Flow of a Bingham Fluid In Contact With a Jeffrey Fluid in a Circular Tube

CH. Badari Narayana, P. Devaki, S. Sreenadh

***Abstract*— The flow of two Immiscible is investigate in a circular tube. The Bingham fluid is surrounded by another immiscible Jeffrey fluid. The two fluids are with different viscosities. The effect of the ration of viscosities is discussed here. The effect of yield stress is consider as a special case, because for a Bingham fluid, yield stress plays a vital role. In this problem velocity, interface velocity are calculated analytically and the effect different parameters like Jeffrey parameter, Yield stress parameter, interface and ratio of viscosity are explained through graphs. These results warrant further investigations in the flow of immiscible fluids in circular tubes and channels.**

***Keywords* — Bingham Fluid, Interface Velocity, Interface, Jeffrey Fluid, Yield Stress.**

I. INTRODUCTION

The study of two-fluid flows is of prime importance in industrial engineering and in the design of artificial physiological systems. Most of the works reported so far deal with single fluid motion in conduits. The constitution of most of the industrial or physiological fluids suggests two fluid modelling. Several works with industrial or physiological applications have been carried out with single component fluid. But in order to have a better understanding of the biofluid flow in a physiological system such as artery, at least two fluid model consideration has become necessary.

It has been observed that whole blood, being predominantly a suspension of erythrocytes in plasma, behaves as a non- Newtonian fluid at low shear rates in microvessels. Existing literature in this area also reveals that the shear rate of blood is low in the stenosed region. These experimental observations suggest that blood behaves like a non-Newtonian fluid in the stenotic region. Further a survey on the existing literature on the experimental and theoretical studies of blood flow of various parts of the arterial tree under normal as well as pathological conditions further indicates that in certain situations the Bingham plastic fluid model gives a better description of the rheological properties of blood. We note that this model takes care of the yield stress property of blood.

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The experiments of Bugliarello and Sevilla (1970), Cockelet (1972) reveal that blood is a two layered fluid. The core layer is a region of suspension of all the erythrocytes. The peripheral layer consists of plasma. In view of this, Srivastava and Srivastava (1984) studied the peristaltic pumping of blood in uniform and non-uniform tubes considering blood as a two layered fluid. Mishra and Ghosh (1997) discussed the blood flow in micro vessels, arterioles and venules using channel and axisymmetric geometries. Comparani and Mannucci (1998) analyzed the flow of a Bingham fluid in contact with a Newtonian fluid in a channel. Existence and uniqueness theorems are proved for the solution of the problem.

Ramachandra Rao and Usha (1995a) discussed the pumping of two immiscible viscous fluids in a circular tube. Peristaltic transport of two immiscible fluids in a circular tube under long wave length approximation is investigated by Rao and Usha (1995b). The effect of permeability of walls on the peristaltic flow of two immiscible fluids bounded by flexible walls is discussed by Sreenadh et al. (1999). Usha et al. (2002) extended Brasseur et al. (1987) work for the Peristaltic flow of two immiscible fluids in a flexible channel with permeable walls. Vajravelu et al. (2006) studied Peristaltic pumping of a Herschel-Bulkley fluid in contact with a Newtonian fluid. Vajravelu et al. (2009) studied peristaltic transport of a Casson fluid in contact with a Newtonian fluid in a circular tube with permeable wall. Unsteady flow of two conducting immiscible fluids between two parallel plates is studied by Mitra (1982). Narahari and Sreenadh (2010) studied Peristaltic transport of a Bingham fluid in contact with a Newtonian fluid.

Peristaltic flow of Jeffrey fluid in a vertical porous stratum is studied in detail by Vajravelu et al.(2011). In view of the complex behavior of several physiological fluids it is necessary to model such fluids as two non-Newtonian fluid systems.

In this paper a two fluid system consisting of two non-Newtonian fluids in a circular tube is considered. The core and peripheral layers of the tube are occupied by Bingham and Jeffrey fluids respectively. The velocity fields in the two immiscible layers are obtained. The effects of yield stress and Jeffrey parameters on the interface and plug flow velocities are discussed through graphs.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider the flow of a biofluid in a circular tube consisting of two immiscible and incompressible fluid layers of different viscosities μ_1 and μ_2 . The core region is occupied by a Bingham fluid and peripheral region by a Jeffrey fluid in a circular tube. The half width of the channel is

h. The flow is in z-direction. The velocity fields in the core and peripheral layers are $(0,0,u_1)$ and $(0,0,u_2)$ respectively. Cylindrical polar coordinate system is used. The basic equations for the problem become

$$\frac{\partial p}{\partial z} = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\tau_{rz}\right), \qquad o \le r \le h_1 \tag{1}$$

