Network Control System of Event-Driven Observer Based on Wirtinrer Inequality

Binbin Shen, Liankun Sun

***Abstract*— This paper concern about two different networked control systems, one is based on event-driven observer linear network control system, one is based on event-driven observer time-varying networked control system.The trigger mechanism of the event is represented by a delay model.Meanwhile, we use of some of the latest technology to dealwith the output system itself has the delay and the closure of the system due to the induced delay. The Wirtinrer's inequality, which has just been proposed in recent years, effectively reduces the conservativeness of the system.Using these techniques to construct the Lyapunov function, the existing stability condition is created under the condition of linear matrix inequality.The validity of the proposed method is given by a given numerical example .**

***Index Terms*— Network control system; event-driven; time-varying delay;transmission delay; Wirtinrer' s inequality**

I. INTRODUCTION

The Network Control System (NCS) consists of a series of system components (sensors, controllers and actuators) and shared networks. Compared with the traditional point-to-point control system, the network control system has some better advantages. Such as easy installation and maintenance and expansion, high reliability and flexibility, and resource sharing. Nowadays, the research problem of network control system has gradually become a hot topic in international control theory research[1-10].However, the network is not a reliable communication medium.Because of the limitations of network bandwidth and physical capacity, data transmission in the network inevitably has a series of problems. The most important is delay and packet loss[.Due to the basic parameters of network communication (network topology, bandwidth and communication protocol), the characteristics of network-induced delay are fixed or random.The main applications of NCS include sensor networks, industrial control networks, multi-autonomous coordinated control and microelectromechanical systems whose common purpose is to control one or more loop systems by deploying shared networks for data exchange [2−5]. Due to the application in the real field and the channel limitation of the network itself, some scholars have focused more on the delay processing in the network bandwidth and the maintenance of the system [6−10].

In today's control theory and application areas, the application and research of time-driven control systems dominates. In the time-driven control system, continuous signals are obtained by fixed cycle sampling. With the passage of time, event-driven gradually developed. Event-driven control systems, also known as non-periodic control systems or asynchronous control systems. Signal sampling and controller operations in the system are driven by specific events, rather than time-driven. Event-driven triggering scheme is based on artificial pre-set, in the system once meet the pre-set trigger conditions, the sampling data will be sent through the network. Compared with the time-driven trigger mode, it is effective to reduce the unnecessary use of network bandwidth]. Due to the application in some areas (LAN, fieldbus, etc.), more research focus and interest are used in this direction . And some of the more common triggering mechanisms are mentioned in the paper [11-22]]. In [20], the absolute error between the current sampled data and the newly triggered data is used as the trigger threshold. In [19], a relative error is used to generate the trigger threshold, which is triggered when the trigger condition of the system is satisfied.Researchers can set different triggering thresholds for triggering conditions, and propose a self-triggering scheme in [18,19]. Based on the above mechanism, most of the studies are focused on stability analysis [15,17,20,21], delay and packet loss and quantification . Today, more and more researchers will focus on research on event-driven networked control systems.

Inspired by the above thesis, this paper extends the triggering threshold of the event. We can adjust the trigger threshold by changing the parameters and the weighting matrix, and then generate the induced delay when the event-driven system is closed-loop. In order to derive the event-driven controller and the observer by the convex function, a new method is used to eliminate the coupling of the control matrix with other variables.Compared with [19], the Wirtinrer inequality is used in the process of functional processing, which reduces the conservativeness .

Notation: In this paper $L>0(L<0)$ denotes that the symmetric matrix L is positive (or negative)。 $\mathbb{R}^{m\times n}$ s defined as a set of $m\times n$ real matrixs。 E^T is the transpose of E 。 * denotes a symmetric term of a symmetric matrix , $\|\bullet\|$ refers to the Euclidean norm , $He(E)$ refers to $E+E^T$ 。 I_n refers to n dimensional unit matrix , $0_{m\times n}$ refers to the $m\times n$ dimension block matrix.The rest of the paper are adapted to the needs of the text of the adaptive dimension matrix

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II. DESCRIPTION OF THE PROBLEM

