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Synchronization Control for Coronary Artery System via Jensen Inequality

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***Abstract*—In this paper, we provide a state feedback control strategy for synchronization of coronary artery system(CAS). In order to obtain less conservative results, we design a state feedback controller to achieve synchronization based on the Jensen integral inequality by constructing Lyapunov-Krasovskii functional (LKF). Finally, the simulation results are illustrated to demonstrate the validity of the proposed state feedback control method.**

***Index Terms*—Coronary artery system,Jensen inequality, Synchronization control.**

I. INTRODUCTION

The synchronization of the chaotic systems has been flourished as an appealing research area owing to its potential applications in the image encryption, communication security, biomedical engineering, chemical reaction, brain disorder and so on [1-6]. The chief purpose of chaos synchronization is to realize the identical behavior between the drive and response systems by means of the feedback control. In the past few years, the synchronization of chaotic systems under uncertainty, noise, disturbance[7-8], has been investigated by different nonlinear control schemes [9], such as fuzzy control[10], sliding mode control[11] and impulse control [12], etc. Applying chaos theory to biology and biomedicine greatly promotes the research in the field of biomedicine, such as cardiovascular diseases, nervous system, pathological phenomena and so on. Medical experts believe that the coronary artery vasospasm is an inducement of some diseases related to myocardial ischemia, such as the angina pectoris, myocardial infarction, sudden death syndrome. Once the coronary artery vessels enter the chaotic state[13], the coronary artery vasospasm will be caused. Therefore, understanding the nonlinear characteristic of coronary vasospasm, suppressing the emergence of chaos phenomenon, achieving the synchronization of the healthy and diseased CAS are of great theoretical significance and practical potentiality for realizing the treatment of vessel-related diseases.

A sliding mode control scheme is investigated for synchronization of CACS in the literature [14], which can stabilize the synchronization error system in finite time.In the work [15], the differential transformation approach is utilized to handle the governing equations of CACS, and the presented state feedback control scheme can make coronary artery system synchronize to any state. Considering that the bound of perturbation is unknown, the literature [16] designs a high-order sliding mode adaptive controller for synchronization of CAS, which can effectively alleviate the chattering influence. Taking the external disturbance and input time-varying delay into account, the work[17] presents the state feedback control scheme for CAS synchronization. Based on the work [17], the literature [18] achieves the synchronization of CAS by delay-partitioning approach, which can effectively reduce the conservation.

Motivated by the above discussions, the paper addresses the controller design for synchronization of CAS with disturbances or perturbations. To the best of my knowledge, fewer literatures involve the issue mentioned above. Via the Jensen inequality, we design a state feedback controller for synchronization of coronary artery system by application of Lyapunov-Krasovskii functional. The simulation results demonstrate that the CAS in the state of vasospasm can synchronize to the normal CAS by the proposed scheme in finite time, which provides certain theoretical basis for the treatment of coronary vascular diseases.

II. PROBLEM FORMULATION AND RELATED KNOWLEDGE

The mathematical model of the coronary artery system is described as follows:

$$\dot{x}_1 = -bx_1 - cx_2$$
$$\dot{x}_2 = -(b+1)\lambda x_1 - (c+1)\lambda x_2 + \lambda x_1^3 + E\cos(\sigma t),$$

where the inner diameter change, inner pressure change and periodical perturbation of the coronary artery vessel are represented by x_1, x_2 and $E\cos(\sigma t)$ respectively. Once the coronary artery vessel occurs the pathological change, the parameter λ will follow change. When λ=-0.5, the coronary artery system will occur the chaotic behavior with E=0.3, σ=1, b=0.15, c=-1.7.

