Abstract— With the advancements in the field of communication technology, it has been seen that many different architectures and configurations of antenna have emerged. One of the most popular type of antenna is the Dielectric Resonator Antenna (DRA) which has been quite prominent in most modern high speed communication applications. Although, the design of DRA in various geometric forms have already been accomplished, the methodology for the design which is followed is not at all systematic. In this paper, we have carefully analyzed the theoretical model of a DRA and have proposed various mathematical methods for its analysis. The methods proposed herein can reduce the complexity of analysis and design of circuits involving DRAs. Furthermore, we also show system level implementation of a cylindrical DRA which can serve as a benchmark for microwave and millimeter wave communication circuit designs.

Index Terms— Dielectric Resonator Array (DRA), Antenna Arrays, Spatial Division Multiple Access (SDMA), Mathematical Modeling, System Level Design (SLD)

I. INTRODUCTION

Since about 1970’s, dielectric resonators have helped achieve the miniaturization of active and passive microwave components, such as oscillators and filters [1, 2] and have proved to be one of the driving forces behind advancement of communication systems such as Spatial Division Multiple Access

In a shielded environment, the resonators built with DRs can reach the unloaded Q factor of 20,000 at frequencies between 2 and 20 GHz. The principle of operation of the dielectric resonator can best be understood by studying the propagation of electromagnetic waves on a dielectric rod waveguide. The mathematical description and the experimental verification of the existence of these waves have been known for a long time. Their massive application, however, began with the introduction of optical fibers.

Consider the lowest modes of propagation on dielectric rod waveguides as shown in Figures 1-3 [3]. The first index denotes the number of full-period field variations in azimuthal direction, and the second one the number of radial variations.

When the first index is equal to zero, the electromagnetic field is circularly symmetric. In the cross sectional view, the field lines can be either concentric circles (e.g. the E field of the TE01 mode), or the radial straight line (e.g. the H field of the same mode). For higher modes, the pure transverse electric or transverse magnetic fields cannot exist, so that both electric and magnetic fields must have non-vanishing longitudinal components. Such modes are called hybrid electromagnetic (HEM), the lowest of them being HEM11. The fields are properly expressed in terms of Bessel functions, and there exist closed form expressions for determining the wavelength and the propagation velocity of these waves.

When only a truncated section of the dielectric rod waveguide is used, one obtains a resonant cavity in which the standing waves appear. Such a device is called dielectric resonator. When a dielectric resonator is not entirely enclosed by a conductive boundary, it can radiate, and so it becomes an antenna. DR antenna was successfully built and described in [4], while the rigorous numerical solution was published in [5]. Review treatments of DR antennas can be found in [6], [7] and [8].
Cylindrical DRAs are usually placed on a ground plane and is excited using a probe or aperture. It usually has an aspect ratio of about 0.5 to 4. The radiation pattern and the feeding method depend on the mode of interest. The DRA can resonate at many different modes. For the cylindrical DRA, the modes are analysed and indexed in a similar manner as the dielectric waveguide.

As is the case with dielectric waveguides, to satisfy the boundary conditions of continuity of tangential fields at the boundary of the dielectric and air, the only transverse fields that can exist are the modes with no azimuthal variation. All the other modes are hybrid modes.

A lot of conventional feeding mechanisms for DRA have been studied which are mostly derived from the theory of feeding structures for waveguides and cavities. In practice, it is typical to see DRAs being excited by a short monopole antenna or half circular current loops.

Another important aspect of the analysis of DRA is the radiation pattern. Fundamental modes of DRA radiate like magnetic or electric dipoles because of the fact that field distributions in the cavity for the low order modes support these terms. To study the radiation pattern, a nice approach is to expand the radiated fields using the multipole expansion technique which involves the decomposition of any arbitrary radiation pattern into a sum of dipole, quadrupole and higher order multiple pole terms.

For low profile antennas operated at low order modes, the contributions of the higher order poles are weak. Generally, the smaller the radiating element is compared to the free space wavelength, the better this approximation is. The radiation pattern as a function of elevation angle for a typical circular DRA with two modes excited in quadrature phase is shown below:

The approximate field distributions inside the cylinder for the first three common modes are shown below:

The far field radiation patterns for the TE01 modes are shown below, the second figure being the 3D equivalent of the first figure. It is clear that the electric field is doughnut-shaped.
The far field radiation patterns for the TM01 modes are shown below.

Clearly, the radiation pattern of the TM01 mode looks like a $z$-directed half-wavelength electric dipole and the TE01 mode looks like a $z$-directed half-wavelength magnetic dipole. These kinds of insights are actually very useful towards building a mathematical model from existing mathematical frameworks.

The radiation pattern of the HE11 mode resembles the radiation pattern of a half-wavelength electric dipole along the $y$ axis and is shown below:

III. MATHEMATICAL MODELLING

The resonant frequency for the TE01 mode in a cylindrical dielectric resonator on a ground plane can be approximated as:

$$f_0 = 2.921 \cdot \frac{c}{2\pi} \left[ \frac{0.691 + 0.319}{2h} \cdot \frac{a}{2h} - 0.035 \cdot \left( \frac{a}{2h} \right)^2 \right]$$

The resonant frequency as a function of height of the cylindrical DRA at different permittivities is shown below:

The resonant frequencies for the TM01 mode can be approximated by the expression:

$$f_0 = 2.933 \cdot \frac{c}{2\pi} \left[ 0.075 - 0.05 \cdot \frac{a}{2h} \right] \left[ 0.048 + 0.3 \frac{a}{2h} \right]$$

The corresponding graph of resonant frequency as a function of height of the cylindrical DRA at different permittivities for TM01 mode is shown below.

