Higher Order Hmm Based Group Replacement Model Considering Macroeconomic Variable

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***Abstract*— Block or group replacement model is developed to the objects that fail completely on utilization. Industrial item/equipment gets worn with time and usage and it functions with decreasing efficiency. The increasing repair and maintenance cost demands the replacement of items. In the present work an intermediate state called repairable state in between working and breakdown state is taken into consideration and Higher Order Hidden Markov Model (HMM) concept is applied in generating the probabilities of items falling in different states. Replacement decision is made considering macroeconomic variable, "inflation".**

***Index Terms*— Group Replacement, HMM, Inflation**

I. INTRODUCTION

The replacement problems are concern with the situation that arise when the efficiency of item decreases, failure or breakdown occurs. The decrease in efficiency or breakdown may be either gradual or sudden. The situation which demands the replacement of items are

1. When the cost of maintenance is increasing considerably
2. The existing equipment fails completely
3. Technologically better equipment is available

Block or Group replacement is concerned with those items that either work or fail completely. It often happens that a system contains a large number of identical low cost items that are increasingly liable to failure with age. In such cases, there is a set-up cost for replacement that is independent of the number replaced and it may be advantageous to replace all items at fixed intervals. Such a policy is called *group replacement* and is particularly attractive when the value of any individual item is so small that the cost of keeping records of individual ages cannot be justified.

II. MACROECONOMIC VARIABLE: INFLATION

Economists have defined the term inflation in different ways. According to Irving Fisher "Inflation occurs when the supply of money actively bidding goods and services increases faster than the available supply of goods". Inflation leads to Inflationary spiral. When prices rise, workers demand higher wages. Higher wages lead to higher costs. Higher costs lead to higher prices. Higher prices again lead to higher wages, higher costs and so on. Thus prices, wages, costs chase each other leading to hyper-inflation. According to Quantity of money theorists, inflation is caused by excessive issue of money. According to demand and supply theorists, it is caused by total demand exceeding the total supply of goods and services. Inflation of future periods considered are shown in table-1 and are used for calculation of real interest rates.

TABLE I
FORECASTED INFLATION FOR FUTURE PERIODS

Period (n)	1	2	3	4	5
Inflation (ϕt)	3.40	3.80	4.04	4.37	4.69

Period **(n)**	6	7	8	9	10
Inflation **(ϕ_t)**	5.00	5.33	5.65	6.03	6.29

Period **(n)**	11	12	13	14	15
Inflation **(ϕ_t)**	6.94	7.26	7.58	7.91	8.23

III. HIDDEN MARKOV MODEL (HMM)

Hidden Markov Model is a doubly stochastic process with an underlying stochastic process that is not observable (Hidden in nature), but can only be observed through another set of stochastic process that produce the sequence of observed symbols. HMM are an extension of Markov process. A Markov process is a random process of discrete valued variable involving a number of states linked by a number of possible transitions.

Various parameters that constitute the structure of HMM are

1) N: Finite number of states that are not visible (hidden) and are represented as

$S = \{S_1, S_2, S_3, \ldots S_N\}$

2) M: Number of distinct observations symbols(states) per hidden state. Individual symbols are represented by

$V = \{V_1, V_2, V_3, \ldots V_M\}$

once the each hidden transition is made, an observation(visible) output state is generated according to a probability distribution which depends on the parent state.

3) A: Hidden state transition probability matrix

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$$A = \{ a_{ij} \} = \begin{Bmatrix} a_{11} & a_{12} & - & - & a_{1N} \\ a_{21} & a_{22} & - & - & a_{2N} \\ - & - & - & - & - \\ - & - & - & - & - \\ a_{N1} & a_{N2} & - & - & a_{NN} \end{Bmatrix}$$

$a_{ij} = P[q_t = S_j \mid q_{t-1} = S_i] \quad 1 \le i, j \le N$

Where q_t denotes the actual hidden states at time 't' ,a_{ij} is the probability of moving from state S_i to S_j at time 't' and $\sum a_{ij} = 1$ for all 'i'

4) B: Observation symbol (visible states) probability distribution or output emission probability

$B = \{ b_{jk} \} = P[o_t = V_m \square q_t = S_j]$

$1 \le k \le M, \; 1 \le j \le N$

where o_t is the observation at time 't' and 'j' is state
After each transition is made a symbol will come out based on the output probability distribution which depends on the resent state. $\sum b_{jk} = 1$ for all 'j'

$$B = \{ b_{jk} \} = \begin{Bmatrix} b_{11} & b_{12} & - & - & b_{1M} \\ b_{21} & b_{22} & - & - & b_{2M} \\ - & - & - & - & - \\ - & - & - & - & - \\ b_{N1} & b_{N2} & - & - & b_{NM} \end{Bmatrix}$$

