

# Polynomial Controller RST with IP Structure for AC-DC Converter with Power Factor Correction

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**Abstract**— This paper treats a study of a new topology using a polynomial controller RST: controller RST with IP structure. It is applied on an AC-DC converter with power factor correction (Boost PFC). This controller is applied on the voltage loop and hysteresis command on the current loop. Simulation results and comparison with a scheme using PI controller show that the new proposal leads to good performances in respect of the total harmonic distortion (TDH) satisfying standard normalizes IEC- 61000-3-2.

**Index Terms**— RST controller, boost PFC, TDH, hysteresis command.

## I. INTRODUCTION

An AC-DC converter is a power electronic device used to convert constant voltage constant frequency AC power to adjustable or constant direct voltage DC power. The most current schemes use a capacitor and diodes. Although with low cost, this classical converter generates harmonics in the network which generate problems of the energy distributor [1]:

- 1) Increase of line losses
- 2) Accelerate ageing of compensation condensers because of their low impedance: their rated current may be exceeded
- 3) Over sizing of the transformers of distribution

The rated of re-injection of these current harmonics can be quantified by the rate of total distortion harmonics (TDH). The power factor is defined by:

$$F_p = P / S = (V \cdot I_1 \cdot \cos \varphi_1) / (V I) = (I_1 \cdot \cos \varphi_1) / I \quad (1)$$

With

S,P indicate respectively, apparent and active powers  
I,  $I_1$ ,  $\varphi_1$ , effective values of AC current and the fundamental of current, dephasing between current and tension.

The effective value of the current is:

$$I = \sqrt{\left(\sum_{k=1} I_k^2\right)} = \sqrt{I_1^2 + \sum_{k=2} I_k^2} \quad (2)$$

$I_k$  is the harmonic current of rank k.

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The TDH is defined as,

$$TDH = \sqrt{(I_2 / I_1)^2 + (I_3 / I_1)^2 + \dots} = (1 / I_1) \sqrt{\sum_{k=2} I_k^2} \quad (3)$$

According relations (1), (2) and (3),

$$F_p = \cos \varphi_1 / (\sqrt{1 + TDH^2}) \quad (4)$$

The power factor  $F_p$  is thus related to the rate of TDH. It means that the TDH may be an adapted parameter to quantify the harmonic degree of pollution on the network. In all that will follow, it will be taken as index of comparison. In practice, a TDH expressed in % of  $I_1$  is used. With a current purely sinusoidal and in phase with the voltage, the factor power approaches the unit value ( $F_p \gg 1$ ).

Figure 1 and Figure 2 show the current and voltage waveforms as well as the output voltage of the classic rectifier ( $C=100[mF]$ ,  $R=200[W]$ ) and the current spectrum.

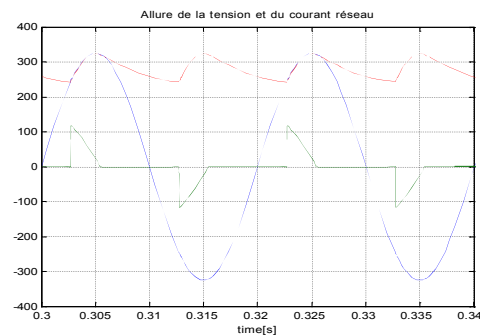


Fig. 1: Current and voltage waveforms

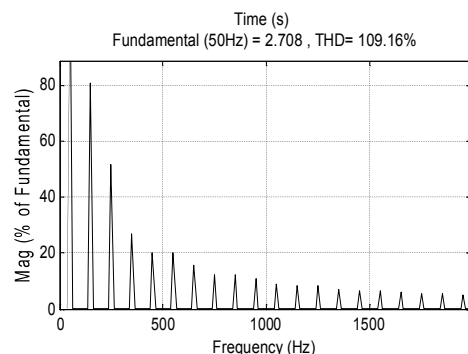


Fig.2: Current spectrum resulting

According to these two figures, it can be viewed that there is an impulse current and not in phase with the voltage. The TDH is too high ( $TDH \gg 109\%$ ).

To bring solution of this problem, various strategies are proposed whose principal objectives are summarized as follows [2], [3], [4], [5], [6]:

- 1) Obtaining a sinusoidal current network and in phase with the voltage
- 2) Or ensuring the smallest possible TDH in order to respect the standard normalizes: IEC- 61000-3-2 for example for the systems of class D.
- 3) Ensuring a voltage output constant

The basic scheme is given by Figure 4.