We consider the line system can be described by:

$$\begin{cases} \dot{x}(t) = Ax(t)) + Bu(t) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

where $x(t) \in ℝ^n$ is the state vector, $u(t) \in ℝ^p$ is the control output, $y(t) \in ℝ^q$ is the system output.A, B, C is symmetry。 A, B, C , is symmetry matrices with adaptive dimensions. We use the state observer to estimate the system state x(t), and the measured output y(t) is modeled as follows:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + L(y(i_k h) - C\hat{x}(i_k h)) \quad t \in [t_k h + \tau_{t_k}, t_k h + \tau_{t_{k+1}}] \quad (2)$$

where $x(t) \in ℝ^{(n)}$ refers to the observed state, L is the designed observer gain, τ_{t_k} is the communication delay.Therefore,taking into account the transmission capacity of the communication channel with the capacity limit is based on the estimated state of the event-driven transmitter is created

$$\begin{cases} \dot{x}(t) = Ax(t)) + Bu(t) \\ y(t) = Cx(t) \\ u(t) = K\hat{x}(t_k h), t \in \Omega \end{cases} \quad (3)$$

where K indicates the controller gain. Considering the limitation of the channel during the information transmission process, we designed an event-driven information transmission mechanism for the system:

$$t_{k+1}h = t_k h + \min_l \{lh \,|\, \{e^T(i_k h)Ve(i_k h) \geq \gamma^2(t)\} \quad (4)$$

where $e(i_k h) = \hat{x}(i_k h) - \hat{x}(t_k h)$ refers to the error between the observer state at the current sampling time $i_k h = t_k h + lh(l \in ℕ)$ and the observer state at the latest triggered time $t_k h$; Φ refers to a symmetric positive definite weighting matrix; $\gamma(t) = \sqrt{\beta\varepsilon^{-\alpha t} + \varepsilon_0}$ is the error threshold with $\varepsilon > 1, \beta > 0, 0 \leq \alpha < 1, \alpha_1 > 0, \varepsilon_0 \geq 0$; h refers to the time sampling period.

The latest trigger signal arrives at the actuator node, and the zeroth order keeper (ZOH) produces a hold time interval for the output signal. We divide the set into $\Omega_1 = [i_k h + \tau_{i_k}, i_k h + h + \tau_{i_k+1}] \Omega = \cup\Omega_1$, where $i_k h = t_k h + lh, l = 0, \ldots, t_{k+1} - t_{k-1}$ 。 We define the sampling time from the current trigger $t_k h$ time to the next trigger $t_{k+1}h$ as sampling time $l = t_{k+1} - t_k - 1$ 。 Immediately-following $\tau_{i_{k+1}} = \tau_{t_{k+1}}$,otherwise $\tau_{i_k} = \tau_{t_k}$ defines $d(t) = t - i_k h$ and $\dot{d}(t) = 1, t \in \Omega_l$. Meanwhlie $0 < d(t) < h + d(t) = d_M$, d_M is the maximum transmission delay. Combing (2) , (4) and ZOH, the resulted closed-loop system can established as:

$$\begin{cases} \dot{x}(t) = Ax(t)) + BKx(t-\tau(t)) - BK\tilde{x}(t-\tau(t)) \\ -BK\tilde{n}(i_k h) \\ \dot{\tilde{x}}(t) = A\tilde{x}(t)) - LC\tilde{x}(t-\tau(t)) \\ u(t) = K\hat{x}(t_k h), t \in \Omega_l \end{cases} \quad (5)$$

$\xi(t) = \begin{bmatrix} x(t) \\ \tilde{x}(t) \end{bmatrix}$. an augmented closed-loop system is rewritten as:

$$\dot{\xi}(t) = A_1\xi(t) + A_2\xi(t-\tau(t)) + B_1 e(i_k h), t \in \Omega_l \text{。} \quad (6)$$

Where $A_1 = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}$, $A_2 = \begin{bmatrix} BK & -BK \\ 0 & -LC \end{bmatrix}$,

Definition 1 [19]: For a given positive integer p, q, a scalar δ at interval (0,1), a given positive definite matrix R belongs to $ℝ^P$. Both matrices M_1 and M_2 belong to $ℝ^{p\times q}$, for all vectors ϑ belonging to $ℝ^q$. function $\Im(\delta, R)$ can be expressed as

$$\Im(\delta, R) = \frac{1}{\delta}\vartheta^T M_1^T R M_1 \vartheta + \frac{1}{1-\delta}\vartheta^T M_2^T R M_2 \vartheta \quad (7)$$

Immediately after the existence of a matrix U belongs to $ℝ^{p\times q}$, and $\begin{bmatrix} R & U^T \\ * & R \end{bmatrix} > 0$. Can get inequalities:

$$\min_{\delta \in (0,1)} \Im(\delta, R) \geq \begin{bmatrix} M_1\vartheta \\ M_2\vartheta \end{bmatrix}^T \begin{bmatrix} R & U^T \\ * & R \end{bmatrix} \begin{bmatrix} M_1\vartheta \\ M_2\vartheta \end{bmatrix} \quad (8)$$