Where $\tau_{rz} = \mu_1\left(-\frac{\partial u_1}{\partial r}\right) + \tau_0$ (2)

$$\frac{\partial p}{\partial z} = -\frac{1}{r}\frac{\partial}{\partial r}\left[-\frac{\mu_2}{1+\lambda_1} r \frac{\partial u_2}{\partial r}\right], \qquad h_1 \le r \le h \tag{3}$$

The boundary conditions are

τ_{rz} is finite at $r = 0$ (4a)

$u_2 = 0 \quad at\ r = h$ (4b)

$\frac{\mu_2}{1+\lambda_1}\frac{\partial u_2}{\partial r} = \mu_1 \frac{\partial u_1}{\partial r} - \tau_0 \qquad at\ r = h_1$ (4c)

$u_1 = u_2 \quad at\, r = h_1$ (4d)

$u_1 = u_p \quad at\, r = r_p$ (4e)

where τ_0 is the yield stress, u_1 is the velocity of the core region and u_2 is the velocity of the peripheral region.

III. Nondimensionalization of the Flow Quantites

The following non-dimensionalized quantities are introduced to make the basic equations and the boundary conditions dimensionless:

$$\bar{r} = \frac{r}{h}, \bar{h}_1 = \frac{h_1}{h}, \bar{z} = \frac{z}{l}, \bar{p} = \frac{h^2 p}{l\mu_1 U}\, \bar{\tau}_{rz} = \frac{h\tau_{rz}}{\mu_1 U}$$

$$\bar{\tau}_0 = \frac{h}{\mu_1 U}\tau_0, \bar{u}_i = \frac{u_i}{U}, i = 1,2, \frac{\mu_2}{\mu_1} = \mu, \bar{u}_p = \frac{u_p}{u}, \bar{r}_p = \frac{r_p}{h}$$

Using these non dimensional quantities into equations (1)-(4), the governing equations and boundary conditions become (dropping the bars)

$$\frac{\partial p}{\partial z} = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\tau_{rz}\right), \qquad o \le r \le h_1 \tag{5}$$

where $\tau_{rz} = -\frac{\partial u_1}{\partial r} + \tau_0$ (6)

$$\frac{\partial p}{\partial z} = -\frac{1}{r}\frac{\partial}{\partial r}\left[-\frac{\mu}{1+\lambda_1} r \frac{\partial u_2}{\partial r}\right], \qquad h_1 \le r \le 1 \tag{7}$$

and the boundary conditions are

$\tau_{rz} is\ finite\ at\ r = 0$ (8a)

$u_2 = 0 \quad at\, r = 1$ (8b)

$\frac{\mu}{1+\lambda_1}\frac{\partial u_2}{\partial r} = \frac{\partial u_1}{\partial r} - \tau_0 \qquad at\ r = h_1$ (8c)

$u_1 = u_2 \quad at\, r = h_1$ (8d)

$u_1 = u_p \quad at\, r = r_p$ (8e)

IV. Solution of the Problem

Solving equations (5) - (7) and using the boundary conditions (8a)-(8e), we get the velocity field in the core and peripheral regions as follows:

$$u_p = \left(r_p - h_1\right)\tau_0 + \left(\frac{h_1^2 - r_p^2}{4}\right)P + \frac{(1+\lambda_1)}{4\mu}P\left(1-h_1^2\right),$$

$o \le r \le r_p$ (9)

$$u_1 = \left(r - h_1\right)\tau_0 + P\left(\frac{h_1^2 - r^2}{4}\right) + \frac{(1+\lambda_1)}{4\mu}P\left(1-h_1^2\right),$$

$r_p \le r \le h_1$ (10)

$$u_2 = \frac{(1+\lambda_1)}{4\mu}P\left(1-r^2\right), \quad h_1 \le r \le 1 \tag{11}$$

Using the condition (8d), we obtain, the interface velocity as

$$u_3 = \frac{(1+\lambda_1)}{4\mu}P\left(1-h_1^2\right) \tag{12}$$

The volume flow rate is given by

$$q = 2\int_0^{r_p} r u_p dr + 2\int_{r_p}^{h_1} r u_1 dr + 2\int_{h_1}^{1} r u_2 dr$$

$$= \frac{r_p^2}{3}\tau_0(r_p - h_1) - \frac{P}{8}(r_p^4 - h_1^4) - \frac{P}{8}\frac{(1+\lambda_1)}{\mu}\left(h_1^4 + 2h_1^2 - 1\right) \tag{13}$$

$$P = -\frac{\partial P}{\partial z} = \frac{8.\left[\frac{rp^2}{3}(rp - h_1)\tau_0 - q\right]}{\left(r_p^4 - h_1^4\right) + \left(\frac{1+\lambda_1}{\mu}\right)\left(h_1^4 + 2h_1^2 - 1\right)}$$