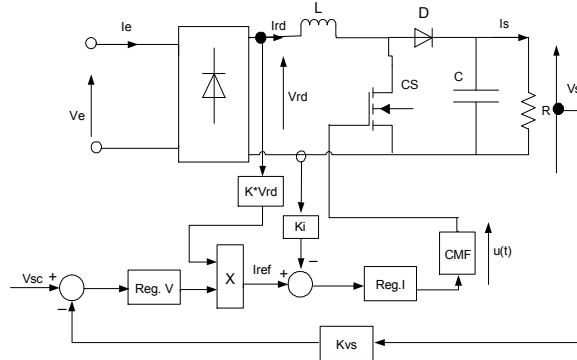


Fig. 4: Basic scheme of a Boost PFC

The existence of two loops is highlighted. The reference of the current  $I_{ref}$  is obtained by multiplying the output voltage regulator by a party ( $K \cdot V_{rd}$ ) of rectified voltage. The output current regulator is treated in a shaping circuit CMF to obtain the command  $u(t)$  used to control the static inverter CS.

In this paper, a new topology using polynomial RST and copying IP structure is proposed for the loop voltage and hysteresis control for the current loop. First this current loop is studied to obtain some conditions having a perfect loop in comparison with loop voltage. Modeling is then done for the RST controller synthesis. The new controller RST with structure IP is finally proposed to be applied on the voltage loop of the boost PFC.

## II. SCHEME WITH PI CONTROLLER

The scheme with PI controller is used for comparison with the new proposed method. PI controller is used for voltage loop and hysteresis control for the current loop one (see Figure 3).

### A. Current loop analysis

The control by hysteresis is selected for the current loop because of the nonlinear model of the static inverter. However, it is necessary to express the quench frequency in order to establish dimensioning of the inductance  $L$ .

The set value of current  $I_{ref}$  (see Figure 5) must be in phase with the tension. It is also needed to ensure  $I_{ref} \gg I_{rd}$  [1]: a fast variation of  $I_{rd}$  around its reference  $I_{ref}$  must be then satisfied which implies a high chopping frequency ( $F_d > 10$ [kHz]). A value of inductance  $L$  according to the undulation of current  $DI$  must be so determined for this

purpose. The value of the output voltage  $V_s$  and the effective value  $V_{rd}$  are considered as constant. When the variation of  $I_{red}$  around its reference  $I_{ref}$  is supposed obtained, the output voltage  $V_s$ , and the effective value  $V_{rd}$  are considered as constant [1], [7].

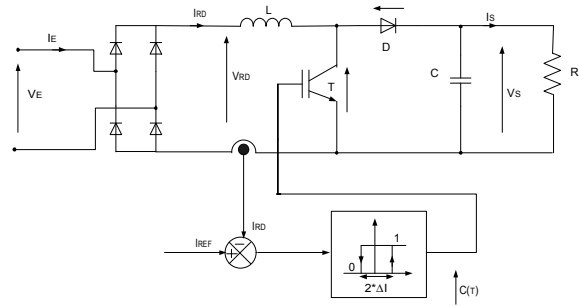


Fig. 5: Control current scheme.

The expression of the frequency is given by:

$$F_d = 1/T_d = [V_{rd}(V_s - V_{rd})] / (2 \cdot L \cdot DI \cdot V_s) \quad (5)$$

The figure 6 shows curves giving  $F_d$  according to the inductance  $L$  for imposed  $\Delta I$ . In this case,  $V_s = 400$  [V],  $V_{rd} = 235$  [V],  $\Delta I = \pm 0,1$  [A],  $\pm 0,2$  [A],  $\pm 0,3$  [A]

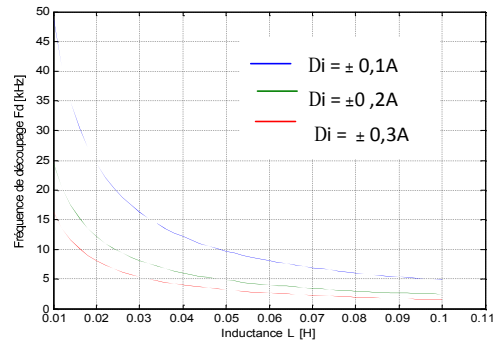


Fig. 6: Curves giving  $F_d$  according to  $L$ .