Definition 2: [19] Given the adaptive dimension matrix D, $E(t)$ and F satisfied $E^T(t)E(t) \leq I$, for any $\varepsilon > 0$, the following inequalities are true:

$$DE(t)F + F^T E^T(t) D^T \leq \varepsilon DD^T + \varepsilon^{-1} F^T F \quad (9)$$

Definition 3 : [19] The following two inequalities are equal:

(a) There is a symmetry and the positive definite matrix P satisfies

$$\begin{bmatrix} -P & A^T \\ A & -P^{-1} \end{bmatrix} < 0. \quad (10)$$

(b)There is a positive definite symmetric P matrix and Y is satisfied :

$$\begin{bmatrix} -P & (YA)^T \\ YA & He(-Y) + P \end{bmatrix} < 0 \quad (11)$$

Definition 4 : [20] For a given matrix $R > 0$, the following inequality applies to all successively differentiable functions ω belongs to $[a, b] \to ℝ^p$

$$\int_b^a \dot{\omega}^T(u)R\dot{\omega}(u)du \ldots \frac{1}{b-a}(\omega(b)-\omega(a))^T R(\omega(b) -\omega(a)+\frac{3}{b-a}\Im^T R\Im \quad (12)$$

Where $\Im = \omega(b)+\omega(a)-\int_b^a \omega(u)du$

III. PROVING PROCESS

The linear matrix inequality is used to prove that the event-driven networked control system is stable and the system state $\xi(t)$ exponential converges to the final bounded set $Bd(\varepsilon_0)$.

Theorem 1:Consider the closed loop(6) system parameters driving mechanism (4)] with $\varepsilon>0,\ 0<\alpha<1,\ \beta>0,\ d_M>0$ Given decay rate $\delta>0$ if there exist matrices $P_1>0, P_2>0, R_1>0,$ $\Psi>0,$ $V>0$, J and $U_{ij}(i=A,B,C,D.\ j=1,2,3,4.)$, ,matrices Z,S,Q with appropriate dimensions such that

$$\begin{bmatrix} \Delta_{11}+J & \Delta_{12} & 0 & 0 \\ * & \Delta_{22} & \Delta_{23} & 0 \\ * & * & He(-B^T BS) & \Delta_{34} \\ * & * & * & -J \end{bmatrix} < 0 \quad (13)$$

where

$W_{11} = He(P_1 A) - 4\varphi R_1 + \delta P_1,$

$W_{17} = (U^T{}_{A1}+U^T{}_{B1})-(U^T{}_{C1}+U^T{}_{D1}),$

$W_{18} = (U^T{}_{A2}+U^T{}_{B2})-(U^T{}_{C2}+U^T{}_{D2}),$

$W_{112} = -2\varphi R_1 - (U_{A1}+U_{B1}+U_{C1}+U_{D1})^T,$

$W_{113} = -(U_{A2}+U_{B2}+U_{C2}+U_{D2})^T,$

$W_{22} = He(P_2 A) - 4\varphi P_2 + \delta P_2,$

$W_{27} = (U^T{}_{A3}+U^T{}_{B3})-(U^T{}_{C3}+U^T{}_{D3}),$

$W_{28} = (U^T{}_{A4}+U^T{}_{B4})-(U^T{}_{C4}+U^T{}_{D4}),$

$W_{212} = (U_{A3}+U_{B3}+U_{C3}+U_{D3})^T,$

$W_{213} = -QC - (U_{A4}+U_{B4}+U_{C4}+U_{D4})^T,$

$W_{312} = 6\varphi R_1 - 2(U_{C1}+U_{D1}),$

$W_{313} = -2(U_{C2}+U_{D2}),$

$W_{413} = 6\varphi P_2 - 2(U_{C4}+U_{D4}),$

$W_{512} = 6\varphi R_1 + 2(U_{A1}+U_{D1})^T,$

$W_{613} = 6\varphi P_1 + 2(U_{B4}+U_{D4})^T,$

$W_{712} = -2\varphi R_1 - U_{A1} + U_{B1} + U_{C1} - U_{D1},$

$W_{713} = -U_{A2} + U_{B2} + U_{C2} - U_{D2},$

$W_{812} = -U_{A3} + U_{B3} + U_{C3} - U_{D3},$

$W_{813} = -2\varphi P_2 - U_{A4} + U_{B4} + U_{C4} - U_{D4},$

$W_{1212} = -8\varphi R_1 + He(U_{A1} - U_{B1} + U_{C1} - U_{D1}),$

$W_{1213} = He(U_{A2} - U_{B2} + U_{C2} - U_{D2}),\ \varphi = \dfrac{e^{d_M}}{d_M}$

