Cases Using the Linear Programming Methodology

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***Abstract*— A comprehensive methodology for facing real optimization world problems is presented, discussed and used in reality. A set of seven cases based on real world problems are thoroughly discussed. The linear programing methodology can be used (and has been used) in reality by transforming the linear programming analyst into an operations research consultant. The methodology proposed here empowers students into acting as professional consultants and widens the range of activities typically undertaken in linear programming or operations research textbooks. The graphical method, which is rarely used, is beautifully illustrated to solve the problem in case 1.**

***Index Terms*— Cases, linear programming, methodology, optimization.**

I. Introduction

Typical textbooks on linear programming make emphasis on the theory and the models that can be used to solve the problem [1]-[8]. In the best case, they go from the "assumed system", which is an explanation of the problem with all the required data in place using tables and any other printed means necessary, to the problem formulation and the solution of some toy problems due to the size of the simplex matrix required [9]-[10].

Nevertheless, these textbooks lose something very important: the ability to do consulting to any given company, organization or situation. This consulting ability requires the knowledge to be able to reach the "assumed system" from the "real system", because typically, it is difficult to go from reality to assumed system, to model and to implementation and, if necessary, back to reality and assumed system. Furthermore, it is also important to implement the solution obtained in reality and see if there is any gain for the operations of the company, organization or situation. In that way, linear programming transcends its academic nature and becomes part of a real world problem solving methodology, allowing students to practice the abilities they will require later on in life when entering the workforce. If students do a good work, it may even become a working opportunity for them, because the application of linear programming to real world problems is a never-ending process.

In this paper, a series of cases carried out by students with my guidance is presented and thoroughly analyzed. These cases come from real life and their data is based on real problems facing companies, organizations or situations, although in some cases the data may have been slightly changed due to specific requests to do so.

The application of the graphical method to solving case 1 beautifully illustrates the advantages of the graphical method for learning purposes: it highlights all the important elements of a linear programming formulation and solving exercise and visually illustrates them. The constraints, the convex feasible region created by the intersection of all the constraints, the nature of the objective function as being a family of curves (a given slope) and how the objective function is optimal by pushing the objective function in the appropriate direction (depending on the type of objective function –maximize or minimize– and the positive or negative nature of the coefficients in the objective function of each variable) into the edge of the convex feasible region and finally how a system of two equations with two unknowns can be used to solve the problem.

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II. Methodology

The use of linear programming to solve problems in the real world is a process with a feedback loop. It goes from "reality", which is the perception each stakeholder has, to the "assumed system", which is the explanation of such realization. Keep in mind that there are several perceptions of what the reality looks like, and all those perceptions need to be taken into account. Once the "assumed system" is clear, it is possible to formulate the problem into a "mathematical model", translate such model to the LINDO (linear programming optimization software) syntax and to come up with solutions to be implemented in the real world (see Figure 1).

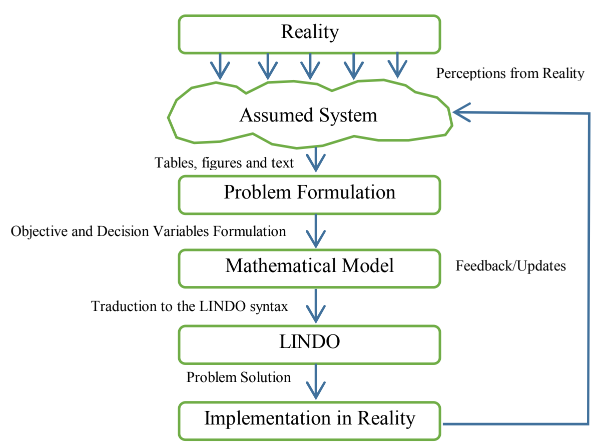

Figure 1. The linear programming approach cycle.

However, knowing how the linear programming (optimization) algorithm works allow the linear programming analysts to bring a novel perspective to the problem being solved. Thus, it may be possible to have the decision-makers in the real world realizing they may be trying to solve the wrong problem. This is the reason for a feedback loop between the "implementation in reality" stage and the "assumed system" stage. It may also be required to update the values used in the optimization model.

III. Practical Case 1: School Trip

A high school prepares a school trip for 400 students. The transport company has 8 busses with 40 seats each (small busses) and 10 busses with 50 seats each (big busses), but it only has 9 bus drivers available. Renting a big bus (50 seats) costs $8,000 and a small bus (40 seats) costs $6,000. It is intended to calculate how many busses of each type are required so that the trip is as economical as possible.