Integrating with respect to $'Z'$ on both sides from 0 to 1, we get

$$\Delta P = \frac{8\left[q - \frac{r_p^2}{3}(r_p - h_1)\tau_0\right]}{\left(r_p^4 - h_1^4\right) + \left(\frac{1+\lambda_1}{\mu}\right)\left(h_1^4 + 2h_1^2 - 1\right)}$$

The shear stress in the peripheral layer is given by

$$\tau_{rz} = -\frac{\mu}{1+\lambda_1}\frac{\partial u_2}{\partial r} = \frac{\Pr}{2}$$

The shear stress at the boundary is

$$\tau_{rz}\big|_{r=1} = \frac{P}{2} \tag{14}$$

V. Results and Discussions

We study the flow of Bingham fluid in contact with a Jeffrey fluid in a circular tube. Here we calculate the velocity of the fluid for the core and the peripheral layers respectively. The velocity of the plug flow and the interface velocity are obtained. The effects of different parameters on these velocities are investigated.

The effects of different parameters on Bingham and Jeffrey fluid velocities are numerically evaluated using eqs (9)-(11) and are shown in Fig. 2 to Fig.8. Fig.2 represents the variation of velocity with radius for different viscosity ratios. Here we observe that as the ratio of viscosity increases the velocity of the fluid is decreasing. The variation of velocity with radius for different Jeffrey parameters is shown in Fig.3. We notice that as the Jeffrey parameter increases, the velocity of the fluid increases. We observe from Fig.4 that as the yield stress increases the velocity of the fluid is decreasing in the non - plug flow of Bingham fluid in the interval $0 \leq r \leq 0.3$, and the yield stress has no effect in the interval $0.3 \leq r \leq 1$, which is the region occupied by Jeffrey fluid. Fig.5 shows the variation of velocity with radius for different values of the interface. Here we observe that as the interface value increases the velocity of the fluid is increasing. Variation of interface velocity with interface for different values of ratio of viscocities is shown in Fig.6. Here we observe that as the ratio of viscosity increases, the interface velocity is decreasing. Fig.7 shows that as the Jeffrey parameter increases the interface velocity of the fluid is increasing. As the yield stress increases the interface velocity decreases which is shown in Fig.8.

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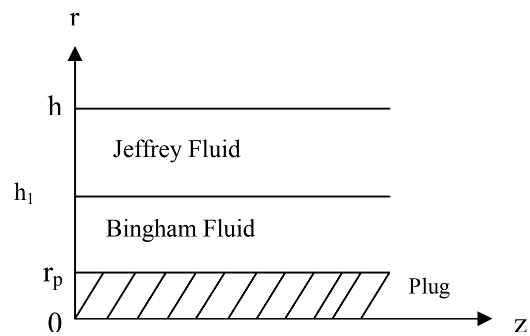

Fig.1: Physical Model

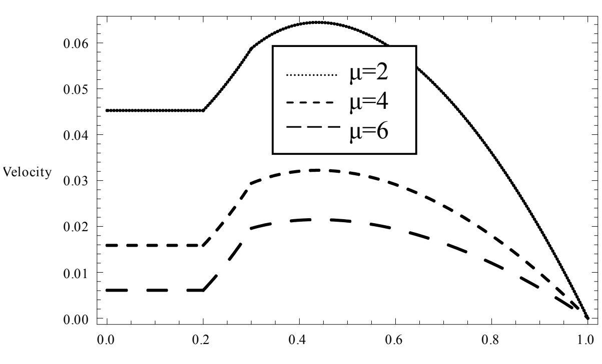

Fig 2: Variation of velocity with radius for different values of viscosity ratio with λ_1=0.05, τ=0.01, h_1=0.3, r_p=0.2.

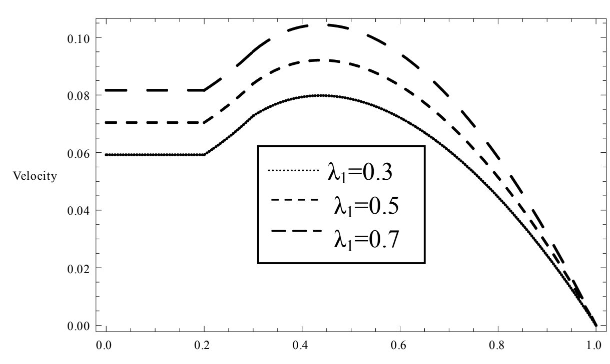

Fig .3: Variation of velocity with radius for different values of Jeffrey parameter with τ=0.01, h_1=0.3, μ=2, r_p=0.2.