$W_{1313} = -8\varphi P_2 + He(U_{A4} - U_{B4} + U_{C4} - U_{D4}),$

$$\Delta_{11} = \left[\begin{matrix} W_{11} & 0 & 2(U_{C1}^T+U_{D1}^T) & 2(U_{C2}^T+U_{D2}^T) & 6\varphi R_1 & 0 & W_{17} & W_{18} & \sqrt{d_M}(R_1A)^T & 0 \\ * & W_{22} & 2(U_{C3}^T+U_{D3}^T) & 2(U_{C4}^T+U_{D4}^T) & 0 & 6\varphi P_2 & W_{27} & W_{28} & 0 & \sqrt{d_M}(P_2A)^T \\ * & * & -12\varphi R_1 & 0 & -4U_{D1} & -4U_{D2} & 6\varphi R_1 & 0 & 0 & 0 \\ * & * & * & -12\varphi P_2 & -4U_{D3} & -4U_{D4} & 0 & 6\varphi P_2 & 0 & 0 \\ * & * & * & * & -12\varphi R_1 & 0 & 2(U_{41}^T-U_{21}^T) & 2(U_{43}^T-U_{23}^T) & 0 & 0 \\ * & * & * & * & * & -12\varphi P_2 & 2(U_{42}^T-U_{22}^T) & 2(U_{44}^T-U_{24}^T) & 0 & 0 \\ * & * & * & * & * & * & -4\varphi R_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -4\varphi P_2 & 0 & 0 \\ * & * & * & * & * & * & * & * & -R_1 & 0 \\ * & * & * & * & * & * & * & * & * & -P_2 \end{matrix}\right] < 0$$

$$\Delta_{22} = \begin{bmatrix} -V & -V_1 & -V_2 \\ * & W_{1212} & W_{1213} \\ * & * & W_{1313} \end{bmatrix}$$

$V = \begin{bmatrix} V_1 & V_2 \end{bmatrix}$

$$\Delta_{23} = \begin{bmatrix} -(B^T BS)^T \\ (B^T BS)^T \\ -(B^T BS)^T \end{bmatrix}$$

$$\Delta_{12}=\begin{bmatrix} -BS & W_{112}+BS & -BS+W_{113} \\ 0 & W_{212} & W_{213} \\ 0 & W_{312} & W_{313} \\ 0 & -2(U_{C2}-U_{D2}) & W_{413} \\ 0 & W_{512} & 2(U_{B2}+U_{D2})^T \\ 0 & 2(U_{B3}+U_{D3})^T & W_{613} \\ 0 & W_{712} & W_{713} \\ 0 & W_{812} & W_{813} \\ -\sqrt{d_M}BS & \sqrt{d_M}BS & -\sqrt{d_M}BS \\ 0 & 0 & -\sqrt{d_M}QC \end{bmatrix}$$

$$\Psi=\begin{bmatrix} R_1 & 0 & 0 & 0 & U_{A1} & U_{A2} & U_{B1} & U_{B2} \\ * & P_2 & 0 & 0 & U_{A3} & U_{A4} & U_{B3} & U_{B4} \\ * & * & 3R_1 & 0 & U_{C1} & U_{C2} & U_{D1} & U_{D2} \\ * & * & * & 3P_2 & U_{C3} & U_{C4} & U_{D3} & U_{D4} \\ * & * & * & * & R_1 & 0 & 0 & 0 \\ * & * & * & * & * & P_2 & 0 & 0 \\ * & * & * & * & * & * & 3R_1 & 0 \\ * & * & * & * & * & * & * & 3P_2 \end{bmatrix}$$

$$\Delta_{34}=\begin{bmatrix}(P_1B-BS)^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\tau_M}(R_1B-BS)^T & 0\end{bmatrix}^T$$

hen the system state index converges to the final bounded set.