5) π: Initial state distribution

$\pi = [\pi_i]$ in which $\pi_j = P(q_i = S_i)$, $1 \le i \le N$
π_j is the initial probability matrix which stores the probability of the system. Starting at state 'i' in an observation. It is the probability of being in state 'i' at t = 1. The complete representation of HMM is made as $\lambda = (A, B, \pi)$

IV. HIGHER ORDER TRANSITION PROBABILITY MATRIX OF THREE - STATE HIDDEN MARKOV MODEL USING SPECTRAL DECOMPOSITION METHOD

If 'A' represents the three-state transition probability matrix, then the higher order transition probabilities are obtained by the following procedure.

Procedure:

1. Determine the eigen values of the transition probability matrix 'A' by solving $| A - \lambda I | = 0$
2. If all eigen values say $\lambda_1, \lambda_2, \ldots. \lambda_k$ are distinct then obtain k-column vectors $(X_1, X_2, \ldots. X_k)$ corresponding to the k-eigen values by solving
   $A X = X$ (where $X \neq 0$)
3. Denote these column vectors (eigen vectors by matrix Q) where

$$Q = \| x_1, x_2, \ldots\ldots, x_k \|$$

and obtain Q^{-1}

4. Compute D,

$$D = \begin{bmatrix} \lambda_1 & 0 & . & . & 0 \\ 0 & \lambda_2 & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & . & . & \lambda_k \end{bmatrix}$$

6. Higher order Transition Probability Matrix (TPM) of Three-State Markov can be computed using the following equation

$$A^n = Q \, D^n \, Q^{-1}$$

V. DEVELOPMENT OF MODEL

In the present work concept of Hidden Markov Model is adopted for group replacement policy. The assumptions involved in development of model are:

- In group replacement decisions, in addition to the functioning state and failure state, an intermediate state called repairable breakdown state is introduced.
- This model can be applied to a system which consists of N_1 items in three different states and to find out the group replacement period. Ex: For maintenance of cutting tools in manufacturing industry
- Repair cost or rectification cost of any item is constant.
- The item has three hidden states and three observable states.
- The failure of items is assumed to follow Weibull distribution

Notations

N_1 = Total number of items.
C_1= Individual replacement cost per item.
C_2= Group replacement cost per item.
C_3= Repair cost of the item.
r_n = Nominal interest rate assumed to be constant during the life span of the item,
ν = Present worth factor = $1 / (1 + r_n)$
X_0^I = Proportion of items in functional state.
X_0^{II} = Proportion of items in repairable breakdown state.
X_0^{III} = Proportion of items in irreparable breakdown state.
X_i^I = Probability of items in functional state in i^{th} period.
X_i^{II} = Probability of items in repairable breakdown state in i^{th} period.
X_i^{III} = Probability of items in irreparable breakdown state in i^{th} period.
a_{jk}= Probability of items transitioned from j^{th} state to k^{th} state
A(t) = Average cost per period in t^{th} group replacement policy.
A = Transition probability matrix

Transition probability matrix or the generator of Hidden Markov Process can be represented as

$$\mathbf{A} = \begin{array}{c} \\ I \\ II \\ III \end{array} \begin{array}{c} \begin{array}{ccc} I & II & III \end{array} \\ \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \\ \theta_{31} & \theta_{32} & \theta_{33} \end{bmatrix} \end{array}$$

I = Functional State
II = Repairable breakdown state
III = Irreparable breakdown state

The considered observable states are Good, Moderate, Worst

Probability of items in different states can be computed as = (probability of items in different states in initial period) $* A^n$,
where n =1,2,3…

$\pi_{i+1} = \pi_i * A^n \quad i = 0,1,2\ldots n-1$

Where $\pi_i = [\, X_i^I \;\; X_i^{II} \;\; X_i^{III} \,]$

$[\, X_1^I \;\; X_1^{II} \;\; X_1^{III} \,] = [\, X_0^I \;\; X_0^{II} \;\; X_0^{III} \,] \; A^n$, Here n = 1
… …
$[\, X_i^I \;\; X_i^{II} \;\; X_i^{III} \,] = [\, X_0^I \;\; X_0^{II} \;\; X_0^{III} \,] \; A^n$, Here n = i

The values of $A^1, A^2, \ldots A^i$ can be calculated by spectral decomposition method and using transition probability matrices, probabilities of items falling in different states i.e. $X_i^I, X_i^{II}, X_i^{III}$ in i^{th} period are to be calculated. By using MATLAB Software probabilities of items falling in different states for future periods are calculated.