### B. Voltage loop

The voltage loop gives the signal which will act to the reference current  $I_{ref}$ . This current must have an sinusoidal form conformed with the network voltage.

It is assumed that current loop is faster than the voltage one and every time  $I_{red} = I_{ref}$ . It is there possible to adopt the following approximation obtained by modelling by assessment of power [1], [4]:

$$V_s(p) / I_{red}(p) \approx V_s(p) / I_{ref}(p) \quad (6)$$

And,

$$V_s(p) / I_{red}(p) = (V_M(p) / 4 \cdot V_s) \cdot [R / (1 + p \cdot RC / 2)] \quad (7)$$

Where,

$V_M$  is the effective value of the network voltage,  $V_s$  the output voltage,  $R$  the load resistance and  $C$  the capacitor.

The opened loop is defined by a first order transfer function.

$$G(p) = K / (1 + pT) \quad (8)$$

With,

$$K = V_M.R / 4.V_S \quad \text{and} \quad T = RC / 2 \quad (9)$$

A PI controller is sufficient to control such system. Its function transfer may be expressed like followed:

$$G_R(p) = 1 + A.pT_i / p.T_i \quad (10)$$

The gain A and the constant integral time  $T_i$  can be determined by imposing a frequency  $F_c$  for the closed loop. Assuming that the transfer function for opened loop  $G_o(p)$  is:

$$G_o(p) = G_R(p).G(p) \quad (11)$$

So,

$$G_o(p) = [(1 + A.pT_i) / p.T_i] / [K / (1 + pT)] \quad (12)$$

By adopting the compensation method, the gain A can be calculated.

$$A.T_i = T \quad (13)$$

Then, the closed loop function transfer is as follows,

$$H(p) = 1 / (1 + p.T_i / K) \quad (14)$$

Imposing frequency  $F_c$  according the relation (14) gives  $T_i$  and then the gain A by relation (13).

Figures 7 and 8 show the simulation results when frequency at closed loop is  $F_c = 5$  [Hz] and  $F_c = 20$  [Hz].

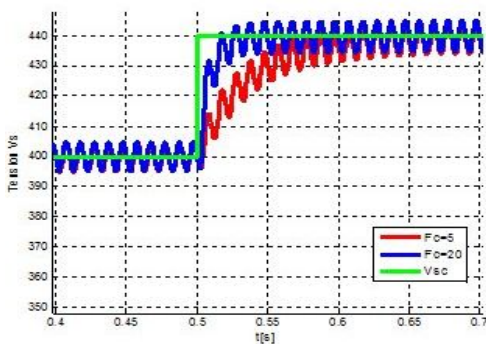


Fig. 7: Step responses of  $V_S$  at  $F_c = 5$  [Hz] and  $F_c = 20$  [Hz]

Following conclusions can be expressed:

- 1) The current is sinusoidal and in phase with the voltage
- 2) More the frequency loop is higher more the regulation is faster but more the TDH is also higher
- 3)  $F_c = 5$  [Hz]      TDH = 3,47%  
 $F_c = 15$  [Hz]      TDH = 10,53 %

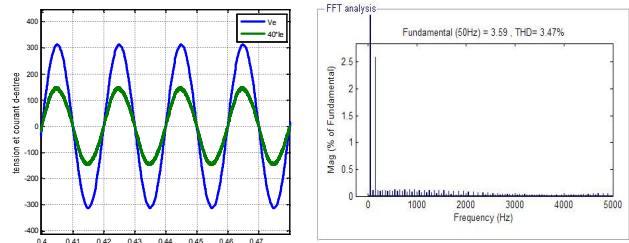


Fig. 8: Current waveform and spectrum for  $F_c = 5$  [Hz]

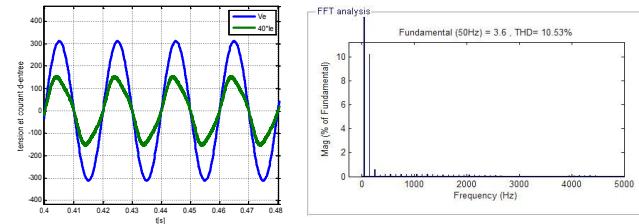


Fig. 9: Current waveform and spectrum for  $F_c = 20$  [Hz]

### III. RST CONTROLLER WITH IP STRUCTURE

In this case, numerical control is applied for the loop voltage because RST controller is primarily numerical. The current loop is always controlled by hysteresis command. Using RST controller, the functional scheme is given by Figure 10. The basic idea is to search the three polynomials  $R(z)$ ,  $S(z)$  and  $T(z)$  to have correspondence. The closed transfer function  $H_m(z)$  is given: it represents the desired model.