In this case, there are only two variables to consider: x, which is the number of small busses and y, which is the number of large busses. The problem is thus formulated as follows:

```
Minimize: z = f(x,y) = 6000x + 8000y
Subject to:
40x + 50y ≥ 400  ①
  x +   y ≤ 9    ②
  x       ≤ 8    ③
        y ≤ 10   ④
x, y ≥ 0
```

This problem is very simple because it has only two variables. Precisely because of that, it can be solved using the graphical method of linear programming [11]. First, let us transform the inequality into equalities while keeping in mind the type of inequality each constraint is:

```
① (≥)40x + 50y = 400
if x = 0, y = 8
if y = 0, x = 10
② (≤) x + y = 9
if x = 0, y = 9
if y = 0, x = 9
③ (≤) x = 8
④ (≤) y = 10
```

The minimum and maximum value for x are 0 and 10, respectively. Also, the minimum and maximum value for y are 0 and 10, respectively. The slope of the objective function is m = -6000/8000 = -6/8 = -3/4. Plotting, results in Figure 2.

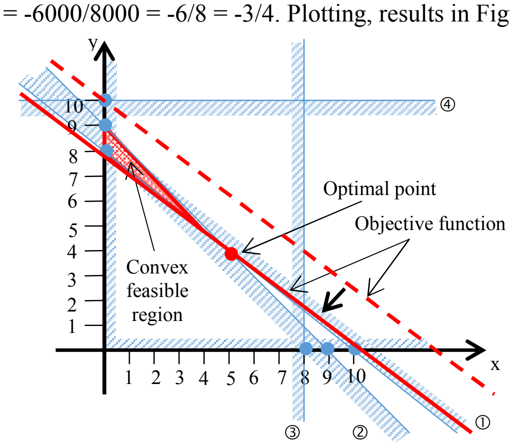

Figure 2. Solution of case problem 1 using the graphical method.

Clealy from Figure 2, the optimal point is at the intersection of constraints ① and ②. Thus, a system of equations for these two constraints can be used to find x*, y* and z*, since there are two equations and two unknowns, which is shown as follows:

```
1)     40x + 50y = 400
2) -40( x +    y = 9)
   ------------------
               10y = 40
y* = 4
2) x = 9 - y
x = 9 - 4
x*  = 5
z* = 6000(5)+8000(4) = $62,000
```

That optimally solves the problem. The students need to rent 5 small busses (x*=5) and 4 big busses (y*=4) at a minimum cost of $62,000 (z*=$62,000).

IV. Practical Case 2: Häagen Dazs Ice Crem Store

Häagen-Dazs is an ice cream company created by Reuben and Rose Mattus in the Bronx, New York, in 1961. It started having only three ice cream flavors: vanilla, chocolate and coffee. The brand name produces regular ice cream, ice cream bars, iced sorbets and yoghurt.

At the Häagen-Dazs ice cream store being considered, the consultants realized that there were two special deserts that can be made with different amounts of the same ingredients: ice cream, waffle cones and whipped cream. Two different deserts are being considered: chocoholic (A) and double temptation (B).

The relevant data for this problem is shown in Table 1.

Deserts	Ice cream (Kilograms)	Cones (Kilograms)	Whipped cream (Kilograms)	Profit ($/unit)
Chocoholic (A)	0.17	0.014	0.0425	$85
Double temptation (B)	0.14	0.028	0.0425	$80
Available (Kilograms)	9	5.6	1.812	

Table 1. Relevant data for the Häagen-Dazs ice cream store.

Let XA be the number of chocoholic deserts made and XB the number of double temptation deserts made. Then the LINDO compatible model follows:

```
MAXIMIZE 85 XA + 80 XB
SUBJECT TO
  ICECREAM)   0.17 XA +   0.14 XB <= 9
  CONES)     0.014 XA +  0.028 XB <= 5.6
  WCREAM)   0.0425 XA + 0.0425 XB <= 1.812
END
GIN 2
```

Appendix A shows the LINDO output. The result is a total profit of $3,624 by producing 42 chocoholic deserts (A) and zero double temptation (B) deserts. Apparently, the double temptation deserts should be eliminated from the menu for not being efficient towards creating profit. Thus, additional alternatives must be considered and see if there is an optimal combination of deserts that becomes possible.