Number of individual replacement

1^{st}period, $f_1 = N_1 \; X_1^{III}$
2^{nd}period , $f_2 = N_1 \; X_2^{III} + f_1 \; X_1^{III}$
3^{rd}period , $f_3 = N_1 \; X_3^{III} + f_1 \; X_2^{III} + f_2 \; X_1^{III}$
4^{th}period , $f_4 = N_1 \; X_4^{III} + f_1 \; X_3^{III} + f_2 \; X_2^{III} + f_3 \; X_1^{III}$

… … … … … …

Number of repairable breakdowns

1^{st}period, $g_1 = N_1 \; X_1^{II}$
2^{nd}period , $g_2 = N_1 \; X_2^{II} + g_1 \; X_1^{II}$
3^{rd}period , $g_3 = N_1 \; X_3^{II} + g_1 \; X_2^{II} + g_2 \; X_1^{II}$
4^{th}period , $g_4 = N_1 \; X_4^{II} + g_1 \; X_3^{II} + g_2 \; X_2^{II} + g_3 \; X_1^{II}$
… … … … … …

Total cost up to 'n' periods

$$F(t) = NC_2 + C_1 [\, f_1 + f_2 \nu + f_3 \nu^2 + \ldots\ldots f_n \nu^{n-1} \,] + C_3 [\, g_1 + g_2 \nu + g_3 \nu^2 + \ldots\ldots g_n \nu^{n-1}]$$

Average cost per period

$$A(t) = F(t) / (\Sigma \nu^{n-1})$$

Maintenance costs in $(n+1)^{th}$ period = R(n+1)
$= C_1 [f_1 + f_2 \nu + \ldots f_n \nu^{n-1} + f_{n+1} \nu^n] + C_3 [g_1 + g_2 \nu + \ldots g_n \nu^{n-1} + g_{n+1} \nu^n]$
R(t+1) > A(t) is equivalent to A(t+1) > A(t)
Sum of individual replacements and repair costs in (t+1) period should be greater than the average cost in 't' period, to group replace in t^{th} period.

VI. CASE STUDY

This model can be applied for maintenance of cutting tools in a big manufacturing unit. The cutting tools have three hidden sates. The cutting edge tool remains sharp and it removes the material from job successfully falls under functional state. In case of repairable state, the cutting edge gets worn-out and can be brought to working condition by regrinding. When it comes to break down state the cutting tool gets damaged completely and can't be brought to working state. The observable states are classified based on the quality and dimensional accuracy obtained on the job machined by the cutting tool . The initial states probabilities, Initial state transition matrix and emission matrix are given. The cutting tool will have

C_1 = Rs.6000; C_2 = Rs. 4000; C_3 = Rs.1500; N_1 = 600; r_n = 20%

The generator for Hidden Markov Process can be given as

$$[\, X_0^I \;\; X_0^{II} \;\; X_0^{III} \,] \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \\ \theta_{31} & \theta_{32} & \theta_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 0.667 & 0.167 & 0.167 \end{bmatrix} \begin{bmatrix} 0.75 & 0.125 & 0.125 \\ 0.6 & 0.1 & 0.3 \\ 0 & 0 & 1 \end{bmatrix}$$

Similarly $[X_i^I \;\; X_i^{II} \;\; X_i^{III} \,] = [\, X_0^I \;\; X_0^{II} \;\; X_0^{III} \,] \, A^n$,
Here n = i

MATLAB Program

```
aij = [0.75 0.125 0.125; 0.6 0.1 0.3; 0 0 1] ;
(Initial Transition probability matrix of states)
bjk = [ 0.8 0.15 0.05; 0.5 0.3 0.2; 0.1 0.3 0.6];
(Initial Emission matrix)
x1 = floor(random('Weibull',2.8,20,1,10)) ;
x2 = [x1,x1,x1,x1,x1,x1];
[uaij,ubjk]= hmmtrain(x2,aij,bjk);
a=[0.667 0.166 0.167];
(Initial probabilities of states)
Uaij
b=a;
for i=1:15
i
 b=b*uaij^i
i=i+1;
end
```

OUTPUT
Modified Transition probability matrix of states

```
uaij =   0.3750    0.1875    0.4375
         0.2000    0.1000    0.7000
```