The resonant frequencies for the HE11 mode can be approximated by the expression:

$$f_0 = 2.735 \cdot \frac{c}{2\pi} \left[ 0.543 + 0.589 \frac{a}{2h} - 0.050 \cdot \left( \frac{a}{2h} \right)^2 \right]$$

The graph of resonant frequency as a function of height of the cylindrical DRA at different permittivities for HE11 mode is shown below.
To demonstrate the suitability of the mathematical approach as elucidated here, the authors of the paper accomplished a system level implementation of a cylindrical DRA for high speed communication system applications. Various parametric characteristics of the DRA are as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td>10.5 microns</td>
</tr>
<tr>
<td>Radius</td>
<td>1.5 microns</td>
</tr>
<tr>
<td>Co-axial feed line</td>
<td>9 microns</td>
</tr>
<tr>
<td>Permittivity</td>
<td>31.5</td>
</tr>
</tbody>
</table>

Based on the above parameters, the graphical representation of the cylindrical DRA can be shown as per figure below:

The DRA is usually operated at one of its lower order modes. At its lower order modes, the field lines show a behavior very similar to the lower order terms of the multipole expansion. Therefore, the radiation pattern is expected to show a radiation pattern that is similar to the corresponding multipole term.

Thus, the DRA is like a tiny aperture sitting in free space that radiates energy away. This view of the circular DRA eases away most of the non-linear behavior due to local perturbations and only leaves terms which essentially capture the local dynamics of the DRA whose understanding is essential.

The DRA has been fabricated in lab and has been tested with the help of microwave test bench. However, before the fabrication of the DRA could be realized, there were several challenges that was needed to be mitigated. One of the inherited advantages of DRA is its use at millimetre wave frequencies and until a very sophisticated process aided by micromachining is available a poor fabrication process is involved.

Fabrication of the DRA has been achieved by using a copper plate as the background plane and then attaching the Dielectric Resonator on the ground plane through an adhesive with very tiny dielectric permittivity as compared to the DRA. The process of etching was used to bond the DRA onto the copper plane with adhesive to fully secure the antenna and later enclosed in a plastic case that does not alter the radiating capabilities of the antenna.

The choice of specific adhesive and plastic enclosure also plays a very important role. If adhesive with very high permittivity is chosen, then it can substantially alter the properties of the DRA and hinder its performance. Ideally, the permittivity of the adhesive must be very very small as compared to the dielectric resonator such that it can be neglected. If it is comparable to that of the dielectric resonator, then the system acts like a segmented DRA and the liquid adhesive heats up and alters its chemical properties which can substantially affect the properties of the dielectric resonator also.

The radiation pattern of the proposed circular DRA has been obtained from both theoretical simulation using HFSS tool from ANSYS and also practical observation in the lab using microwave testbench. The radiation pattern shows that the circular DRA is capable of performing well even in the absence of Etched Background Plane. Although, the radiation pattern with Etched Background Plane is better and shows more directivity. The plot of the radiation pattern can be seen below:

Various other associated operational parameters of the DRA were also observed in the microwave lab using the testbench and exciting the antenna using Klystron tube. The various parameters are relevant for the system designer to know before he can use it for communication purposes. These operational parameters and their values often form part of the...
datasheet of an antenna which is always supplied when one buys it as a standalone device. These operational parameters for the circular DRA are as follows:

<table>
<thead>
<tr>
<th>Operational Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beamwidth</td>
<td>72/60</td>
</tr>
<tr>
<td>VSWR</td>
<td>&lt; 1.5</td>
</tr>
<tr>
<td>Impedance</td>
<td>50 Ohms</td>
</tr>
</tbody>
</table>

These operational parameters were ascertained in the microwave lab using microwave test bench and by conducting series of tests to remove any ambiguity and tolerance factors. The above values are exact as measured in the tests and has an average tolerance or variation factor of around 0.5%. Such precise and accurate manufacturing of DRA is necessary for industrial deployment where the environment is such that precision and accuracy plays a very important role in the commercial success of the project.

CONCLUSION AND FUTURE WORK

In this work, we elucidated how cylindrical DRAs can be efficiently analysed with rigorous numerical techniques which allowed us to reproduce the graphs of the variation of their resonant frequencies as a function of cylinder height and at different permittivities. This serves as a step towards development of a rigorous mathematical algorithm for the design and analysis of cylindrical DRAs for the purpose of real-time high speed communication systems required to be operated at micro and millimetre wave ranges.

We further showed how a DRA can be implemented at the system level and its various operational characteristics can be obtained for high speed communication system design. Future extension of this work will involve generalizing the numerical technique employed here to develop a software for design and analysis of cylindrical DRAs for a number of parameter changes and varied applications.

REFERENCES