The first and second Order Polynomial Models with Double Scalar Quantization for Image Compression

Prof. Dr. Ghadah Al-Khafaji, Sara A. Abdulrahman

***Abstract*— This paper introduced a mixed separated polynomial coding technique of linear and nonlinear base of two stage scalar quantizer to compress grayscale images. The test results of the proposed compression method showed highly performance compared to traditional polynomial technique of linear and nonlinear base**

***Index Terms*— Image compression, first & second order models order polynomial model, and double scalar quantizer**

I. Introduction

Image compression can be recognized as an enabling technology that looks like the art of science and becomes urgently needed with the immense revolution in computers, smart phones and communication. People daily communicate and convey information easily through images using phones and computers that characterized by the limited memory size and communication band width, so compression process successfully reducing the data amount required to increase the phones/computers storage and communication efficiency [1].

The compression process is basically based on removing image redundancy, where the techniques generally fall into two categories: lossless and lossy depending on the redundancy type exploited [2].

Polynomial coding is a modern image compression technique based on modelling concept to remove the spatial redundancy embedded within the image effectively. The basic idea of polynomial coding is the utilization of mathematical model to represent each nonoverlapping partitioning block with a small number of coefficients of low error (residual) [3], that can be classified simply into linear and nonlinear base, also refereed to as first and second order model respectively, depending on Taylor series expansion [3-5].

The use of double scalar quantizer that known as two stage quantizer, simply based on join quantizer error which is further quantized by a finer second stage uniform quantizer, namely quantized the data by two uniform quantizers whose cells ate staggered by an offset of *q/2*, where *q* is the quantization step corresponding to the first stage uniform quantizer, for more details see [6-7].

This paper investigates the mixed between polynomial models along with the double scalar quantizer compression system, that organized as follows; section 2 discussed the proposed compression system. Section 3 explained experimental results and discussion.

Manuscript received July 31, 2018

Prof. Dr. Ghadah Al-Khafaji, Dept. of Computer Science, University of Baghdad, College of Science

Sara A. Abdulrahman, Dept. of Computer Science, University of Baghdad, College of Science

II. The Proposed System

The suggested technique combined between the polynomial models of linear and nonlinear based, where the two modelling formula utilized along with the two stage scalar quantizer base to enhancing the polynomial coding techniques. The linear polynomial model characterized by simplicity of computational coefficients with less efficiency estimation around the edge that leads to high residual size compared to nonlinear model where the complexity of computational coefficients with more accurate estimation of small residual size [8].

Put simply, the idea relies on using two polynomial coding models (linear and nonlinear) of separate base of two stage scalar quantizer, as illustrated in the following steps. Figure (1) shows the proposed mixed techniques structure.

Step 1: Load the original uncompressed gray image *I* of *BMP* format of size *N*×*N*.

Step 2: Start by applying the linear polynomial technique, where the following sub steps are applied [9-10]:

1- Partition the image (*I*) into nonoverlapped blocks of fixed size *n*×*n*, such as (4×4) or (8×8) then compute the coefficients according to equation (1) bellow.

$$a_0 = \frac{1}{n \times n} \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} I(i,j)$$

$$a_1 = \frac{\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} I(i,j) \times (j - x_c)}{\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (j - x_c)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$a_2 = \frac{\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} I(i,j) \times (i - y_c)}{\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (i - y_c)^2}$$

Where a_0 coefficient corresponds to the mean (average) of block of size ($n\times n$) of original image I. The a_1 and a_2 coefficients represent the ratio of sum pixel multiplied by the distance from the center to the squared distance in i and j coordinates respectively, and the (j-xc) and (i-yc) corresponds to measure the distance of pixel coordinates to the block center (xc, yc)[3].

$$xc = yc = \frac{n-1}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

2- Quantize/dequantize the computed coefficients uniformly,

$$\begin{aligned} a_0Q &= round\left(\frac{a_0}{QS_{a0}}\right) \rightarrow a_0D = a_0Q \times QS_{a0} \\ a_1Q &= round\left(\frac{a_1}{QS_{a1}}\right) \rightarrow a_1D = a_1Q \times QS_{a1} \quad \ldots\ldots\ldots\ldots(3) \\ a_2Q &= round\left(\frac{a_2}{QS_{a2}}\right) \rightarrow a_2D = a_2Q \times QS_{a2} \end{aligned}$$

where each coefficient is quantized using different quantization step.

