International Journal of Engineering Research And Management (IJERM)
ISSN: 2349- 2058, Volume-05, Issue-08, August 2018

# Significance of Data Assimilation Approach with special Reference to Ordinary Differential Equations

**Mr. Rakesh Kumar Verma**

***Abstract*— Environmental systems can be realistically described by mathematical and numerical models of the system dynamics. These models can be used to predict the future behaviour of the system, provided that the initial states of the system are known. Complete data defining all of the states of a system at a specific time are, however, rarely available. Moreover, both the models and the available initial data contain inaccuracies and random noise that can lead to significant differences between the predicted states and the actual states of the system. A variety of models is used to describe systems arising in environmental applications, as well as in other physical, biological and economic fields. These range from simple linear, deterministic, continuous ordinary differential equation models to sophisticated non-linear stochastic partial-differential continuous or discrete models. The data assimilation schemes, with minor modifications, can be applied to any general model. Data assimilation schemes are described here for a system modelled by the discrete non-linear equations**

$$\mathbf{x}_{k+1} = \mathcal{M}_{k,k+1}(\mathbf{x}_k), \quad k = 0, \ldots, N-1$$

**where$x_k \in R^n$ denotes the vector of n model states at time $t_k$ and $M_{k,k+1} : R^n \to R^n$ is a non-linear operator describing the evolution of the states from time $t_k$ to time $t_{k+1}$. The operator contains known inputs to the system including known external forcing functions that drive the system and known parameters describing the system.**

***Index Terms*— Ordinary differential equation, Kalman filter (ET KF), Kalman smoother (EnKS), Axisymmetric and Non-Axisymmetric Coupled model (ANAC).**

**Broad Area : Mathematics**

## I. INTRODUCTION

This is deliberately general; such statements can take many forms. Examples include categorical or discrete statements (e.g. “including isopycnal mixing improves ocean circulation models”), logical propositions (e.g. “increasing soil temperatures lead to increased soil respiration”) or quantitative statements like “the Amazon forest is a sink of between 1 and 2 pgCy$^{-1}$ ”. The choice of the set of events we consider is the first one we make setting up any data assimilation problem. We require that any events which are logically incompatible are disjoint (mutually exclusive) and that the set of events is complete.

**Manuscript received Aug 12, 2018**

**Mr. Rakesh Kumar Verma**, HOD- Applied Sciences, Yoganarda College of Engineering & Technology, Akhnoor Road, Gurha Brahmana (Patoli)Jammu180205 (J & K) )-India

The concept of probability is so simple and universal that it is hard to find a definition which is more than tautology. It is a function mapping the set of events onto the interval (0,1). Its simplest properties, often called the Kolmogorov axioms reflect the definition of events, i.e that the probability of the whole space is 1, and that the probability of the union of two disjoint events is the sum of their individual probabilities. If the events are part of 30 a continuous space we can also define a probability density function (PDF) so that the probability that x ∈ (a, b) is the integral or its multi-dimensional counterpart.

## II. ORDINARY DIFFERENTIAL EQUATION

In mathematics, an ordinary differential equation (ODE) is a differential equation containing one or more functions of one independent variable and its derivatives. The term *ordinary* is used in contrast with the term partial differential equation which may be with respect to *more than* one independent variable. Among ordinary differential equations, linear differential equations play a prominent role for several reasons. Most elementary and special functions that are encountered in physicsand applied mathematics are solutions of linear differential equations (see Holonomic function). When physical phenomena are modeled with non-linear equations, they are generally approximated by linear differential equations for an easier solution. The few non-linear ODEs that can be solved explicitly are generally solved by transforming the equation into an equivalent linear ODE. Ordinary differential equations (ODEs) arise in many contexts of mathematics and science (social as well as natural). Mathematical descriptions of change use differentials and derivatives. Various differentials, derivatives, and functions become related to each other via equations, and thus a differential equation is a result that describes dynamically changing phenomena, evolution, and variation.

