

# Improved Fuzzy Control for Coronary Artery Time-Delay System Synchronization

Haoliang Cui

**Abstract**—This paper investigates the problem of fuzzy state-feedback control for a category of coronary artery system(CAS) with state time-delay. T-S fuzzy model is employed to close in CAS which exists unknown nonlinear characteristics. An improved Parallel Distributed Compensation(PDC) controller is constructed to synchronize diseased CAS and normal CAS that both have state time-delay. Improved Jensen inequality is used to reduce the conservatism of the system. A simulation example is given to validate the effectiveness of the new design strategy.

**Index Terms**—Chaos synchronization,PDC controller, Improved Jensen inequality.

## I. INTRODUCTION

Chaos synchronization has become an important topic in the field of nonlinearity, it exists widely in communication, chemistry and medicine and other fields. Confidential communication, chemical reactions, cardiovascular disease treatment are some examples which use chaos synchronization [1-5].With the changes of blood vessel and blood pressure,the coronary artery system will produce non-linear chaotic dynamic behavior.Thus,The coronary artery system is also a special chaotic system.Recently years, many researchers studies on the coronary artery system.Xu in 1986 put forward the mathematical model of the coronary artery system and the main parameter values, mathematically prove that coronary artery spasm will occur chaotic behavior [6].

Fuzzy control[7-10] is an effective method for modeling and controlling nonlinear uncertain system. Through the fuzzy logic, the language information is constructed on the control system, so it is needn't to establish the precise mathematical model of controlled plant in the design, so that the control strategy is easy to design and apply to practical system. Wang et al proposed the PDC method to design the fuzzy controller in 1966[11], and based on linear matrix inequality, the sufficient conditions for the stability of fuzzy systems were given. The control rules of PDC controller have the relative independence, and the fuzzy connection of these control rules can be used to find the suitable choice, so that the control effect is better than the conventional controller.

Considering to the time-delay of CAS, we designed an improved PDC controller which integrates memoryless controller and delay state feedback controller. By selecting

the appropriate Lyapunov-Krasovskii function and using the linear matrix inequality technique,we obtained the conditions that make the system stable.Improved Jensen's inequality [12] was used to reduce the conservatism of the CAS.Theoretical analysis and numerical simulation results confirm the effectiveness of this method.

## II. PROBLEM FORMULATION AND RELATED KNOWLEDGE

In this paper, let us use T-S fuzzy model to describe CAS as follows:

$$\dot{x}_m(t) = A_i x_m(t) + A_i^d x_m(t - h(t)) + E \cos(\omega t)$$

$$x = \varphi(t)$$

where  $\phi_j(t)(j=1\dots r)$  is the premise variable,  $\mu_{ij}(i=1\dots k, j=1\dots r)$  is the fuzzy set, r represents the number of the fuzzy rule,  $x_m(t) \in R^n$  are the state vector,

$h(t) \in [0, \bar{h}]$  denotes a time-varying delay with  $\dot{h}(t) \leq \eta$  and  $\bar{h}$  is a positive constant.  $A_i, A_i^d \in R^{n \times n}$  are constant real matrices.

The dynamic fuzzy model in (1) can be represented by:

$$\dot{x}_m(t) = \sum_{i=1}^r h_i(\phi(t)) \{A_i x_m(t) + A_i^d x_m(t - h(t)) + E \cos(\omega t)\}$$

Similar to the drive system, we can get the fuzzy response system as follows:

$$\dot{x}_s(t) = \sum_{i=1}^r h_i(\phi(t)) \{A_i x_s(t) + A_i^d x_s(t - h(t)) + E \cos(\omega t)\} + B u(t)$$

According to master and slave system, synchronization error is defined as  $e = x_s - x_m$ . We can describe the error system as follows:

$$e(t - d(t)) = x_s(t - h(t)) - x_m(t - h(t))$$

$$\dot{e}(t) = \sum_{i=1}^r h_i(\phi(t)) \{A_i e(t) + A_i^d e(t - h(t)) + B u(t)\}$$

**Lemma** (Improved Jensen inequality) For given symmetric positive definite matrix Z, x is a continuously differentiable function  $x \in [p, q] \rightarrow R^n$ , we can obtain:

$$\int_p^q x^T(u) Z \dot{x}(u) du \geq \frac{1}{q-p} \Omega \text{diag}(Z, 3Z) \Omega$$

Where

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$$\Omega = \begin{bmatrix} x(q) - x(p) \\ x(q) + x(p) - \frac{2}{q-p} \int_p^q x(u) du \end{bmatrix}$$

III. SYNCHRONIZATION CONTROL

**Theorem.** Coronary artery error system is asymptotically stabilized by using the new designed controller for any

time-delay that satisfies  $0 < h(t) < \bar{h}$  and  $\bar{h} \leq \eta$ , if there exist appropriate dimensions positive-definite symmetric matrices  $P, Q_i, R_j (i=1,2,3, j=1,2)$ , and the scalars

$h_1, h_2, \nu$ , so that the following LMI hold:

$$\begin{bmatrix} \Phi_1 & \Xi & -2R_1 & & R_2 & 6R_1 & 0 & 0 & \Gamma_1 & \Gamma_2 \\ * & \Phi_2 & -2R_2 & & -3R_3 & 0 & 6R_2 & 6R_2 & \Gamma_3 & \Gamma_4 \\ * & * & \Phi_3 & & 0 & 6R_1 & 6R_2 & 0 & 0 & 0 \\ * & * & * & \Phi_4 & 0 & 0 & 6R_2 & 0 & 0 & 0 \\ * & * & * & * & -12R_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -12R_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -12R_2 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -R_1 & 0 & 0 \\ * & * & * & * & * & * & * & * & -R_2 & 0 \end{bmatrix} < 0$$