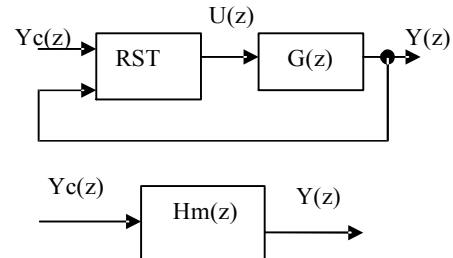


Fig. 10: Basic scheme for RST synthesis.

The RST controller generalizes the law of command obtained by a classic one.

$$R(z).U(z) = T(z).Y_c(z) - S(z).Y(z) \quad (14)$$

Having  $G(p)$ , the discrete transfer function  $G(z)$  can be calculated with a sampling time  $h$  :

$$G(z) = (1 - z^{-1}) \mathbf{Z} \left\{ L^{-1} [G(p) / p] \right\} \quad (15)$$

Where  $\mathbf{Z}$  denotes the discretization operation and  $L^{-1}$  the inverse operation of Laplace's transformation.

Using the relations (8) and (15),  $G(z)$  is as follows:

$$G(z) = K.(1 - e^{-h/T}) / (z - e^{-h/T}) \quad (16)$$

It is more practical to use the following form for  $G(z)$ :

$$G(z) = b_o / (z - z_o) = B(z) / A(z) \quad (17)$$

The desired closed loop function transfer is :

$$H_m(z) = B_m(z) / A_m(z) \quad (18)$$

Here, because of the expression of G(z), no zero cancellation is needed.

All steps to calculate R(z), T(z) and S(z) are resumed in [8] an [9]. Here G(z) and Hm(z) are known.

A. RST controller synthesis

According figure 9,

$$Y(z) = U(z).G(z) \quad (19)$$

Using relations (14), (17), and (19), the closed loop transfer function is,

$$Y(z) / Y_c(z) = [B(z).T(z)] / [A(z).R(z) + B(z).S(z)] \quad (20)$$

Assume that the desired closed loop is:

$$Y_c(z) / Y(z) = H_m(z) = B_m(z) / A_m(z) \quad (21)$$

Relations (20) and (21) give,

$$[B(z).T(z)] / [A(z).R(z) + B(z).S(z)] = B_m(z) / A_m(z) \quad (22)$$

To obtain relation (22), it can be posed,

$$\begin{cases} B(z).T(z) = A_o(z).B_m(z) \\ A(z).R(z) + B(z).S(z) = A_o(z).A_m(z) \end{cases} \quad (23)$$

Where A<sub>o</sub>(z) is defined as the observant polynomial.

The polynomial B(z) is defined as follow:

$$B(z) = B^+(z). B^-(z) \quad (24)$$

There are several methods to calculate the polynomials RST: with zero cancellation or without zero cancellation. Both methods can be completed by compensation of disturbance. All these methods lead in the resolution of Diophantine equation:

$$A_1(z).R_1(z) + B_1(z).S_1(z) = C(z) \quad (25)$$

Where R<sub>1</sub>(z) and S<sub>1</sub>(z) are the unknown polynomials.

There is a causal RST when these conditions are satisfied [8]:

$$\begin{cases} \deg(A_m) - \deg(B_m) \leq \deg(A) - \deg(B) \\ \deg(A_o) \leq 2.\deg(A) - \deg(A_m) - \deg(B^+) - 1 \end{cases} \quad (26)$$

Here deg(X) denotes the degree of the polynomial X(z)

In several cases, H<sub>m</sub>(z) is selected as followed:

$$H_m(z) = [B^-(z).P(1) / B^-(1)] / [z^d .P(z)] \quad (27)$$

Where z<sup>d</sup> is chosen to respect (23). The polynomial P(z) is generally as followed:

$$\begin{cases} P(z) = z + c \\ P(z) = z^2 + c_1z + c_2 \end{cases} \quad (28)$$

In these expressions, the different coefficients c, c<sub>1</sub>, c<sub>2</sub> are selected to ensure the absolute and relative conditions of damping.