Where a_0Q, a_1Q, a_2Q are the polynomial quantized values, $QS_{a0}, QS_{a1}, QS_{a2}$ are the quantization steps of the polynomial coefficients, and a_0D, a_1D, a_2D are polynomial dequantized values.

3- Create the predicted image value $\tilde{I}$ using the dequantized polynomial coefficients for each encoded block representation:

$$\tilde{I} = a_0D + a_1D(j - x_c) + a_2D(i - y_c) \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

4- Find the residual or prediction error as difference between the original I and the predicted one $\tilde{I}$.

$$Res_{Linear}(i,j) = I(i,j) - \tilde{I}(i,j) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

5- Quantize/dequantize the residual uniformly.

$$Res_{Linear}Q = round\left(\frac{Res_{Linear}}{QS_{Res_{Linear}}}\right) \rightarrow Res_{Linear}D = Res_{Linear}Q \times QS_{Res_{Linear}} \ldots\ldots(6)$$

6- Use Huffman coding techniques to remove the coding redundancy that embedded between the quantized values of the residual and the linear polynomial coefficients.

Step 3: Followed by utilizing the nonlinear base that consists of the same basic steps of partitioning, quantization the coefficients, create the prediction, finding the residual that quantized and coded lossily according to equations below [3-5].

$$\begin{aligned} a_1 &= \frac{\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} I(i,j)(j-xc)}{\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} (j-xc)^2} \\ a_2 &= \frac{\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} I(i,j)(i-yc)}{\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} (i-yc)^2} \quad \ldots\ldots\ldots\ldots(7) \\ a_5 &= \frac{\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} I(i,j)(j-xc)(i-yc)}{\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} (j-xc)^2(i-yc)^2} \end{aligned}$$

$$v_1 = a_0 w_1 + a_3 w_2 + a_4 w_2$$
$$v_2 = a_0 w_2 + a_3 w_3 + a_4 w_4$$
$$v_3 = a_0 w_2 + a_3 w_4 + a_4 w_3$$
$$w_1 = n \times n$$
$$w_2 = \sum_{j=0}^{n-1} (j - xc)^2 = \sum_{i=0}^{n-1} (i - yc)^2$$
$$w_3 = \sum_{j=0}^{n-1} (j - xc)^4 = \sum_{i=0}^{n-1} (i - yc)^4$$
$$w_4 = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (j - xc)^2 (i - yc)^2 \text{.............}(8)$$
$$v_1 = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} I(i,j)$$
$$v_2 = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (j - xc)^2 I(i,j)$$
$$v_3 = \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (i - yc)^2 I(i,j)$$

To find out the coefficients estimation values, one simple method is to use the Crammers rule, such as [4]:

$$a_0 = \frac{\begin{vmatrix} v_1 & w_2 & w_2 \\ v_2 & w_3 & w_4 \\ v_3 & w_4 & w_3 \end{vmatrix}}{\begin{vmatrix} w_1 & w_2 & w_2 \\ w_2 & w_3 & w_4 \\ w_2 & w_4 & w_3 \end{vmatrix}}$$

$$a_3 = \frac{\begin{vmatrix} w_1 & v_1 & w_2 \\ w_2 & v_2 & w_4 \\ w_2 & v_3 & w_3 \end{vmatrix}}{\begin{vmatrix} w_1 & w_2 & w_2 \\ w_2 & w_3 & w_4 \\ w_2 & w_4 & w_3 \end{vmatrix}} \text{................}(9)$$

$$a_4 = \frac{\begin{vmatrix} w_1 & w_2 & v_1 \\ w_2 & w_3 & v_2 \\ w_2 & w_4 & v_3 \end{vmatrix}}{\begin{vmatrix} w_1 & w_2 & w_2 \\ w_2 & w_3 & w_4 \\ w_2 & w_4 & w_3 \end{vmatrix}}$$