V. Practical Case 3: Wood Workshop

There is a family owned wood workshop that produces individual platform beds, matrimonial platform beds, individual headboards, matrimonial headboards, drawers, tables and chairs. The variables used to denote the latter furniture (the number of each type of furniture produced in a given season) are x_1, x_2, x_3, x_4, x_5, x_6, and x_7, respectively.

Table 2 summarizes the relevant information.

Variable	Product	Price	Cost	Profit
x_1	Individual platform beds	$1,800	$1,200	$600
x_2	Matrimonial platform beds	$2,100	$1,400	$700
x_3	Individual headboards	$685	$400	$285
x_4	Matrimonial headboards	$900	$600	$300
x_5	Drawers	$400	$200	$200
x_6	Tables	$1180	$700	$480
x_7	Chairs	$500	$300	$200

Table 2. Wood workshop furniture data.

The company has a production budget of $37,000 for each season. Also, the sum of the cost of all bed-related furniture (platform beds and headboards) must not exceed $17,000 due to company policy. Clearly, for each individual platform bed made an individual headboard must be made. The same occurs with matrimonial platform beds and headboards. For each table, four chairs must be made. It is policy to make at least 8 individual platform beds and 2 matrimonial platform beds. Not more than 12 drawers must be made. Finally, at least 40 chairs must be made.

The LINDO compatible formulation of the problem follows:

```
MAXIMIZE 600x1+700x2+285x3+300x4+200x5+480x6+200x7
SUBJECT TO
  COST)
1200x1+1400x2+400x3+600x4+200x5+700x6+300x7 <=
37000
  BEDS)   1200x1+400x3+1400x2+600x4 <= 17000
  IND)    x1-x3 = 0
  MAT)    x2-x4 = 0
  TABCH)  4x6-x7 = 0
  IPLATB) x1 >= 8
  MPLATB) x2 >= 2
  DRAWER) x5 <= 12
  CHAIRS) x7 >= 40
END
```

After solving the problem using LINDO (results are included in Appendix B) yields an optimal solution in which the maximized utility is z* = $23,080, by producing 8 individual platform beds (x_1*=8), 2 matrimonial platform beds (x_2*=2), 8 individual headboards (x_3*=8), 2 matrimonial headboards (x_4*=2), 6 drawers (x_5*=6), 10 tables (x_6*=10) and 40 chairs (x_7*=40). The solution was very helpful to the workshop people because so far they relied on trial and error and the solution obtained gave them an optimal allocation of resources.

VI. Practical Case 4: Holanda Ice Cream Company

Holanda ice cream company is considering placing ice cream packages in a distribution center. They are interested in obtaining the maximum sale and knowing how many packages to place of each one available for each season. There are five package combinations. The first package includes 2 magnums, 2 soleros, 1 classic and 2 max. The suggested sales price is $90. The second package includes 2 soleros, 1 classic and 3 max. The suggested sales price is $70. The third package includes 3 soleros, 3 classics and 1 max. The suggested sales price is $74. The fourth package includes 2 magnums, 2 classics and 2 soleros. The suggested sales price is $92. Finally, the fifth package includes 4 max, 2 classics and 2 soleros. The suggested sales price is $56. The variables are P1, P2, P3, P4 and P5 indicating the number of packages 1, 2, 3, 4 and 5 to be sold, respectively. Table 3 indicates the number of products each package requires (magnum, solero, classic or max) and the totals of each product available.

Product	Package 1 (P1)	Package 2 (P2)	Package 3 (P3)	Package 4 (P4)	Package 5 (P5)	Product Availability
Magnum	2	0	0	2	0	600
Solero	2	2	3	2	2	500
Classic	1	1	3	2	2	400
Max	2	3	1	0	4	400

Table 3. Holanda ice cream factory data for one region.

The LINDO compatible problem formulation follows:

```
MAXIMIZE 90P1+70P2+74P3+92P4+56P5
SUBJECT TO
  MAGNUMS)  2P1+2P4 <= 600
  SOLEROS)  2P1+2P2+3P3+2P4+2P5 <= 500
  CLASSICS) P1+P2+3P3+2P4+2P5 <= 400
  MAXS)     2P1+3P2+P3+4P5 <= 400
END
```

Once given the problem to LINDO and solving it (results included in Appendix C), the total sales equal z*=$22,800. The optimal values are selling 100 units of package 1 (P1*=100), 0 units of package 2 (P2*=0), 0 units of package 3 (P3*=0), 150 units of package 4 (P4*=150) and 0 units of package 5 (P5*=0). The results are not surprising since packages 1 and 4 are the ones with the highest sales value. However, the people at the company were very surprised to hear the results. They could not believe it was better not to sell a single unit of packages 2, 3 and 5. Thus, they asked the consultants to change the model in order to force selling at least one unit of the other three packages not being sold. That was done and the resulting sale was lower ($22,680). Clearly, the results will make the company rethink their selling strategies and perhaps even changing the packages mix.