## III. LITERATURE REVIEW

Sakov et al reviewed the problem of assimilation of asynchronous observations, or four-dimensional data assimilation, with the ensemble Kalman filter (EnKF). We show that for a system with perfect model and linear dynamics the ensemble Kalman smoother (EnKS) provides a simple and efficient solution for the problem: one just needs to use the ensemble observations (that is, the forecast observations for each ensemble member) from the time of observation during the update, for each assimilated observation. Cartis et al present DFO-GN, a derivative-free version of the Gauss-Newton method for solving nonlinear least-squares problems. As is common in derivative-free optimization, DFO-GN uses interpolation of function values to build a model of the objective, which is then used within a trust-region framework to give a globally-convergent algorithm requiring O(ϵ−2) iterations to reach approximate first-order criticality within tolerance ϵ. Bishop et al called the ensemble transform Kalman filter (ET KF) is introduced. Gerrit et al discusses an important issue related to the implementation and interpretation of the analysis scheme in the ensemble Kalman filter. It is shown that the observations must be treated as random variables at the analysis steps. That is, one should add random perturbations with the correct statistics to the observations and generate an ensemble of observations that then is used in updating the ensemble of model states. Cao et al describe Four-dimensional variational data assimilation (4DVAR) is a powerful tool for data assimilation in meteorology and oceanography. However, a major hurdle in use of 4DVAR for realistic general circulation models is the dimension of the control space (generally equal to the size of the model state variable and typically of order $10^7$–$10^8$) and the high computational cost in computing the cost function and its gradient that require integration model and its adjoint model. Romina et al describe a novel technique for imaging and data assimilation of the topside ionosphere and plasmasphere. The methodology is based upon the three-dimensional variational technique (3DVAR), where an empirical background model is utilized. Griffith et al describe Data assimilation aims to incorporate measured observations into a dynamical system model in order to produce accurate estimates of all the current (and future) state variables of the system. Johnson et al describe the very large nonlinear dynamical systems that arise in a wide range of physical, biological and environmental problems, the data needed to initialize a numerical forecasting model are seldom available. To generate accurate estimates of the expected states of the system, both current and future, the technique of 'data assimilation' is used to combine the numerical model predictions with observations of the system measured over time.

## IV. Proposed Methodology

The sawtooth instability is a relaxation oscillation in the centre of the plasma at large electric currents, mainly observed through oscillations in electron temperature and density, followed by subsequent movement of particles and energy as a heat pulse from the centre of the plasma to the boundary. Edge-localised modes occur during sufficient increase of input power, when the edge of the plasma, characterised by large differences in electron density and temperature, undergoes short heat and particle eruptions. In addition to instabilities being prone to nonlinear interactions, a wide range of spatial and time scales also make simulations of large scale behaviour of the tokamak plasma at high temperatures difficult and computationally demanding. However, simplifications of tokamak geometries under symmetry considerations enables the study of sawteeth and ELMs via simple ordinary differential equation (ODE) models that reproduce their behaviour as outlined in [3].

The simplest coupled equations called Axisymmetric and Non-Axisymmetric Coupled model (ANAC), observed to qualitatively fit the experimental data, are the following

$$\begin{aligned} \ddot{a} &= \gamma a + 2\mu a^3 \\ \dot{b} &= \alpha - \beta b^2 - (1 + \delta_r b)a^2 \end{aligned}$$

where the dot notation represents derivatives with respect to time t. The final goal is to perform data assimilation with this model. However, due to complexity of the coupled system, simpler models are first considered for purposes of testing the data assimilation algorithm and the parameter estimation optimisation scheme, as well as to gradually build up complexity and understanding of the problem.