To ensure permanent error e<sub>p</sub> equal to zero, H<sub>m</sub>(z) must verify:

$$H_m(1) = 1 \quad (29)$$

This condition is obtained by the relation (27). Because of the expression of G(z), no zero cancellation is needed. So,

$$B^+(z) = 1 \quad \text{and} \quad B^-(z) = B(z) = b_o \quad (30)$$

The Diophantine equation gives R(z) and S(z). The relations (24), (27) and (30) imply that:

$$B_m(z) = B(z).B_m^1(z) \quad (31)$$

Where,

$$B_m^1(z) = P(1) / B^-(1) \quad (32)$$

The polynomial T(z) is calculated as follow,

$$T(z) = B_m^1(z).A_o(z) \quad (33)$$

B. Construction of the RST controller with IP structure

In [10], it is shown that IP controller gives slower regulation in respect of the step response than the PI one but it gives better performance in respect of disturbance rejection. This conclusion is extended for a second order system in this reference [10].

Figure 10 shows the basic scheme using IP controller in the analogical version. It may be noted that this kind of controller has no defined function transfer as the PI one.

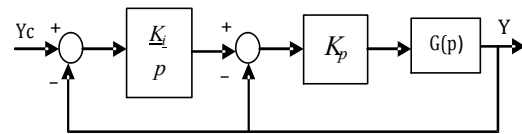


Fig.11: Basic scheme for analogical IP controller

The advantages of IP controller are used here to be applied with RST controller. To obtain the RST controller with IP structure, the relation (14) giving the command law is used:

$$R(z).U(z) = T(z).Y_c(z) - S(z).Y(z)$$

The method consists to remark that,

$$T(z).Y(z) - T(z).Y(z) = 0 \quad (34)$$

So,

$$\begin{aligned}
 & R(z).U(z) = T(z).Y_c(z) - S(z).Y(z) \\
 & T(z).Y(z) - T(z).Y(z) = 0 \\
 & R(z).U(z) = T(z).Y_c(z) - S(z).Y(z) + T(z).Y(z) - T(z).Y(z) \\
 & R(z).U(z) = [T(z).Y_c(z) - T(z).Y(z)] + [T(z).Y(z) - S(z).Y(z)] \\
 & R(z).U(z) = [Y_c(z) - Y(z)].T(z) + [T(z) - S(z)].Y(z)
 \end{aligned}
 \tag{35}$$

Figure 11 shows the result by translating the last expression of relation (35) in functional scheme.

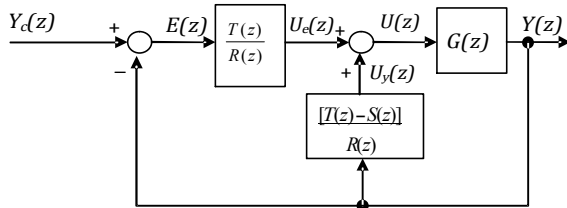


Fig. 12: RST controller with IP structure.

The command  $U(z)$  has two components:

$$U(z) = U_e(z) + U_y(z) \tag{36}$$

#### IV. APPLICATION FOR A BOOST PFC

This new controller will be applied on a voltage loop of the AC-DC converter with power factor correction. Hysteresis command is used for the current loop.

It is already said that zero cancellation is not necessary because of the expression of the function transfer  $G(z)$ .

The condition tests are:

$$V_M = 220\sqrt{2} [V] \quad R = 325 [W] \quad C = 470 [mF]$$

$$\begin{aligned}
 & t = 1,5 [s] \quad R \otimes R/2 \\
 & t = 3 [s] \quad V_{SC} : 400[V] \otimes 450[V]
 \end{aligned}
 \tag{37}$$

A polynomial  $P(z)$  with degree 2 is chosen and one effect of disturbance compensation is used ( $m=1$ ). The sampling time is  $h$ .

$$\begin{aligned}
 & h = 5 [ms] \quad z = 0,707 \quad w_n = 600 [rd.s^{-1}] \quad \deg(P) = 2 \quad m = 1 \\
 & R(z) = z - 1 \\
 & S(z) = 0,5149z - 0,2304 \\
 & T(z) = 0,2804
 \end{aligned}$$

Simulation results are resumed in following figures.