VII. Practical Case 5: Cinépolis Movie Theater Store

The store at the Cinepolis movie theater sells popcorn, sodas, hot dogs, nachos, ice drinks and M&Ms chocolates as part of a set of eight different combos. Combo 1 includes 1 popcorn, 1 hot dog and a soda for $120. Combo 2 includes 2 sodas and 1 popcorn for $124. Combo 3 includes a soda, 1 porporn and 1 nachos for $122. Combo 4 includes 2 sodas, 1 popcorn, 1 hot dog and 1 nachos for $199. Combo 5 includes 1 M&Ms, 1 soda and 1 popcorn for $107. Combo 6 includes 2 ice and 1 popcorn for $133. Combo 7 includes 2 sodas, 1 popcorn and 2 hot dogs for $195. Finally, combo 8 includes 2 sodas, 1 popcorn and 2 nachos for $200.

The variables being used are CMB1, CMB2, CMB3, CMB4, CMB5, CMB6, CMB7 and CMB8 for each of the eight combos. Table 4 summarizes the information and shows product availability. Each kilogram of popcorn corn yields 12 portions of popcorns.

Product	CMB1 $120	CMB2 $124	CMB3 $122	CMB4 $199	CMB5 $107	CMB6 $133	CMB7 $195	CMB8 $200	Availability
Popcorn	1	1	1	1	1	1	1	1	600 portions
Soda	1	2	1	2	1	0	2	2	150 liters
Hot dog	1	0	0	1	0	0	2	0	100 pieces
Nachos	0	0	1	1	0	0	0	2	100 pieces
Ice	0	0	0	0	0	2	0	0	150 liters
M&Ms	0	0	0	0	1	0	0	0	100 pieces

Table 4. Cinépolis movie theater store data.

The LINDO compatible formulation follows:

```
MAXIMIZE 120 CMB1 + 124 CMB2 + 122 CMB3 + 199 CMB4 +
107 CMB5 + 133 CMB6 + 197 CMB7 + 200 CMB8
SUBJECT TO
  POPCORN) 1 CMB1 + 1 CMB2 + 1 CMB3 + 1 CMB4 +
           1 CMB5 + 1 CMB6 + 1 CMB7 + 1 CMB8 <= 600
  SODAS) 1 CMB1 + 2 CMB2 + 1 CMB3 + 2 CMB4 +
         1 CMB5 + 2 CMB7 + 2 CMB8 <= 150
  HOTDOGS) 1 CMB1 + 1 CMB4 + 2 CMB7 <= 100
  NACHOS) 1 CMB3 + 1 CMB4 + 2 CMB8 <= 100
  ICE) 2 CMB6 <= 150
  M&Ms) 1 CMB5 <= 100
END
```

The optimal sales value is $28,175 (see Appendix D). In this case the variables were not restricted to be integer values so that a sensitivity analysis could be performed. Luckily the results obtained still were integer numbers (keep in mind that is not possible to have fractional numbers of combos). The optimal solution was to sell 50 units of combo 1 (CMB1* = 50), 0 units of combo 2 (CMB2* = 0), 100 units of combo 3 (CMB3* = 100), 0 units of combos 4 and 5 (CMB4* = 0 and CMB5* = 0), 75 units of combo 6 (CMB6* = 75) and 0 units of combos 7 and 8 (CMB7* = 0 and CMB8* = 0).

The slack of each constraint indicates which resources are being used fully. Popcorns have a slack of 375 portions, hotdogs have a slack of 50 pieces and M&Ms have a slack of 100 pieces. That means that it is not necessary to have that much of those resources. The same sales value would be obtained with only 600-375 = 275 portions of popcorns, 100-50 = 50 hotdogs and 0 M&Ms. Thus, management could obtain considerable savings by using only the required amounts of popcorns, nachos and M&Ms (in fact no M&Ms need to be bought). The sensitivity analysis shows precisely the amount of popcorns, nachos and M&Ms that can be reduced shown by the slack section.