$$Bd(\varepsilon_0)=\{\xi(t)\,|\,\|\xi(t)\|\le\sqrt{\frac{\varepsilon_0}{\delta\lambda_{\min}(p_1)}}\} \quad (14)$$

where $K=G^{-1}S, L=P_2^{-1}Q$

Proof we choose a candidate Lyapunov function as:

$$V(t)=\xi(t)^T H\xi(t)+(d_M-d(t)) \times\int_{t-d_M}^{t}e^{\sigma(s-t)}\dot{\xi}^T(s)R\dot{\xi}(s)ds \quad (15)$$

Where $P=\begin{bmatrix}P_1 & 0\\ 0 & P_{2,}\end{bmatrix}, R=\begin{bmatrix}R_1 & 0\\ 0 & P_{2,}\end{bmatrix}$。

Deriving (15) and substituting $\dot{d}(t)=1$, then according to (3) can gain

$$\begin{aligned}\dot{V}(t)+\delta V(t) &\le 2\dot{\xi}^T(t)P\dot{\xi}^T(t) \\ &-\int_{t-d_M}^{t}e^{-\delta d_M}\dot{\xi}^T(s)R\dot{\xi}(s)ds \\ &+(d_M-d(t))\dot{\xi}^T(t)R\dot{\xi}(t) \\ &-(d_M-d(t))e^{-\delta d_M}\dot{\xi}^T(t-d_M)R\dot{\xi}(t-d_M) \\ &+\delta\xi^T(t)P\xi(t)+\tilde{n}^T(i_kh)V\tilde{n}(i_kh) \\ &-\tilde{n}^T(i_kh)V\tilde{n}(i_kh) \\ &\le 2\dot{\xi}^T(t)P\dot{\xi}^T(t)-e^{-\delta d_M}\int_{t-d_M}^{t}\dot{\xi}^T(s)R\dot{\xi}(s)ds \\ &+\tau_M\dot{\xi}^T(t)R\dot{\xi}(t)+\delta\xi^T(t)P\xi(t) \\ &+\alpha x^T(t-d(t))Vx(t-d(t)) \\ &-\alpha x^T(t-d(t))V\tilde{n}(i_kh) \\ &+\tilde{n}^T(i_kh)Vx(t-d(t))+(\alpha-1)\tilde{n}^T(i_kh)V\tilde{n}(i_kh) \\ &+\gamma^2(t) \end{aligned} \quad (16)$$

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According to the definition 4 and the definition 3, you can gain:

$$\begin{aligned}&-e^{d_M}\int_{t-\tau_M}^{t}\dot{\xi}^T(s)R\dot{\xi}^T(s)ds\le-\frac{e^{d_M}}{d_M} \\ &\times\frac{d_M}{d_M-d(t)}\eta^T(t)\begin{bmatrix}e_1^T & e_2^T\end{bmatrix}\begin{bmatrix}R & 0\\ 0 & 3R\end{bmatrix}\begin{bmatrix}e_1\\ e_2\end{bmatrix}\eta(t) \\ &-\frac{e^{d_M}}{d_M}\times\frac{d_M}{d(t)}\eta^T(t)\begin{bmatrix}e_3^T & e_4^T\end{bmatrix}\begin{bmatrix}R & 0\\ 0 & 3R\end{bmatrix}\begin{bmatrix}e_3\\ e_4\end{bmatrix}\eta(t) \\ &\le\eta^T(t)\Gamma^T\Xi\Gamma\eta(t)\end{aligned} \quad (17)$$

Define $\varphi=\frac{e^{d_M}}{d_M}$,

where $\Gamma^T=\begin{bmatrix}e_1^T & e_2^T & e_3^T & e_4^T\end{bmatrix}$

$$\Xi=\begin{bmatrix}R & 0 & U_A & U_B\\ * & 3R & U_C & U_D\\ * & * & R & 0\\ * & * & * & 3R\end{bmatrix},$$

$$U_i=\begin{bmatrix}U_{i1} & U_{i2}\\ U_{i3} & U_{i4}\end{bmatrix} i=(A,B,C,D)$$

$$e_1=\begin{bmatrix}0 & 0 & 0 & -I & I\end{bmatrix}$$

$$e_2=\begin{bmatrix}0 & -2I & 0 & I & I\end{bmatrix} e_3=\begin{bmatrix}I & 0 & 0 & 0 & -I\end{bmatrix}$$

$$e_4=\begin{bmatrix}I & 0 & -2I & 0 & I\end{bmatrix}$$

Followed by formula (17) into (16):

$$\dot{V}(t)+\delta V(t)\le\varsigma^T(t)(\Pi+\Sigma^TR^{-1}\Sigma)\varsigma(t)+\gamma^2(t) \quad (18)$$