## Conclusion

The new approach developed for initialisation of EnKF involves using observations over the first period for parameter and initial state estimation. The optimised values of parameters are used to initialise the model to be assimilated, whereas the optimised initial state is used for initialisation of the initial ensemble of state vectors. Synthetic data is generated by perturbing the solutions obtained by numerical integration at each time step, which is assumed to be fixed. The perturbations are randomly generated from the uniform or Gaussian distribution with variance σ2. Observations are subsampled from synthetic data so that the observation time step is a positive integer multiple of the numerical integration time step.

## References


[1] K. Law, A. Stuart, and K. Zygalakis, Data assimilation: a mathematical introduction. Springer, 2015, vol. 62.

[2] T. Craciunescu, M. Murari, E. Peluso, M. Gelfusa, and J. Contributors", "A New Approach to Bolometric Tomography in Tokamaks," in This proceedings, 2018.

[3] Murari, W. Arter, D. Mazon, M. Gelfusa, V. Ferat, and J.-E. Contributors, "Symmetry based analysis of macroscopic instabilities in Tokamak plasmas," JET-EFDA, Tech. Rep. PR(11)07, 2011, http://www.iop.org/Jet/fulltext/EFDP11007.pdf

[4] E. Lorenz, "Deterministic Nonperiodic Flow," Journal of the Atmospheric Sciences, vol. 20, no. 2, pp. 130–141, 1963.

[5] A.Majda and J. Harlim, Filtering complex turbulent systems. Cambridge University Press, 2012.

[6] G. Evensen, Data assimilation: the ensemble Kalman filter. Springer, 2009.

[7] P. Sakov, G. Evensen, and L. Bertino, "Asynchronous data assimilation with the EnKF," Tellus A, vol. 62, no. 1, pp. 24–29, 2010.

[8] C. Cartis and L. Roberts, "A Derivative-Free Gauss-Newton Method," Technical Report, University of Oxford, arXiv:1710.11005, 2017.

[9] Johnson, C., B.J. Hoskins and N.K. Nichols, 2005a. A singular vector perspective of 4-DVar: Filtering and interpolation. Q. J. R. Meteorol. Soc., 131, 1–20.

[10] Griffith, A.K. and N.K. Nichols, 2000. Adjoint techniques in data assimilation for estimating model error. J. Flow, Turbulence Combustion, 65, 469–488.

[11] Houtekamer, P.L. and H.L. Mitchell, 1998. Data assimilation using an ensemble Kalman filter technique. Mon. Weather Rev., 126, 796–811.

[12] Gratton, S., A.S. Lawless and N.K. Nichols, 2007. Approximate Gauss-Newton methods for nonlinear least-squares problems. SIAM J. Optim., 18, 106–132.

[13] Martin, M.J., M.J. Bell and N.K. Nichols, 2001. Estimation of systematic error in an equatorial ocean model using data assimilation. In Numerical Methods for Fluid Dynamics VII, Baines, M.J. (ed.), ICFD, Oxford, pp 423–430

[14] Nerger, L., W. Hiller and J. Schroeter, 2005. A comparison of error subspace Kalman filters. Tellus, 57A, 715–735.
[15] Nichols, N.K., 2003a. Data assimilation: Aims and basic concepts. In Data Assimilation for the Earth System, NATO Science Series: IV. Earth and Environmental Sciences 26, Swinbank, R., V. Shutyaev and W.A. Lahoz (eds.), Kluwer Academic Publishers, Dordrecht, The Netherlands, pp 9–20, 378pp
[16] Ott, E., B.R. Hunt, I. Szunyogh, A.V. Zimin, E.J. Kostelich, M. Corazza, E. Kalnay, D.J. Patil and J.A.lYorke, 2004. A local ensemble Kalman filter for atmospheric data assimilation. Tellus, 56A, 415–428.

## BIOGRAPHY

**Corresponding author:** Mr. Rakesh Kumar Verma is working as an Assistant Professor-HOD, in the Department of Applied Science in the **Yogananda College of Engineering & Technology, Jammu-180205 (J & K).** He has published 6-research paper in an National /International Journal/Conference before this paper.