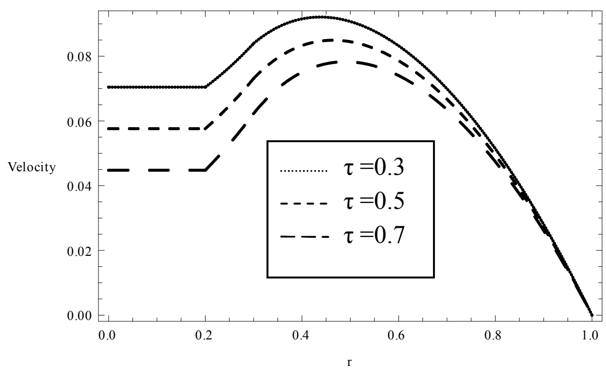

Fig 4: Variation of velocity with radius for different values of yield stress with λ_1=0.05, r_p=0.2, h_1=0.3, μ=2

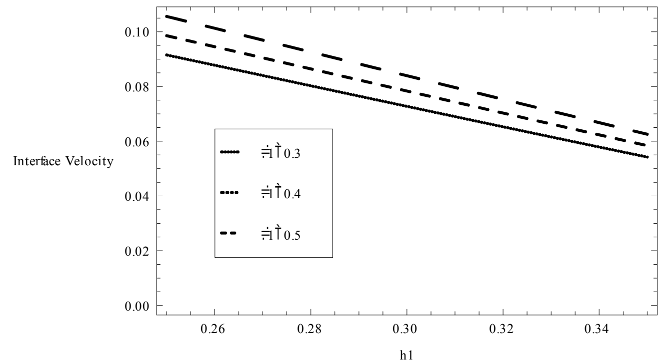

Fig 7: Variation of interface velocity with interface for different values of Jeffrey parameter with μ=2, τ=0.01, r_p=0.2.

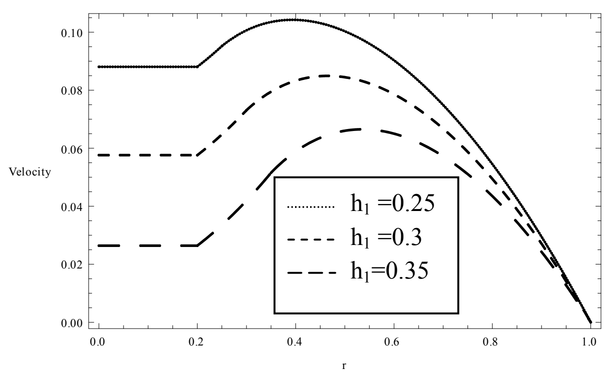

Fig.5: Variation of velocity with radius for different values of h_1 with λ_1=0.05, r_p=0.2, τ=0.01, μ=2

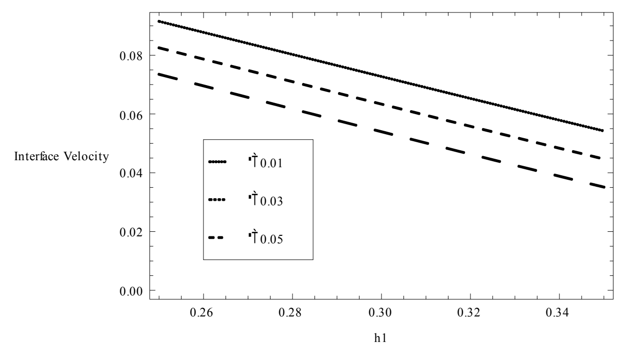

Fig 8: Variation of interface velocity with interface for different values of yield stress with μ=2, λ_1=0.3, r_p=0.2.

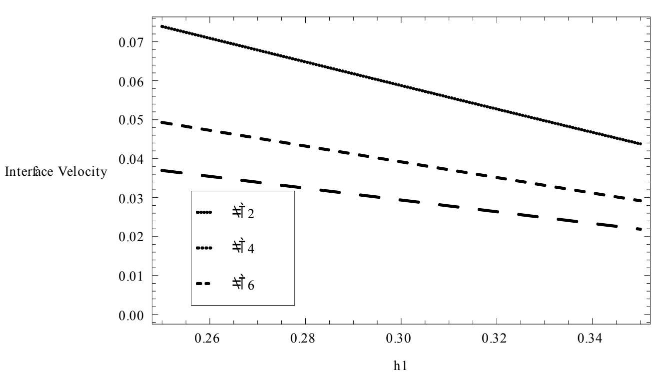

Fig.6: Variation of interface velocity with interface for different values of viscosity ratio with λ_1=0.3, τ=0.01, r_p=0.2.