$$a_0Q = round(\frac{a0}{QS_{a0_{coff}}}) \rightarrow a_0D = a_0Q \times QS_{a0_{coff}}$$
$$a_1Q = round(\frac{a1}{QS_{a_{1coff}}}) \rightarrow a_1D = a_1Q \times QS_{a_{1coff}}$$
$$a_2Q = round(\frac{a2}{QS_{a_{2coff}}}) \rightarrow a_2D = a_2Q \times QS_{a_{2coff}} \text{........}(10)$$
$$a_3Q = round(\frac{a3}{QS_{a_{3coff}}}) \rightarrow a_3D = a_3Q \times QS_{a_{3coff}}$$
$$a_4Q = round(\frac{a4}{QS_{a_{4coff}}}) \rightarrow a_4D = a_4Q \times QS_{a_{4coff}}$$
$$a_5Q = round(\frac{a5}{QS_{a_{5coff}}}) \rightarrow a_5D = a_5Q \times QS_{a_{5coff}}$$

$$\tilde{I} = a_0DW_1 + a_1D(x - xc) + a_2D(y - yc) + a_3D(x - xc)^2 + a_4D(y - yc)^2 + a_5D(x - xc).(y - yc) \text{...}(11)$$

$$\text{Re}sNonLinear(i,j) = I(i,j) - \tilde{I}(i,j) \text{..................................}(12)$$

$$\text{Re}sNonLinearQ = round(\frac{\text{Re}sNonLinear}{QS_{\text{Re}sNonLinear}}) \rightarrow \text{Re}sDNonLinear = \text{Re}sQNonLinear \times QS_{\text{Re}sNonLinear} \text{........}(13)$$

Step 4: Find the average of the two dequantized residual images (i.e., resulting of linear and nonlinear models respectively from the step above) that added to find the reconstructed image as in equation (15).

$$\text{Re}sDAv(i,j) = (\text{Re}sD_{Linear}(i,j) + \text{Re}sD_{NonLinear}(i,j))/2 \text{ .... } (14)$$

Where $\text{Re}sD_{Linear}$, $\text{Re}sDNonLinear$ are the dequantized residual images of two separate linear and nonlinear models.

$$\hat{I}(i,j) = \tilde{I}(i,j) + \text{Re}sDAv(i,j) \text{ .... } (15)$$

Step 5: Find the second residual image as a difference between the original image and the reconstructed from Step 4 above, which also quantized using the scalar base that corresponds to the second stage quantizer and reconstruct the compressed image according to equation (15). As mentioned previously the Huffman coding techniques utilized to remove the coding redundancy.

I. EXPERIMENTAL RESULTS

For testing the proposed system performance; it is applied on two standard gray (256 gray levels or 8 bits/pixel) square (256×256) images of varying details (see Fig. 2for an over view), also the compression ratio utilized along with the objective fidelity criteria of *PSNR* measure according to equations below.

(a)

(b)

Fig. (2): The tested images of size 256×256, gray scale images, (a) Lena and (b) Rose.

$$CR = \frac{Size\ \ of\ \ Original\ \ Image\ \ in\ \ Byte}{Size\ \ of\ \ Compressed\ \ Image\ \ Information\ \ in\ \ Byte} \ldots\ldots\ldots\ldots\ldots\ldots(16)$$

$$PSNR(dB) = 10\ \ \log_{10}[\frac{(maximum\,gray\ scale\ \ of\ \ image)^2}{MSE}] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

$$MSE(I,\hat{I}) = \frac{1}{N \times N} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} [\hat{I}(i,j) - I(i,j)]^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

The results shown in tables (1 & 2) illustrate the comparison between the traditional linear & nonlinear polynomial coding and proposed two stage scalar quantizer polynomial coding techniques of mixed base between the first and second order polynomial models of two tested images using block sizes of *4x4*.