0 0 1.0000

Probabilities of states for 15 periods

For i = 1, b = 0.2833 0.1417 0.5750
Fori = 2, b = 0.1346 0.0673 0.7981
For i = 3, b = 0.0639 0.0320 0.9041
For i = 4, b = 0.0304 0.0152 0.9545
For i = 5, b = 0.0144 0.0072 0.9784
For i = 6, b = 0.0069 0.0034 0.9897
For i = 7, b = 0.0033 0.0016 0.9951
For i = 8, b = 0.0015 0.0008 0.9977
For i = 9, b = 0.0007 0.0004 0.9989
For i =10, b = 0.0003 0.0002 0.9995
For i = 11, b = 0.0000 0.0000 0.0000
For i = 12, b = 0.0000 0.0000 0.0000
For i = 13, b = 0.0000 0.0000 0.0000
For i = 14, b = 0.0000 0.0000 0.0000
For i = 15, b = 0.0000 0.0000 0.0000

TABLE 2
EFFECT OF GRADUAL INCREASE OF INFLATION ON DECISION

1	2	3	4	5	6	7	8	9 = (7 + 8)	10	11	12 = (10 + 11)	13 = (12 / 6)
Period **n**	a) Inflation ϕ_t (%)	Real interest rate $r_t = \frac{(r_n - \varphi_t)}{(1+\varphi_t)}$	Present worth factor **ν = 1/(1 + r_t)**	Discount factor ν^{n-1}	Dividing discount factor $\sum \nu^{n-1}$	Individual Replacement $f_i \times C_1 \times \nu^{n-1}$ (Rs)	Repairable Replacement $g_i \times C_3 \times \nu^{n-1}$ (Rs)	Maintenance Cost R_i (Rs)	$\sum R_i$ (Rs)	Group Replacement $N \times C_2 \times \nu^{n-1}$ (Rs)	Total cost **TC(t)** (Rs)	Average Cost **A(t)** (Rs)
1	3.4	0.1605	0.8617	1.0000	1.0000	1019880.00	127530.000	1147410.0000	1147410.0000	2400000.0000	3547410.0000	3547410.0000
2	3.8	0.1561	0.8650	0.8650	1.8650	448910.783	40543.4159	489454.1993	1636864.1993	2076000.0000	3712864.1993	1990811.9031
3	4.04	0.1534	0.8670	0.7517	2.6167	178176.677	10360.2144	188536.8922	1825401.0916	1804053.6000	3629454.6916	1387040.9099
4	4.37	0.1498	0.8698	0.6579	3.2746	71340.9497	2675.4721	74016.4218	1899417.5134	1579045.1715	3478462.6849	1062247.8081
5	4.69	0.1462	0.8724	0.5793	3.8539	28820.8596	696.3805	29517.2401	1928934.7535	1390295.3069	3319230.0603	861262.0543
6	5.00	0.1429	0.8750	0.5129	4.3668	11701.8986	182.9564	11884.8550	1940819.6085	1230981.4453	3171801.0538	726340.6270
7	5.33	0.1393	0.8778	0.4573	4.8241	4784.4752	48.2915	4832.7666	1945652.3751	1097580.2168	3043232.5919	630833.1440
8	5.65	0.1358	0.8804	0.4100	5.2342	5738.5424	137.1395	5875.6819	1951528.0570	984076.8812	2935604.9382	560852.8543
9	6.03	0.1318	0.8836	0.3715	5.6057	9924.2801	260.8278	10185.1079	1961713.1649	891643.7395	2853356.9044	509010.0569
10	6.29	0.1290	0.8858	0.3356	5.9413	17887.0751	472.4466	18359.5217	1980072.6866	805400.1214	2785472.8081	468833.6406
11	6.94	0.1221	0.8912	0.3159	6.2572	33677.3309	889.6247	34566.9556	2014639.6422	758229.3282	2772868.9704	443147.7665
12	7.26	0.1188	0.8938	0.2910	6.5482	62029.7934	1638.5870	63668.3804	2078308.0226	698285.8700	2776593.8926	424026.3731
13	**7.58**	**0.1154**	**0.8965**	**0.2695**	**6.8177**	**114924.294**	**3035.8550**	**117960.1490**	**2196268.1716**	**646866.6610**	**2843134.8326**	**417023.1304**
14	7.91	0.1120	0.8993	0.2514	7.0691	214429.295	5664.3919	220093.6870	2416361.8586	603471.8911	3019833.7497	427185.5949

CONCLUSIONS

From decision making table-2 it is observed that the average cost is gradually decreasing up to thirteenth period and for the next period it is increasing suddenly. Economics can be achieved if the group replacement is done at the end of thirteenth period. Since the model is developed using Higher Order Hidden Markov Model better probabilities of various states (Functional, Repairable, and Breakdown) are obtained. Unlike conventional group replacement models this model permits an intermediate state like repairable breakdown stage of item.

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