Considering two chaotic system, the nonlinear master and slave time-delay system can be written as follows:

$$\dot{x}_m = Ax_m + A_1 x_m(t-\tau(t)) + Cf(x_m) + g$$
$$\dot{x}_s = Ax_s + A_1 x_s(t-\tau(t)) + Cf(x_s) + g + \omega + u,$$

According to master and salve system, synchronization error is defined as $e = x_s - x_m$. By defining the controller u=Ke, we can describe the error system as follows:

$$\dot{e} = (A+K)e + A_1 e(t-\tau(t)) + Cf(x_s, x_m) + \omega,$$

Lemma (Jensen inequality) Suppose that R>0 is symmetric matrix, x is a continuously differentiable function $x \in [a,b] \to R^n$, we can obtain:

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$$\int_a^b \dot{x}^T(\varphi)\, R\, \dot{x}(\varphi) \geq \frac{1}{b-a}\begin{pmatrix}\omega_1\\ \omega_2\end{pmatrix}^T \begin{bmatrix} R & 0\\ * & 3R\end{bmatrix}\begin{pmatrix}\omega_1\\ \omega_2\end{pmatrix}$$

Where

$$\omega_1 = x(b) - x(a)$$
$$\omega_2 = x(b) + x(a) - \frac{2}{b-a}\int_a^b x(\varphi)\, d\varphi$$

III. SYNCHRONIZATION CONTROL

Theorem. Consider the error system obtained by drive and response systems, the error e asymptotically converge to zero if there exist positive-definite symmetric matrices $P \in R^{n*n}$, $Q_s \in R^{n*n}$, $Z_v \in R^{n*n}$, the appropriate dimensions matrices S_v , s=1,2,3, v=1,2, and the scalars τ_m, τ_M, τ_d such that LMIs:

$$\begin{bmatrix} \Pi & \tau_m \bar{\Lambda}^T & \tau_{Mm}\bar{\Lambda}^T & {h_1}^T P & {h_1}^T P L^T \\ * & H_1 & 0 & 0 & 0 \\ * & * & H_2 & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0$$

Where

$$\Pi = h_1^T \Phi + \Phi^T h_1 + R + \Upsilon - \Lambda_1^T Z_1 \Lambda_1 - {\Lambda_2}^T 3Z_1 \Lambda_2 - \Lambda_3^T Z_2 \Lambda_3 - \Lambda_4^T 3Z_2 \Lambda_4 - \Lambda_5^T Z_2 \Lambda_5 - \Lambda_6^T 3Z_2 \Lambda_6 - \Lambda_3^T S_1 \Lambda_5$$
$$H_1 = -P\overline{Z_1}^{-1}P$$
$$H_2 = -P\overline{Z_2}^{-1}P$$
$$V_1 = e^T P e$$
$$V_2 = \int_{t-\tau(t)}^{t} e^T R_1 e d\varphi + \int_{t-\tau_1}^{t} e^T R_2 e d\varphi + \int_{t-\tau_2}^{t} e^T R_3 e d\varphi$$
$$V_3 = \tau_1 \int_{-\tau_1}^{0}\int_{t+\varphi}^{t} \dot{e}^T Z_1 \dot{e}\, d\mu d\varphi + \tau_{12}\int_{-\tau_2}^{-\tau_1}\int_{t+\varphi}^{t} \dot{e}^T Z_2 \dot{e}\, d\mu d\varphi$$

IV. SIMULATION

To exhibit the validity of the presented synchronization control strategy, the simulation results of CAS are given as follows. The parameters of system master and slave are considered:

$$A = \begin{bmatrix} -0.1 & 1.5 \\ 0.75 & -0.25 \end{bmatrix}$$
$$A_1 = \begin{bmatrix} -0.05 & 0.2 \\ 0.025 & -0.1 \end{bmatrix}$$

(1) Suppose $\tau(t) = 0.4 + 0.3\sin t$, $\gamma = 0.8$, the controller gain matrix K1 is computed from Theorem as:

$$K_1 = \begin{bmatrix} -2.5837 & 0.7684 \\ -0.2663 & -3.3596 \end{bmatrix}$$

By employing the controller, the healthy and diseased CAS are synchronized as depicted by Figure1.