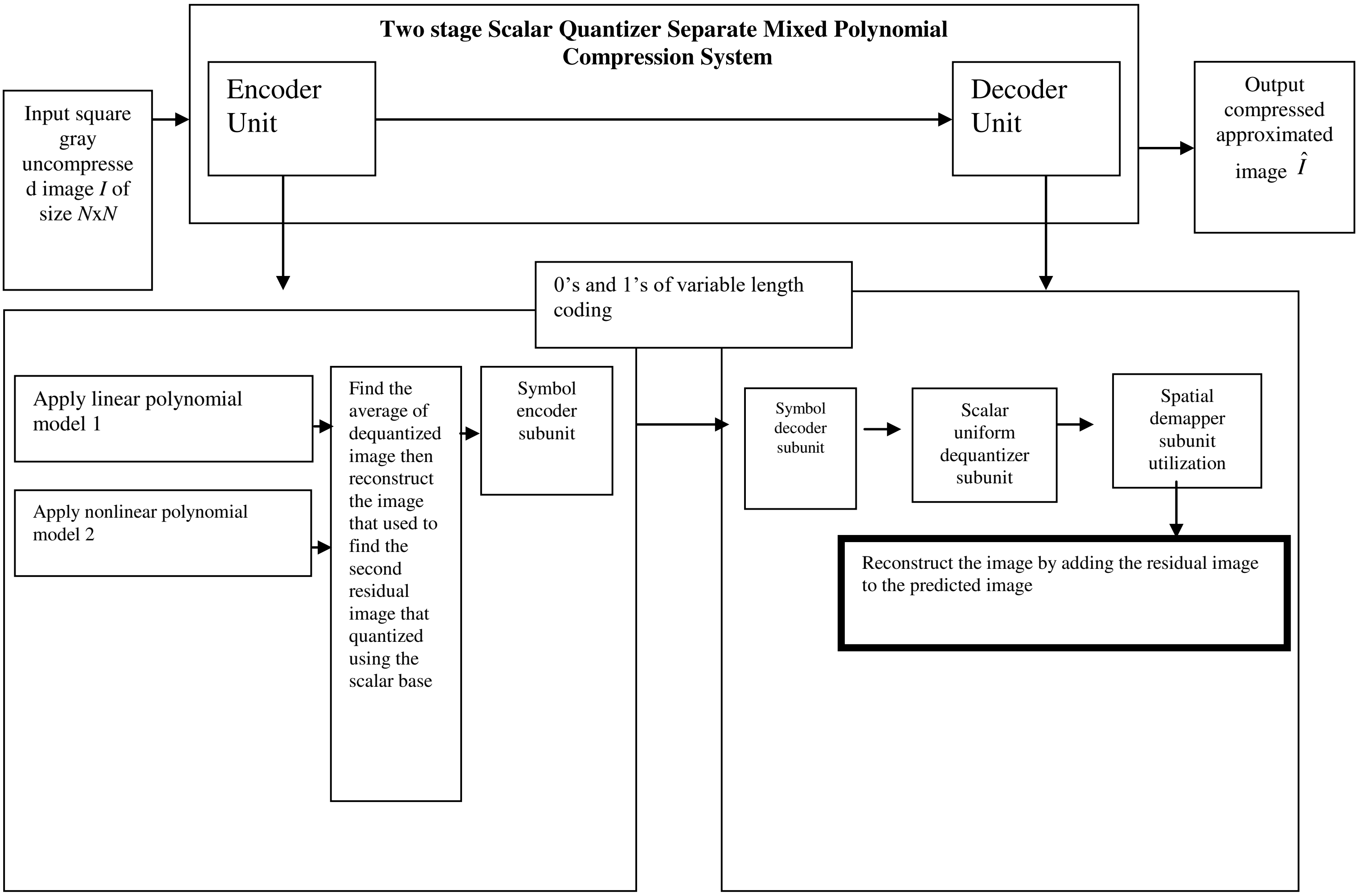

Fig. (1): Two stage Scalar Quantizer Separate Mixed Polynomial Model Structure.

Table (1): Comparison performance between polynomial coding and proposed techniques of two stages scalar quantizer of Lena image.

Tested image	Linear Polynomial Coding with Qcoff=1,2,2		Nonlinear Polynomial Coding with Qcoff=1,2,3,3,5,5		Mixed polynomial with two stage SC Qcoff1=1,2,2 Qcoff2=1,2,3,3,5,5	
Lena	Q Res=20		Q Res=20,60,20		Q Res=20,60,20	
	CR	PSNR	CR	PSNR	CR	PSNR
	4.2413	34.9135	4.6211	35.5098	8.2249	36.1671
	Q Res=50		Q Res=20,60,50		Q Res=20,60,50	
	4.4667	30.0366	4.7956	31.4396	8.9043	34.9315

Table (2): Comparison performance between polynomial coding and proposed techniques of two stages scalar quantizer of Rose image