VIII. Practical Case 6: TEPSA Electrical Products Company

TEPSA is an electrical products company subsidiary of the company Mine Power. It has been assigned the task to produce five electrical products for mining operations, but it needs to do so optimally in order to be competitive enough to remain in the market. That is where linear programming optimization came handy to them. They produce five products: 1) Mini sentinel 250 Amperes (Mini sentinel), 2) Cabinet with emergency stop switch and siren (Cabinet), 3) Basic sentinel 225 Amperes without plug (Basic sentinel), 4) Starting keypad with selective stop (Keypad), and 5) Capacitors bench of 40,000 Volts and 100 Amperes (Capacitors bench). The assembly line process consists of 4 stages: Drilling, Mounting, Connecting, and Packaging. The production of each product (units) per hour as well as the number of hours available per month for each production stage is indicated in Table 5.

Production Stage	Mini sentinel	Cabinet	Basic sentinel	Keypad	Capacitors bench	Hours per month available
Drilling	4	1	0.8	2.9	2.9	56.3
Mounting	2.9	2	2	5	5	102.9
Connecting	0.8	0.8	0.6	0.9	1	46.3
Packaging	5	4	2	3.3	2.2	105.9

Table 5. TEPSA products per working hours and production stage.

The sales price and production costs are very sensitive information for TEPSA. Thus the figures provided in Table 6 have been changed. Nevertheless, the nature of the problem remains the same. Also, there are monthly minimum and maximum sales values included in Table 6.

Product	Sales price	Production cost	Monthly demand (units)	
			Sales minimum	Sales maximum
Mini sentinel	$25,000	$20,000	6	15
Cabinet	$3,000	$2,500	8	12
Basic sentinel	$2,500	$1,800	7	14
Keypad	$2,000	$1,600	8	16
Capacitors bench	$5,500	$5,000	7	17

Table 6. TEPSA sales, costs and monthly minimum and maximum demands.

For the next month, there is a total amount of cable of caliber 10 of 1,800 meters. The mini sentinel requires 9 meters; the cabinet requires 10 meters; the basic sentinel requires 25 meters; and the capacitors bench requires 1 meter.

The LINDO compatible model formulation follows. In this case, the variables have been set to be integers [12]. The variables are x_1 (units of the mini sentinel to be produced per month), x_2 (units of the cabinet), x_3 (units of the basic sentinel), x_4 (units of the keypad) and x_5 (units of the capacitors bench). Keep in mind that the reciprocal of the products produced per working hours figures from Table 3 need to be calculated in order to be consistent with the units used.

```
MAXIMIZE 5000x1+500x2+700x3+400x4+500x5
SUBJECT TO
  DRILLING)
0.2500x1+1.0000x2+1.2500x3+0.3448x4+0.3448x5 <=
56.3
  MOUNTING)
0.3448x1+0.5000x2+0.5000x3+0.2000x4+0.2000x5 <=
102.9
  CONNECT)
1.2500x1+1.2500x2+1.6667x3+1.1111x4+1.0000x5 <=
46.3
  PACKING)
0.2000x1+0.2500x2+0.5000x3+0.3030x4+0.4545x5 <=
105.9
  MINSX1)    x1 >= 6
  MAXSX1)    x1 <= 15
  MINSX2)    x2 >= 8
  MAXSX2)    x2 <= 12
  MINSX3)    x3 >= 7
  MAXSX3)    x3 <= 14
  MINSX4)    x4 >= 8
  MAXSX4)    x4 <= 16
  MINSX5)    x5 >= 7
  MAXSX5)    x5 <= 17
  CABLE)     9x1+10x2+25x3+x5 <= 1800
END
GIN 5
```

After solving in LINDO (output included in Appendix E), the results are to produce 6 mini sentinels (x_1*=6), 8 cabinets (x_2*=8), 7 basic sentinels (x_3*=7), 8 keypads (x_4*=8), and 8 capacitors benches (x_5*=8) by obtaining an optimal profit of \$50,577.20 (z*=50,577.20)[1].

IX. Practical Case 7: Three Nations Industrial Plant Construction Company

The industrial plant construction company Three Nations is building an industrial plant. They have had delays in the construction efforts of 150 square meters. They have three weeks of construction ahead of them. Before such time, the linear programming consultants analyzed the problem to see if they can offer a solution to the delays and allow Three Nations to complete the project in time. The normal construction progress is 120 square meters per week in one week. That means the whole construction effort including the 150 square meters lacking so far should be 150+120x3 = 510 square meters in the three weeks available.