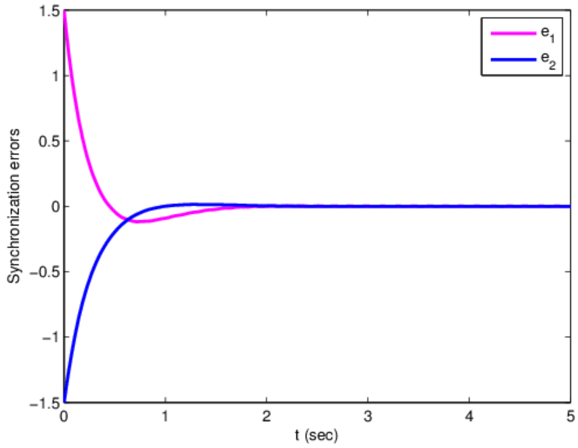

Figure1: The error system with K1

(2) Suppose $\tau(t) = 0.04 + 0.1\sin t$, $\gamma = 0.02$, the controller gain matrix K2 is computed from Theorem as:

$$K_2 = \begin{bmatrix} -1.0427 & 0.0004 \\ -0.2655 & -3.3563 \end{bmatrix}$$

As we can see from Figure 2, the error system converges to zero in finite time, which indicates that the diseased and healthy CAS can produce the identical behaviors with the control under disturbances, and also shows that the synchronization controller has the desirable robust performance.

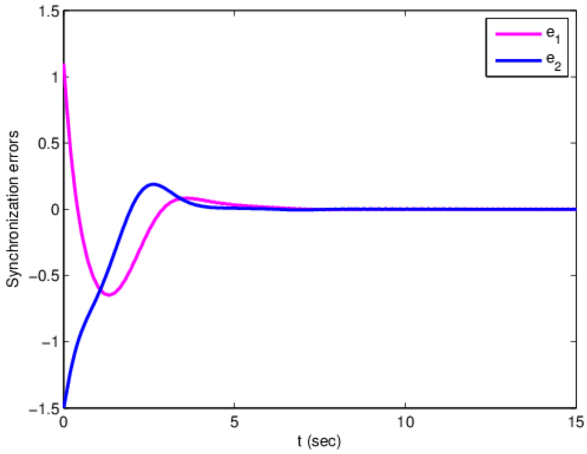

Figure2: The error system with K2

(3) Suppose $\tau(t) = 0.15 + 0.05\sin t$, $\gamma = 0.01$, the controller gain matrix K3 is computed from Theorem as:

$$K_3 = \begin{bmatrix} -4.3837 & 0.0084 \\ -0.0003 & -2.3346 \end{bmatrix}$$

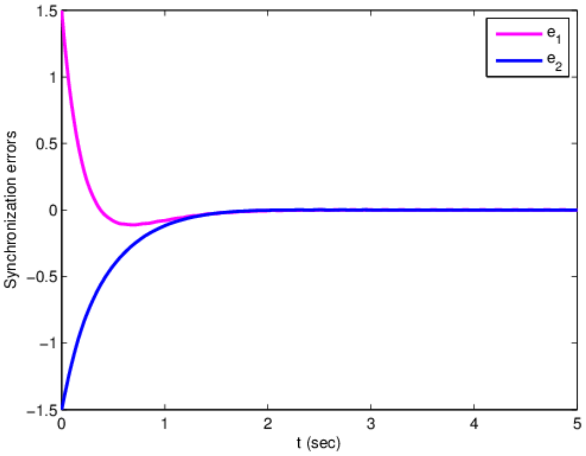

Figure3: The error system with K3

Figure3 shows the synchronous behavior withthe control input.By using the different figures,Figure3 also shows the trajectory of the error system.

V. Conclusion

The paper addresses the synchronization for the healthy and diseased CAS subjected to the time-delays under disturbances and perturbations. The time-delays can be handled by employing the delay-rangedependent strategy for LKF. Based on the Jensen inequality, the synchronization of CACS is achieved, which ensures the regional stability of the error system.