Where

$$\Phi_1 = XA_i^T + A_i X + Xk_j^T B^T + BK_j X + Q_1 + Q_2 + Q_3 - 4R_1$$

$$\Phi_2 = -(1-\nu)Q_3 - 8R_2$$

$$\Phi_3 = -Q_1 - 4R_1 - 4R_2$$

$$\Gamma_1 = h_1 XA_i^T + h_1 Xk_j^T B^T$$

$$\Gamma_2 = h_{12} XA_i^T + h_{12} Xk_j^T B^T$$

$$\Gamma_3 = h_1 XA_i^{dT} + h_1 Xk_j^{dT} B^T$$

$$\Gamma_4 = h_{12} XA_i^{dT} + h_{12} Xk_j^{dT} B^T$$

$$\Xi = XA_i^{dT} + Xk_j^{dT} B^T$$

We select Lyapunov function as

$$V(t) = V_1(t) + V_2(t) + V_3(t)$$

Where

$$V_1(t) = e^T P e$$

$$V_2(t) = \int_{t-h_1}^t e^T(s) Q_1 e(s) ds + \int_{t-h_2}^t e^T(s) Q_2 e(s) ds + \int_{t-h(t)}^t e^T(s) Q_3 e(s) ds$$

$$V_3(t) = h_1 \int_{-h_1}^0 \int_{t+s}^t e^T(u) R_1 e(u) du ds + h_{12} \int_{-h_2}^{-h_1} \int_{t+s}^t e^T(u) R_2 e(u) du ds$$

IV. SIMULATION

In this section, we demonstrate the effectiveness of our proposed controller by synchronizing the coronary artery system.

Considering coronary artery system with time-varying delays and disturbances, the common parameters are listed as following:

$$A_1 = \begin{bmatrix} -0.15 & 1.7 \\ 0.575 & -0.35 \end{bmatrix} \quad A_2 = \begin{bmatrix} -0.15 & 1.7 \\ 100 & -0.35 \end{bmatrix}$$

$$A_1^d = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix} \quad A_2^d = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ -0.5 & -0.5 \end{bmatrix}$$

Initial conditions are  $h(t) = 0.3 + 0.1 \sin(t)$ , The initial state diagram of the system is as follows:

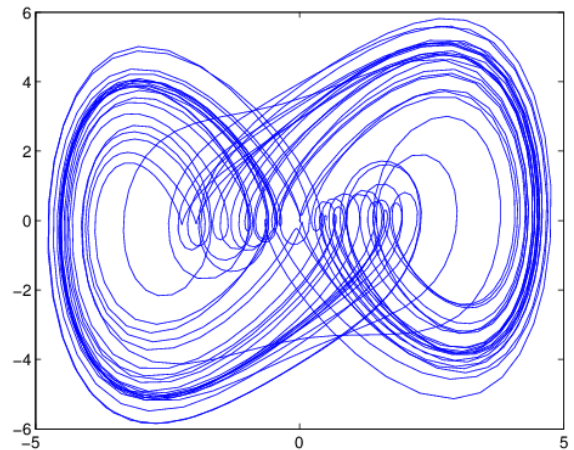


Figure1: The initial state diagram

The error diagram of the system without controller is as follow:

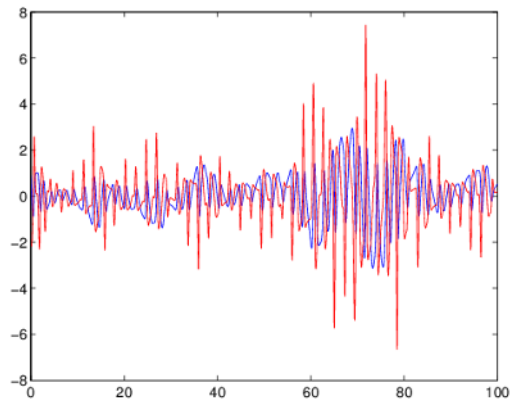


Figure2: The error system without controller

As we can see from Figure 2, the error system is in a chaotic state, which indicates that the diseased and healthy CAS can produce the different behavior without the control under disturbances.

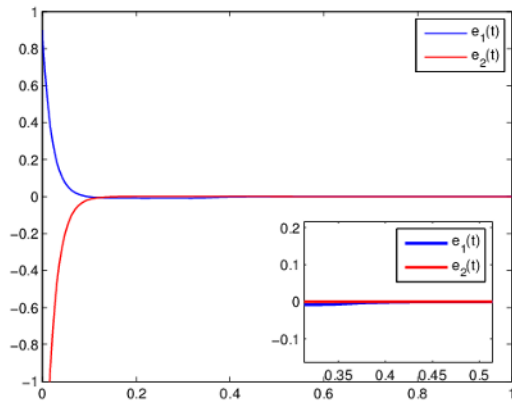


Figure3: The error system with controller

The error system with controller are illustrated in Figure3. From Figure3, we can find that with the time changes, the error system is approaching zero under the controller. That is, we prove its valid, at the same time, we can make the patients get a good treatment.

#### V. CONCLUSION

We designed a new PDC controller which includes memoryless and delayed state feedback for CAS of fuzzy models. By choosing the appropriate Lyapunov Krajevskii function and improved integral inequality, we obtained new stability conditions. The simulation diagram shows the good performance of the controller we designed. Recently, Lam proposed Interval Type-2 Fuzzy-Model-Based model which can be regarded as a collection of a number of type-1 T-S fuzzy models. Our future work will use the proposed results and methods to consider the analysis and synthesis of the control problems for CAS.

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