where

$$\eta^T(t)=\begin{bmatrix} x^T(t) & H_1 & H_2 & x^T(t-d_M) & x^T(t-d(t)) \end{bmatrix}$$

$$H_1=\frac{1}{d_M-\tau(t)}\int_{t-d_M}^{t-d(t)} x^T(s)ds$$

$$H_2=\frac{1}{d(t)}\int_{t-d(t)}^{t} x^T(s)ds$$

$$\varsigma^T(t)=\begin{bmatrix} \eta^T(t) & \tilde{n}^T(i_k h) \end{bmatrix}$$

$$F_{11}=He(PA_1)+\delta P-4\varphi R$$

$$F_{12}=2(U_A^T+U_D^T)$$

$$F_{14}=2(U_A^T+U_B^T)-(U_C^T+U_D^T)$$

$$F_{15}=-2\varphi R-U_A^T-U_B^T-U_C^T-U_D^T+PA_2$$

$$F_{25}=6\varphi R-2(U_C-U_D)$$

$$F_{35}=6\varphi R+2(U_2^T+U_4^T)$$

$$F_{45}=-2\varphi R-U_A+U_B+U_C-U_D$$

$$F_{55}=-8\varphi R+He(U_A-U_B+U_C-U_D)$$

$$\Sigma=\begin{bmatrix} \sqrt{d_M}RA_1 & 0 & 0 & 0 & \sqrt{d_M}RA_2 & \sqrt{d_M}RB_1 \end{bmatrix}$$

$$\Pi=\begin{bmatrix} F_{11} & F_{12} & 6\varphi R & F_{14} & F_{15} & PB_1 \\ * & -12\varphi R & -4U_D & 6\varphi R & F_{25} & 0 \\ * & * & -12\varphi R & 2(U_D^T-U_B^T) & F_{35} & 0 \\ * & * & * & -4\varphi R & F_{45} & 0 \\ * & * & * & * & F_{55} & 0 \\ * & * & * & * & * & -V \end{bmatrix} \quad (19)$$

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r (13) applying the Soul's complement theorem and definition 3 can be obtained

$$\begin{bmatrix} \Delta_{11}+J & \Delta_{12} & 0 \\ * & \Delta_{22} & \Xi_{23} \\ * & * & -(\Delta_{34}J^{-1}\Delta_{34}^T)^{-1} \end{bmatrix}<0 \quad (20)$$

Where $\Xi_{23}=\begin{bmatrix} -G^{-1}S & G^{-1}S & -G^{-1}S \end{bmatrix}^T$

$$\begin{bmatrix} \Delta_{11} & \Delta_{12} \\ * & \Delta_{22} \end{bmatrix}+\begin{bmatrix} I_{10} \\ 0_{3\times10} \end{bmatrix} J \begin{bmatrix} I_{10} \\ 0_{3\times10} \end{bmatrix}^T +\begin{bmatrix} 0_{10\times1} \\ \Xi_{23} \end{bmatrix}\Delta_{34}J^{-1}\Delta_{34}^T\begin{bmatrix} 0_{1\times10} \\ \Xi_{23}^T \end{bmatrix}<0 \quad (21)$$

By definition 2, the following inequality is established. :

$$\begin{bmatrix} I_{10} \\ 0_{3\times10} \end{bmatrix} J \begin{bmatrix} I_{10} \\ 0_{3\times10} \end{bmatrix}^T +\begin{bmatrix} 0_{10\times1} \\ \Xi_{23} \end{bmatrix}\Delta_{34}J^{-1}\Delta_{34}^T\begin{bmatrix} 0_{1\times10} \\ \Xi_{23}^T \end{bmatrix}\geq He\left[\begin{bmatrix} I_{10} \\ 0_{3\times10} \end{bmatrix}\begin{bmatrix} P_1B-BG \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \sqrt{d_M}(R_1B-BG) \\ 0 \end{bmatrix} G^{-1}S\begin{bmatrix} 0_{1\times10} & -I & I & -I \end{bmatrix}\right] \quad (22)$$

Union (22) and $L=P_2^{-1}Q$, (21) can be written as: :

$$\begin{bmatrix} F_{11} & F_{12} & 6\varphi R & F_{14} & \sqrt{d_M}(RA_1)^T & PB_1 & F_{15} \\ * & -12\varphi R & -4U_D & 6\varphi R & 0 & 0 & F_{25} \\ * & * & -12\varphi R & 2(U_D^T-U_B^T) & 0 & 0 & F_{35} \\ * & * & * & -4\varphi R & 0 & 0 & F_{45} \\ * & * & * & * & -R & \sqrt{d_M}(RB_1) & \sqrt{d_M}(RA_2) \\ * & * & * & * & * & -V & 0 \\ * & * & * & * & * & * & F_{55} \end{bmatrix}<0 \quad (23)$$