The problem is that having more workers laboring extra hours causes inefficiencies in the construction effort. In order to be able to use linear programming to solve the problem, five construction strategies have been proposed:

- Strategy 1: 80 workers required, \$80,000 per week, 0 extra hours, and 120 square meters construction in one week (that is: 120/80 = 1.5 square meters per worker).
- Strategy 2: 90 workers required, \$90,000 per week, 12 extra hours, and 155 square meters construction in one week (that is: 155/90 = 1.7222 square meters per worker).
- Strategy 3: 100 workers required, \$100,000 per week, 18 extra hours, and 192 square meters construction in one week (that is: 192/100 = 1.92 square meters per worker).
- Strategy 4: 110 workers required, \$110,000 per week, 24 extra hours, and 197 square meters construction in one week (that is: 197/110 = 1.7909 square meters per worker).
- Strategy 5: 120 workers required, \$120,000 per week, 30 extra hours, and 182 square meters construction in one week (that is: 182/120 = 1.5167 square meters per worker).

Notice that in all cases the cost per worker is \$1,000 per week. The variables for this problem are x_{ij}, were i is the week being considered (i = 1, 2, and 3) and j is the strategy being used in any given week (j = 1, 2, 3, 4, and 5). In this case x_{ij} is the number of workers hired in week i by following strategy j. Keep in mind that the strategy depends exclusively on the number of workers hired, so the results obtained from the LINDO solver may need interpretation. Also, consider that the variables need to be integer values.

Following is the LINDO compatible formulation:

```
MINIMIZE 1000x11+1000x12+1000x13+1000x14+1000x15+
         1000x21+1000x22+1000x23+1000x24+1000x25+
         1000x31+1000x32+1000x33+1000x34+1000x35
SUBJECT TO
  CATCHING) 1.5000x11+1.7222x12+1.9200x13+
            1.7909x14+1.5167x15 >= 120
  NORMAL)   1.5000x11+1.7222x12+1.9200x13+
            1.7909x14+1.5167x15+1.5000x21+
            1.7222x22+1.9200x23+1.7909x24+
            1.5167x25+
            1.5000x31+1.7222x32+1.9200x33+
            1.7909x34+1.5167x35 >= 510
  STRAT1W1) x11 <= 80
  STRAT2W1) x21 <= 80
  STRAT3W1) x31 <= 80
  STRAT1W2) x12 <= 90
  STRAT2W2) x22 <= 90
  STRAT3W2) x32 <= 90
  STRAT1W3) x13 <= 100
  STRAT2W3) x23 <= 100
  STRAT3W3) x33 <= 100
  STRAT1W4) x14 <= 110
  STRAT2W4) x24 <= 110
  STRAT3W4) x34 <= 110
  STRAT1W5) x15 <= 120
  STRAT2W5) x25 <= 120
  STRAT3W5) x35 <= 120
END
GIN 15
```

The results obtained (see Appendix F) are x_{13}* = 65, x_{14}*=1, x_{23}* = 100 and x_{33}*=100 where z*=\$266,000. These results have to be interpreted. They mean that for week 1, 65 workers from strategy 3 and 1 worker from strategy 4 need to be hired. However, there is no such strategy with those worker figures. Thus, these results need to be considered as having the strategy with the lowest number of workers (80) for week 1. That means having 80 workers laboring under strategy 1 for week 1 (x_{11}* = 80). The case for weeks 2 and 3 are nevertheless clear: having 100 workers both in weeks 2 and 3 working under strategy 3 (x_{23}* = 100 and x_{33}* = 100). The latter is not surprising considering that strategy 3 is the most efficient one (it provides the maximum number of square meters built per worker: 192/100 = 1.92 square meters per worker). The actual objective function is z* = (80+100+100)x\$1,000 = \$280,000. The company has a total budget of \$300,000, which means savings of \$300,000-\$280,000 = \$20,000.

Nevertheless, changing from strategies 3 and 4 to strategy 1 for week 1 means changing the workers efficiency: is there enough construction performed? Let us see: 80x1.5000+(100+100)x1.9200 = 504 square meters, which is lower than the actual total requirement of 510. This means it would be necessary for the company to put a little extra effort in order to build the remaining 6 square meters necessary to accomplish the goal set.

X. Discussion and Conclusion

All the cases presented here are real cases. The students that faced them had to follow the entire linear programming methodology, going from the real situation to the implementation in reality stage and sometimes even going back again to the assumed system. This is precisely what real consultants actually do.