REFERENCES

[1] Wang JL, Xu M, Wu HN, et al. (2017) Finite-time passivity of coupled neural networks with multiple weights. IEEE Transactions on Network Science and Engineering. Epub ahead of print 31 August 2017. DOI: 10.1109/TNSE.2017.2746759.
[2] Yang JQ, Chen YT and Zhu FL. Singular reduced-order obsever-based synchronization for uncertain chaotic systems subject to channel disturbance and chaos-based secure communication. Applied Mathematics and Computation. (2014) 229(6): 227-238.
[3] Zhou JH, Liu DD, Li X, et al. Pink noise: Effect on complexity synchronization of brain activity and sleep consolidation. Journal of Theoretical Biology. (2012) 306: 68-72.
[4] Yadav VK, Das S, Bhadaduria BS, et al. Stability analysis, chaos control of a fractional order chaotic chemical reactor system and its function projective synchronization with parametric uncertainties. Chinese Journal of Physical. (2017).55(3): 594-605.
[5] Sun HB and Guo L. Neural Network-Based DOBC for a Class of Nonlinear Systems With Unmatched Disturbances. IEEE Transactions on Neural Networks and Learning Systems. (2017) 28(2): 482-489.
[6] Soriano DC, Abib GA and Eisencraft M (2012) Using an evolutionary denoising approach to improve the robustness of chaotic synchronization. \emph{IFAC Proceeding Volumes} 45: 35-39.
[7] Ray A, Chowdhhury RA and Ghosh D. Effect of noise on chaos synchronization in time-delayed systems: Numercial and experimental observations. Physica A: Statistical Mechanics and its Applications. (2013) 392(20): 4837-4849.
[8] Mohammad PA, Aiuob H . Chaos synchronization between two different chaotic systems with uncertainies, external disturbances, unknown parameters and input nonlinearities.Applied Mathematical Modelling.(2012).36(4): 1639-1652.
[9] Wang JL, Li HY and Zhou Q Adaptive fuzzy control for nonstrict feedback systems with unmodeled dynamics and fuzzy dead zone via output feedback.IEEE Transactions on Cybernetics. (2017) 47(9): 2400-2412.
[10] Hsu CF Adaptive fuzzy wavelet neural controller design for chaos synchronization. Expert systems with Applications (2011) 38(8): 10475-10483.
[11] Ding SH and Li SH. Second-order sliding mode controller design subject to mismatched term. Automatica. (2017) 77(3): 388-392.
[12] Yang CH, Ge ZM and Chang CM . Chaos synchronization and chaos control of quantum-CNN chaotic system by variable structure control and impulse control. Nonlinear Analysis: Real World Applications. (2010) 11(3): 1977-1985.
[13] Moiss OV and Hector P . A cascade control approach for a class of biomedical system. Proceedings of the 28th IEEE EMBS Annual International Conference.(2006) 4420-4423.
[14] Lin CJ, Yang SK and Yau HT. Chaos suppression control of a coronary artery system with uncertainties by using variable structure control. Computers & Mathematics with Applications. (2012) 64(5): 988-995.
[15] Cheng CW and Her TY. Chaos Analysis and Synchronization Control of Coronary Artery Systems. \Abstract and Applied Analysis,(2013),(2013-11-13) 2013: 764-787.
[16] Zhao ZS, Zhang J, Ding G, et al. Chaos synchronization of coronary artery system based on higher order sliding mode adaptive control. Acta Physica Sinica (2015) 64(21).
[17] Li XM, Zhao ZS, Zhang J, et al. H∞ synchronization of the coronary artery system with input time-varying delay.Project supported by the National Natural Science Foundation of China Chinese Physics B. (2016) 25(6): 109-117.
[18] Wu WS, Zhao ZS, Zhang J, et al. State feedback synchronization control of coronary artery chaos system with interval time-varying delay. Nonlinear Dynamics. (2017) 87(3): 1773-1783.