(25) left by Λ and right by Λ^T to get (26)

$$\begin{bmatrix} F_{11} & F_{12} & 6\varphi R & F_{14} & F_{15} & PB_1 & \sqrt{d_M}(RA_1)^T \\ * & -12\varphi R & -4U_D & 6\varphi R & F_{25} & 0 & 0 \\ * & * & -12\varphi R & 2(U_D^T-U_B^T) & F_{35} & 0 & 0 \\ * & * & * & -4\varphi R & F_{45} & 0 & 0 \\ * & * & * & * & F_{55} & 0 & \sqrt{d_M}(RA_2)^T \\ * & * & * & * & * & -V & \sqrt{d_M}(RB_1)^T \\ * & * & * & * & * & * & -R \end{bmatrix} < 0 \qquad (24)$$

where $\Lambda = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I & 0 & 0 \end{bmatrix}$

We can see that (24) was derived from the Schur supplement of Π. So we can get the conclusion: for any non-zero matrix $\zeta(t)$ can get $\varsigma^T(t)(\Pi+\Sigma^T R^{-1}\Sigma)\varsigma(t)<0$, Union (16) and (4) can gain：

$$\dot{V}(t)+\delta V(t) \leq \gamma^2(t) \qquad (25)$$

Apply the conclusion of [28] and substituting the formula into (25)

$$:V(t)\leq e^{-\delta t}V(0)+\int_0^t e^{-\delta(t-s)}\gamma^2(s)ds \qquad (26)$$

The above $\gamma^2(t)=\beta\varepsilon^{-\alpha t}+\varepsilon_0=\beta e^{(-\alpha\ln\varepsilon)}+\varepsilon_0$ can be rewritten as:

$$\begin{aligned} V(t) &\leq e^{-\delta}V(0)+e^{-\delta t}\int_0^t e^{(\delta-\alpha\ln\varepsilon t)}ds \\ &+\varepsilon_0\int_0^t e^{-\delta(t-s)}ds = e^{-\delta t}(V(0)-\frac{\varepsilon_0}{\sigma})+\frac{\varepsilon_0}{\delta} \qquad (27) \\ &+\beta e^{-\delta t}\int_0^t e^{(\delta-\alpha\ln\varepsilon t)}ds \end{aligned}$$

Let's discuss the classification below:

if $\delta-\alpha\ln\varepsilon=0$ can gain：

$$V(t)\leq e^{-\delta t}(V(0)-\frac{\varepsilon_0}{\sigma}+\beta t)+\frac{\varepsilon_0}{\delta} \qquad (28)$$

if $\delta-\alpha\ln\varepsilon>0$ can gain：

$$\begin{aligned} V(t) &\leq e^{-\delta t}(V(0)-\frac{\varepsilon_0}{\delta}) \\ &+\frac{\beta e^{-\delta t}}{\delta-\alpha\ln\varepsilon}(e^{(\sigma-\alpha\ln\varepsilon)t}-1) \qquad (29) \\ &= e^{-\delta t}(V(0)-\frac{\varepsilon_0}{\delta}-\frac{\delta}{\alpha-\ln\varepsilon}) \\ &+\frac{\varepsilon_0}{\delta}+\frac{\beta\varepsilon^{-\alpha t}}{\delta-\alpha\ln\varepsilon}. \end{aligned}$$

if $\delta-\alpha\ln\varepsilon<0$, can gain

$$\begin{aligned} V(t) &\leq e^{-\delta t}(V(0)-\frac{\varepsilon_0}{\delta}-\frac{\delta}{\alpha-\ln\varepsilon}) \\ &+\frac{\varepsilon_0}{\delta}+\frac{\beta e^{-\delta t}}{\alpha-\delta\ln\varepsilon} \qquad (30) \end{aligned}$$

Combining the above formula, so regardless of the value of $\delta-\alpha\ln\varepsilon$, the event-driven network control system is exponentially convergent to the bounded set.

$$Bd(\varepsilon_0)=\{\xi(t)\,|\,\|\xi(t)\| \leq \sqrt{\frac{\varepsilon_0}{\delta\lambda_{\min}(p_1)}}\}$$

Ⅳ Simulation

Consider system (3) with parameters

$$A=\begin{bmatrix} -0.2 & 0.1 \\ 0.2 & -0.5 \end{bmatrix}, C=\begin{bmatrix} 1 \\ 1 \end{bmatrix}, D=\begin{bmatrix} 1 & 1 \end{bmatrix},$$

Applying the theorem1 with h=0.01 $d_M=0.5$, α_1=0.1。

With the change of δ, the corresponding controller gain K, the observer gain L and event-driven matrix V are listed in the table.