The experience was highly valuable for the students and forced them to use not just their technical abilities in problem solving but also their social and managerial skills. They informed that it was often difficult to gain enough trust from the decision makers in the companies considered. This is a typical situation for consultants. Also, in some cases, students requested that some of the information being used be changed in order to maintain the confidentiality requested by the company, which in this paper was duly done.

It can be concluded that the linear programming methodology presented here is an accurate description of what actually happens in real scenarios in which operations

[1] Keep in mind that the utility (sales - cost) figures have been changed due to the company's request and thus the results are similar to the real ones but not the same.

research consultants need to offer an optimal solution to a given problem. The order of the cases is not casual, they go from the lowest complexity level in case 1, which was actually solved using the graphical method to the highest complexity level in case 7, where the solution provided by the optimization software needed to be reinterpreted (changed) according to the characteristics of the problem being faced and even such solution required additional comments to be made to the decision makers in the company of interest.

It also follows that even having the trust of the decision makers, it is usually difficult to create an assumed system such that it can be translated into a linear programming model (consider, specifically, case 7). Case 4 is also interesting because it cost a great deal of effort to the consultants to gain the trust from the decision makers of "Holanda Ice Cream Company". Even after actually solving the problem, the decision makers were not willing to provide the actual profit (sales-costs) obtained in their products, which would have been much more valuable to them when receiving the results from the optimization solver, since they could not believe the recommendation derived from the results obtained.

Appendix A: Case 2 LINDO Output

```
LP OPTIMUM FOUND AT STEP      1
OBJECTIVE VALUE =   3624.00000

FIX ALL VARS.(     1)  WITH RC >  0.000000E+00

NEW INTEGER SOLUTION OF     3570.00000      AT BRANCH
0 PIVOT        2
BOUND ON OPTIMUM:  3570.000
ENUMERATION COMPLETE. BRANCHES=     0 PIVOTS=      2

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)      3570.000

 VARIABLE        VALUE          REDUCED COST
       XA        42.000000        -85.000000
       XB         0.000000        -80.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
ICECREAM)         1.860000          0.000000
   CONES)         5.012000          0.000000
  WCREAM)         0.027000          0.000000

NO. ITERATIONS=       2
BRANCHES=     0 DETERM.=  1.000E    0
```

Appendix B: Case 3 LINDO Output

```
LP OPTIMUM FOUND AT STEP      5

        OBJECTIVE FUNCTION VALUE

        1)      23080.00

  VARIABLE        VALUE          REDUCED COST
        X1         8.000000          0.000000
        X2         2.000000          0.000000
        X3         8.000000          0.000000
        X4         2.000000          0.000000
        X5         6.000000          0.000000
        X6        10.000000          0.000000
        X7        40.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
     COST)         0.000000          1.000000
     BEDS)       200.000000          0.000000
      IND)         0.000000        115.000000
      MAT)         0.000000        300.000000
    TABCH)         0.000000        -55.000000
   IPLATB)         0.000000       -715.000000
   MPLATB)         0.000000      -1000.000000
   DRAWER)         6.000000          0.000000
   CHAIRS)         0.000000       -155.000000

NO. ITERATIONS=       5
```

Appendix C: Case 4 LINDO Output

```
LP OPTIMUM FOUND AT STEP      2

        OBJECTIVE FUNCTION VALUE

        1)      22800.00

 VARIABLE        VALUE          REDUCED COST
       P1       100.000000          0.000000
       P2         0.000000         20.000000
       P3         0.000000         64.000000
       P4       150.000000          0.000000
       P5         0.000000         36.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
 MAGNUMS)       100.000000          0.000000
 SOLEROS)         0.000000         44.000000
CLASSICS)         0.000000          2.000000
    MAXS)       200.000000          0.000000

NO. ITERATIONS=       2
```