Tested image	Linear Polynomial Coding with Qcoff=1,2,2		Nonlinear Polynomial Coding with Qcoff=1,2,3,3,5,5		Mixed polynomial with two stage SC Qcoff1=1,2,2 Qcoff2=1,2,3,3,5,5	
Rose	Q Res=20		Q Res=20,60,20		Q Res=20,60,20	
	CR	PSNR	CR	PSNR	CR	PSNR
	4.3743	36.3577	4.8776	38.0097	10.5160	38.8828
	Q Res=50		Q Res=20,60,50		Q Res=20,60,50	
	4.4943	32.5447	4.9312	35.9031	10.7401	38.4386

Clearly, the linear polynomial coding by their simplicity and limited redundancy removal due to insufficient model of three coefficients of large residual that implicitly means poor prediction model, on the other hand the nonlinear one better performance due to low residual (small error) resultant, and high prediction model, in spite of more coefficients utilization and complex computations.

The results show that the mixed technique is better than the other techniques that based on combination between first and second polynomial models that efficiently utilize the small residual of nonlinear model along with the large residual of linear model. Figure (3 a&b) illustrates the performance results for tested images using first, second and mixed polynomial coding techniques.

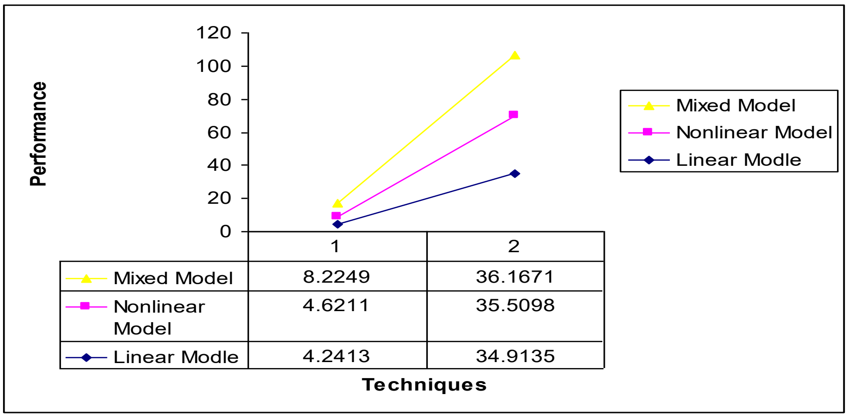

Fig (3a): Performance of linear model based of Lena image, using quantization polynomial coefficients of 1,2,2 and quantization residual of 20 of traditional linear model, quantization polynomial coefficients of 1,2,3,3,5,5 and quantization residual of 20of nonlinear model, and quantization polynomial coefficients of 1,2,2, 1,2,3,3,5,5 and quantization residual of 20,60,20 of mixed model.

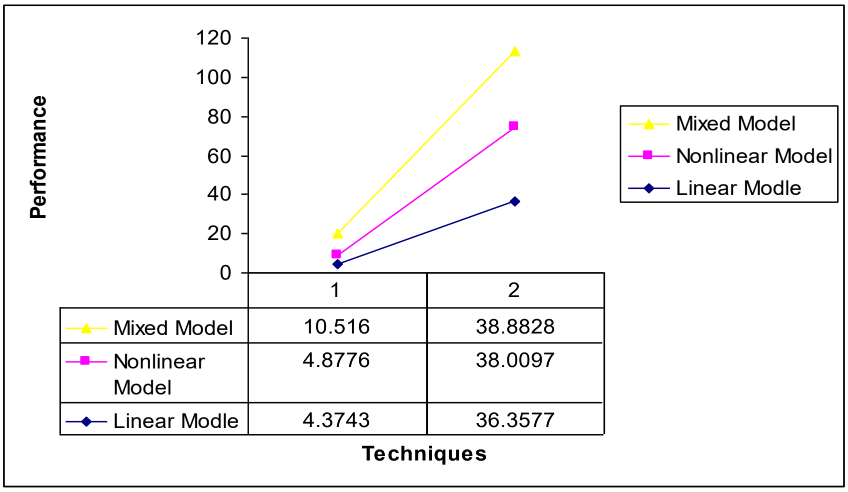

Fig (3 b): Performance of linear model based of Rose image, using quantization polynomial coefficients of 1,2,2 and quantization residual of 20 of traditional linear model, quantization polynomial coefficients of 1,2,3,3,5,5 and quantization residual of 20of nonlinear model, and quantization polynomial coefficients of 1,2,2, 1,2,3,3,5,5 and quantization residual of 20,60,20 of mixed model.

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