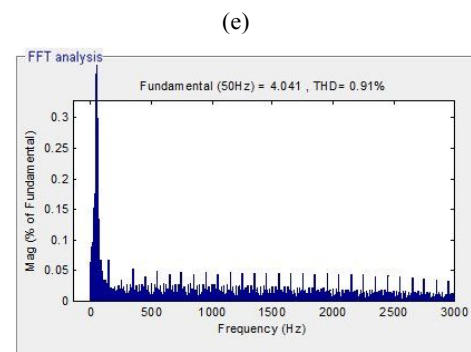
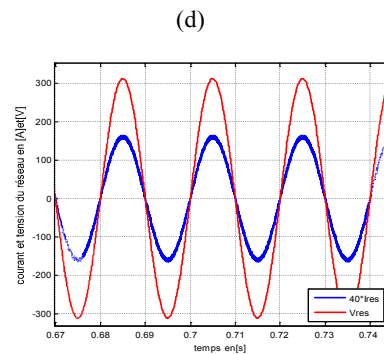
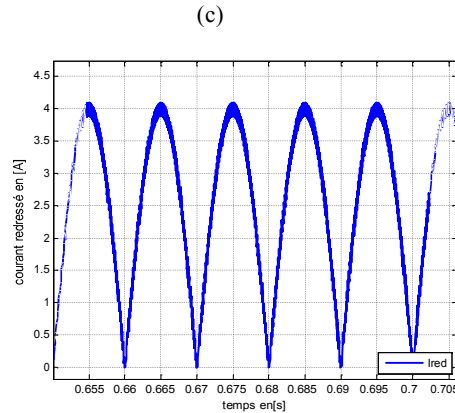
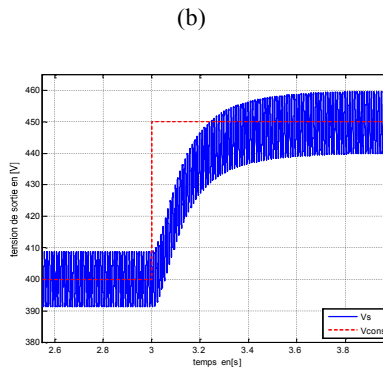
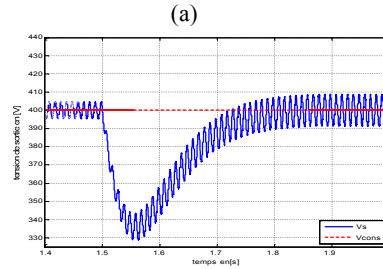
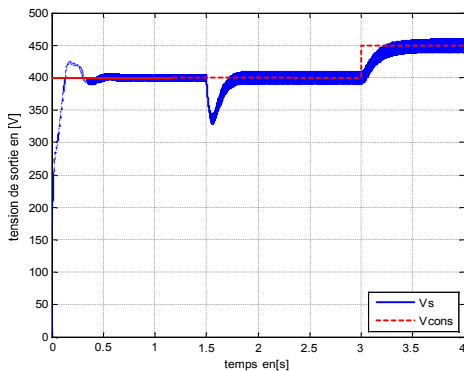


Fig.13: Simulation results

At the beginning, there is an overshoot.

$$D_1=3 \% \quad t_p = 0,2 [s]$$

Figure 13-(b) shows that there is a good disturbance rejection.

$$DV= 37 [V] \quad Dt = 0,3 [s]$$

According figure 13-(c), there is a good tracking of the reference for the output voltage. Here, the condition test is severe because the resistance R is still decreased by half ( $R \otimes R/2$ ). However, it should be noted that the response is slower than with a PI controller at  $F_c = 5 [Hz]$ .

In Figure 13-(d), the rectified current follows well the voltage waveform.

According figure 13-(e), the current is sinusoidal and in phase with the voltage.

The spectrum current analysis [figure 13-(f)] shows that the TDH = 0,91%. It means that  $F_p$  has an unit value ( $F_p \gg 1$ ). All the main objectives are obtained. By comparing the results obtained by PI controller, it can be concluded that using RST with IP structure is better by far in respect of TDH taken as criteria.

PI $F_c = 5 [Hz]$	TDH = 3,74 %
RST with IP structure	TDH = 0,91%

## V. CONCLUSION

In this paper, a new topology using polynomial RST controller with IP structure (called RST – IP) is proposed. Simulation results show that this proposal is realizable and leads to good performances as tracking test and disturbance rejection. The TDH is less than 1% ensuring a factor power equal to the unity. In the future, this RST with IP structure will be also used to build a hybrid controller with fuzzy logic.

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