Appendix D: Case 5 LINDO Output

```
LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      28175.00

 VARIABLE        VALUE          REDUCED COST
     CMB1        50.000000          0.000000
     CMB2         0.000000        116.000000
     CMB3       100.000000          0.000000
     CMB4         0.000000         43.000000
     CMB5         0.000000         13.000000
     CMB6        75.000000          0.000000
     CMB7         0.000000         43.000000
     CMB8         0.000000         44.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
 POPCORN)       375.000000          0.000000
   SODAS)         0.000000        120.000000
 HOTDOGS)        50.000000          0.000000
  NACHOS)         0.000000          2.000000
     ICE)         0.000000         66.500000
    M&MS)       100.000000          0.000000

NO. ITERATIONS=       1


RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
VARIABLE        CURRENT     ALLOWABLE       ALLOWABLE
                  COEF      INCREASE        DECREASE
    CMB1     120.000000      2.000000       13.000000
    CMB2     124.000000    116.000000        INFINITY
    CMB3     122.000000      INFINITY        2.000000
    CMB4     199.000000     43.000000        INFINITY
    CMB5     107.000000     13.000000        INFINITY
    CMB6     133.000000      INFINITY      133.000000
    CMB7     197.000000     43.000000        INFINITY
    CMB8     200.000000     44.000000        INFINITY

                           RIGHTHAND SIDE RANGES
     ROW        CURRENT     ALLOWABLE       ALLOWABLE
                  RHS       INCREASE        DECREASE
 POPCORN     600.000000      INFINITY      375.000000
```

```
   SODAS    150.000000      50.000000      50.000000
 HOTDOGS    100.000000       INFINITY      50.000000
  NACHOS    100.000000      50.000000      50.000000
     ICE    150.000000     750.000000     150.000000
    M&MS    100.000000       INFINITY     100.000000
```

APPENDIX E: CASE 6 LINDO OUTPUT

```
LP OPTIMUM FOUND AT STEP      8
 OBJECTIVE VALUE =    50577.2031

 NEW INTEGER SOLUTION OF    46100.0000     AT BRANCH
0 PIVOT      19
 BOUND ON OPTIMUM:  46100.00
 ENUMERATION COMPLETE. BRANCHES=    0 PIVOTS=     19

 LAST INTEGER SOLUTION IS THE BEST FOUND
 RE-INSTALLING BEST SOLUTION...

        OBJECTIVE FUNCTION VALUE

        1)      46100.00

  VARIABLE        VALUE          REDUCED COST
        X1         6.000000      -5000.000000
        X2         8.000000       -500.000000
        X3         7.000000       -700.000000
        X4         8.000000       -400.000000
        X5         8.000000       -500.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
 DRILLING)        32.533199         0.000000
 MOUNTING)        90.131203         0.000000
  CONNECT)         0.244300         0.000000
  PACKING)        93.139999         0.000000
   MINSX1)         0.000000         0.000000
   MAXSX1)         9.000000         0.000000
   MINSX2)         0.000000         0.000000
   MAXSX2)         4.000000         0.000000
   MINSX3)         0.000000         0.000000
   MAXSX3)         7.000000         0.000000
   MINSX4)         0.000000         0.000000
   MAXSX4)         8.000000         0.000000
   MINSX5)         1.000000         0.000000
   MAXSX5)         9.000000         0.000000
    CABLE)      1483.000000         0.000000

 NO. ITERATIONS=      19
 BRANCHES=    0 DETERM.=  1.000E    0
```

APPENDIX F: CASE 7 LINDO OUTPUT

```
RE-INSTALLING BEST SOLUTION...

        OBJECTIVE FUNCTION VALUE

        1)      266000.0

  VARIABLE        VALUE          REDUCED COST
       X11         0.000000       1000.000000
       X12         0.000000       1000.000000
       X13        65.000000       1000.000000
       X14         1.000000       1000.000000
       X15         0.000000       1000.000000
       X21         0.000000       1000.000000
       X22         0.000000       1000.000000
       X23       100.000000       1000.000000
       X24         0.000000       1000.000000
       X25         0.000000       1000.000000
       X31         0.000000       1000.000000
       X32         0.000000       1000.000000
       X33       100.000000       1000.000000
       X34         0.000000       1000.000000
       X35         0.000000       1000.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
 CATCHING)         6.590897         0.000000
   NORMAL)         0.590889         0.000000
 STRAT1W1)        80.000000         0.000000
 STRAT2W1)        80.000000         0.000000
 STRAT3W1)        80.000000         0.000000
 STRAT1W2)        90.000000         0.000000
 STRAT2W2)        90.000000         0.000000
 STRAT3W2)        90.000000         0.000000
 STRAT1W3)        35.000000         0.000000
 STRAT2W3)         0.000000         0.000000
 STRAT3W3)         0.000000         0.000000
 STRAT1W4)       109.000000         0.000000
 STRAT2W4)       110.000000         0.000000
 STRAT3W4)       110.000000         0.000000
 STRAT1W5)       120.000000         0.000000
 STRAT2W5)       120.000000         0.000000
 STRAT3W5)       120.000000         0.000000

 NO. ITERATIONS=       9
 BRANCHES=    1 DETERM.=  1.000E    0
```

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