δ	0.1	0.2	0.3
K	$\begin{bmatrix} -0.1079 \\ -0.0771 \end{bmatrix}^T$	$\begin{bmatrix} -0.1205 \\ -0.0793 \end{bmatrix}^T$	$\begin{bmatrix} -0.1418 \\ -0.0861 \end{bmatrix}^T$
L	$\begin{bmatrix} 0.6132 \\ 0.6321 \end{bmatrix}$	$\begin{bmatrix} 0.6145 \\ 0.6271 \end{bmatrix}$	$\begin{bmatrix} 0.6187 \\ 0.6240 \end{bmatrix}$
V	$\begin{bmatrix} -14.197 & 17.204 \\ 117.59 & 14.064 \end{bmatrix}$	$\begin{bmatrix} 13.87 & 1.44 \\ -0.97 & 13.67 \end{bmatrix}$	$\begin{bmatrix} 13.03 & 43.38 \\ -42.81 & 12.74 \end{bmatrix}$

table： K, L, V

Given h=0.01, $\delta=0.2$ Controller $K=\begin{bmatrix} -0.1205 \\ -0.0793 \end{bmatrix}^T$

Observer Gain $L=\begin{bmatrix}0.6145\\0.6271\end{bmatrix}$ and Weight Matrix

$V=\begin{bmatrix}13.03 & 43.38\\-42.81 & 12.74\end{bmatrix}$ By the above known conditions we can conclude that the linear system (1) based on the event-driven observer is stable. initial conditions are

$$x(0)=\begin{bmatrix}1\\-1\end{bmatrix} \text{ if } \delta=0.2,\alpha_1=0,\beta=0.1,$$

$\alpha=0.5,\ \varepsilon=e,\varepsilon_0=0.01$, Then according to the parameters we set above, some simulation images are displayed.

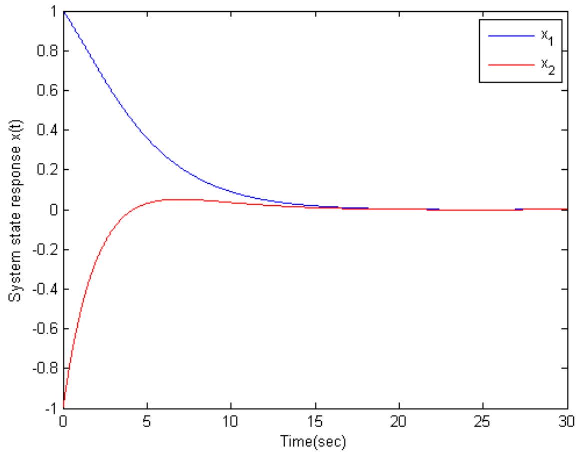

Figure 1: System State Diagram

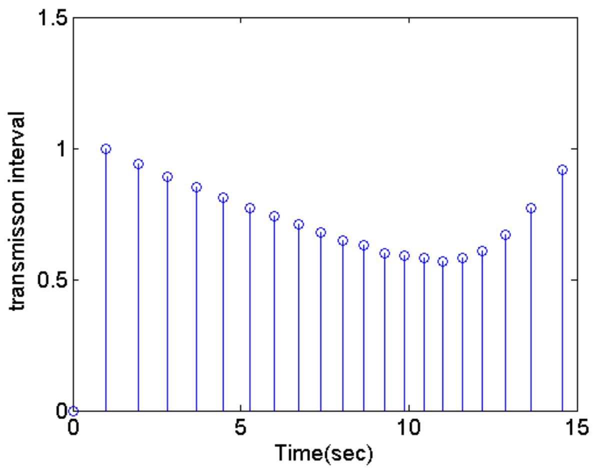

Figure 2: Event Driven Diagram

CONCLUSION

This paper is improved on the basis of [19]. The Wirtinger inequality is chosen for the expansion of the integral term after the Lyapunov function is derived. This method reduces the conservativeness of the system and speeds up the system's time to reach a stable state. In the end, the experimental results prove the validity of our ideas.7

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