(x)\ \ \ f(x)$. We can see that $\bar{f}$ is the inverse map of $\bar{g}$. Hence, g is an IF homeomorphism.

PROPOSITION 2.6

Let $f : X \to Z$ be a surjective intuitionistic fuzzy continuous map. Set $D = \{f^{-1}(z) / z \in Z\}$ and it is an IF-quotient topology. Let $p : X \to D$ be the natural map. The map f induces an intuitionistic fuzzy continuous bijection $g : D \to Z$. Moreover g is an IF homeomorphism $\Leftrightarrow$ f is an IF-quotient map.

Proof

Clearly f induces a bijection $g : D \to Z$ and $g\ \ p = f$ is an intuitionistic fuzzy continuous. Then by **Proposition 2.1** g is an intuitionistic fuzzy continuous.

$(\Rightarrow)$ Assuming g is an IF homeomorphism, $U \subset Z$ is an intuitionistic fuzzy open set in Z $\Leftrightarrow$ $g^{-1}(U)$ is an intuitionistic fuzzy open in D $\Leftrightarrow$ $f^{-1}(U) = p^{-1}(g^{-1}(U))$ is an intuitionistic fuzzy open in X. Thus f is an IF-quotient map.

$(\Leftarrow)$ Assuming f is an IF-quotient map, $S \subset D$ is an intuitionistic fuzzy open set in D $\Leftrightarrow$ $p^{-1}(S)$ is an intuitionistic fuzzy open in X. But $p^{-1}(S) = f^{-1}(g(S))$, so S is an intuitionistic fuzzy open in D $\Leftrightarrow$ $g(S)$ is an intuitionistic fuzzy open in Z. Thus $g^{-1}: Z \to D$ is an intuitionistic fuzzy continuous and g is an IF homeomorphism.

COROLLARY 2.2

Let $f: X \to Z$ be a surjective intuitionistic fuzzy continuous map. Set $D = \{f^{-1}(z) / z \in Z\}$ and it is the IF-quotient topology. If X is an intuitionistic fuzzy compact and Z is an intuitionistic fuzzy Hausdorff then $g: D \to Z$ is an IF homeomorphism.

Proof

If C is an intuitionistic fuzzy closed set in X then since X is an intuitionistic fuzzy compact so is C. Thus $f(C)$ is an intuitionistic fuzzy compact subset of Z and since Z is an intuitionistic fuzzy Hausdorff, $f(C)$ is an intuitionistic fuzzy closed set in Z. Thus f is an intuitionistic fuzzy closed map and hence an IF-quotient map. Then by **Proposition 2.6**, g is an intuitionistic fuzzy homeomorphism.

REMARK 2.1

If $f: X \to Y$ is a surjection, let R_f be the equivalence relation on X defined by $(a, b) \in R$ if and only if f (a) = f (b). Let $X|R_f$ be the set of equivalence classes and $p: X \to X|R_f$ the natural projection. The function $fp^{-1}: X|R_f \to Y$ is one-to-one and onto.

PROPOSITION 2.7

Let $f: X \to Y$ be an intuitionistic fuzzy continuous surjection of IFTS. Then $X|R_f$ (with the IF-quotient topology) is IF homeomorphic to Y by fp^{-1} if and only if f is an IF-quotient map.

Proof

$(\Rightarrow)$ Assume $fp^{-1}: X|R_f \to Y$ is an IF homeomorphism. Since an IF homeomorphism is an intuitionistic fuzzy closed, fp^{-1} is an IF-quotient map. By **Proposition 2.4**, $(fp^{-1})p = f$ is an IF-quotient map.

$(\Leftarrow)$ Now let f be an IF-quotient map. By **Proposition 2.1**, fp^{-1} is intuitionistic fuzzy continuous. If U is an intuitionistic fuzzy open set in $X|R_f$, then $p^{-1}[U]$ is an intuitionistic fuzzy open in X. Thus $f^{-1}\left[fp^{-1}[U]\right] = f^{-1}f\left[p^{-1}[U]\right] = p^{-1}[U]$ is an intuitionistic fuzzy open in X. This means that $fp^{-1}[U]$ is an intuitionistic fuzzy open in Y, fp^{-1} is an intuitionistic fuzzy open, and hence $(fp^{-1})^{-1}$ is an intuitionistic fuzzy continuous.

PROPOSITION 2.8

Suppose $f: X \to Y$ is an intuitionistic fuzzy continuous surjective function. If X is an intuitionistic fuzzy compact and Y is an intuitionistic fuzzy Hausdorff, then f is an IF-quotient map.

Proof

We show that f is an intuitionistic fuzzy closed map. Let C be an intuitionistic fuzzy closed subset of X. An intuitionistic fuzzy closed subset of an intuitionisic fuzzy compact space is an intuitionistic fuzzy compact, C is an intuitionistic fuzzy compact. The continuous image of an intuitionistic fuzzy compact set is intuitionistic fuzzy compact, $f(C)$ is an intuitionistic fuzzy compact subset of Y. An intuitionistic fuzzy compact subset of an intuitionistic fuzzy Hausdorff space is an intuitionistic fuzzy closed. Hence $f(C)$ is an intuitionistic fuzzy closed in Y.

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