Interaction Solutions of the Whitham–Broer–Kaup Equations

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***Abstract*— In this paper, a new auxiliary equation method is presented. A nalytical multiple function solutions including trigonometric functions, exponential functions and hyperbolic functions can be easily obtained. Using this method，we obtain the interaction solutions of the Whitham–Broer–Kaup equations. It is significant to help physicists to analyze special phenomena in their relevant fields accurately.**

***Index Terms*— auxiliary equation method; interaction solution; Whitham–Broer–Kaup equations**

I. INTRODUCTION

It is well known that the complicated nature phenomena are often well described by nonlinear partial differential equations. In this study, we consider WBK equations which are introduced by Whitham, Broer and Kaup[1]. The equations describe propagation of shallow water waves with different dispersion relation.

Modern algebraic method is direct and effective in finding many kinds of exact solutions for nonlinear evolution equations (NLEEs) and it can be systematically implemented by computer algebra system. These methods are the tanh-function method[2], the homogeneous balance method[3], the F-expansion method[4], the Sine-Cosine function method[5], the Exp-function method[6], the simplest equation method[7], and other algebraic methods.

Recently, we introduced a method called the extended simplest equation method as an extension of the simplest equation method, to look for the exact traveling wave solutions of NLEEs[8]. In this paper, we apply the auxiliary equation method to construct the interaction solutions for the Whitham-Broer-Kaup equations[9].

II. NEW SOLUTIONS OF THE NOVEL AUXILIARY EQUATION

The novel auxiliary equation reads:

$$\phi'' + \delta\phi = v \qquad (1)$$

where $\phi'' = \phi''(\xi)$ and δ, v are constants. We obtain new multiple solutions of eq. (1) in the following:

$$\phi_1 = -\frac{(\tanh(\xi)+\tan(\xi))e^{\xi}}{\delta(1+\tan(\xi)\coth(\xi)+\tanh(\xi)+\tan(\xi))} - \frac{v}{\delta} \qquad (2)$$

$$\phi_2 = -\frac{(\tanh(\xi)+\coth(\xi))e^{\xi}}{\delta(1+\tanh(\xi)\tan(\xi)+\tan(\xi)+\coth(\xi))} - \frac{v}{\delta} \qquad (3)$$

$$\phi_3 = -\frac{(\tanh(\xi)+\cot(\xi))e^{\xi}}{\delta(1+\cot(\xi)\coth(\xi)+\tanh(\xi)+\cot(\xi))} - \frac{v}{\delta} \qquad (4)$$

$$\phi_4 = -\frac{(\cot(\xi)+\coth(\xi))e^{\xi}}{\delta(1+\tanh(\xi)\cot(\xi)+\cot(\xi)+\coth(\xi))} - \frac{v}{\delta} \qquad (5)$$

$$\phi_5 = -\frac{(1+\tan(\xi)\coth(\xi))e^{\xi}}{\delta(1+\tanh(\xi)+\tan(\xi)+\tan(\xi)\coth(\xi))} - \frac{v}{\delta} \qquad (6)$$

$$\phi_6 = -\frac{(1+\cot(\xi)\coth(\xi))e^{\xi}}{\delta(1+\tanh(\xi)+\cot(\xi)+\cot(\xi)\coth(\xi))} - \frac{v}{\delta} \qquad (7)$$

$$\phi_7 = -\frac{(1+\tanh(\xi)\tan(\xi))e^{\xi}}{\delta(1+\tan(\xi)+\coth(\xi)+\tanh(\xi)\tan(\xi))} - \frac{v}{\delta} \qquad (8)$$

$$\phi_8 = -\frac{(1+\tanh(\xi)\cot(\xi))e^{\xi}}{\delta(1+\cot(\xi)+\coth(\xi)+\tanh(\xi)\cot(\xi))} - \frac{v}{\delta} \qquad (9)$$

III. THE AUXILIARY EQUATION METHOD

Step1: For a given nonlinear partial differential equation with independent variables x,t…:

$$\begin{cases} P(u_t, u_x, H_x \cdots) = 0 \\ Q(H_t, H_{xx}, u_x \cdots) = 0 \end{cases} \qquad (10)$$

We make a transformation as follow:

$$\begin{cases} u(x,t) = u(\xi) \\ H(x,t) = H(\xi) \end{cases}, \quad \xi = x - ct, \qquad (11)$$

where c are constants to be determined.

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Step2: Inserting (11) into eq. (10), we get the ordinary differential equation:

$$\begin{cases} P(u',u'',u''',H\cdots)=0 \\ Q(H',H'',H''',u\cdots)=0 \end{cases}. \qquad (12)$$

Step3: We assume the solutions of eq. (12) in the following:

$$\begin{cases} u(\xi)=\sum_{i=0}^{m} a_i f^i(\xi) \\ H(\xi)=\sum_{j=0}^{n} b_j f^j(\xi) \end{cases}. \qquad (13)$$

Where m,n are positive integers determined by the balance principle in eq. (12). $f(\xi)$ satisfies eq. (1). Substituting (13) into eq. (12), we obtain a set of algebra equations when we set all coefficients of $f^i(\xi)$ to zeros. Therefore a_i, b_j will be determined by solving the set of algebra equations. We will apply the method to the Whitham-Broer-Kaup equations.

IV. New Interaction Solutions of the WBK Equations

The WBK equations are

$$u_t + uu_x + H_x + \beta u_{xx} = 0, \qquad (14)$$

$$H_t + (uH)_X + \alpha u_{xxx} - \beta H_{xx} = 0, \qquad (15)$$

where the field of horizontal velocity is represented by u(x,t). H(x,t) is the height that deviate from equilibrium position of liquid. And α,β are constants which represent different diffusion power[10].

We get ordinary differential equations (ODEs)

$$-cu' + uu' + H' + \beta u'' = 0, \qquad (16)$$

$$-cH' + Hu' + uH' - \beta H'' + \alpha u''' = 0, \qquad (17)$$

We assume the solutions of eqs. (16) and (17) as the following,

$$\begin{cases} u=\sum_{i=0}^{m} a_i f^i(\xi) \\ H=\sum_{j=0}^{n} b_i f^i(\xi) \end{cases}, \quad \xi = x - ct, \qquad (18)$$

where c is a constants to be determined.

We get $m=1, n=2$ by the balance principle,then we assume the solutions of eqs. (16) and (17) as the following,

$$u(\xi)=a_0 + a_1\left(\frac{\phi'}{\phi}\right) + a_2\left(\frac{1}{\phi}\right), \qquad (19)$$

$$H(\xi)=b_0 + b_1\left(\frac{\phi'}{\phi}\right) + b_2\left(\frac{\phi'}{\phi}\right)^2 + \left[b_3 + b_4\left(\frac{\phi'}{\phi}\right)\right]\left(\frac{1}{\phi}\right), \qquad \textbf{(20)}$$

where $a_i(a_2 \neq 0)$, $b_i(b_2 b_4 \neq 0)$ are constants. They could be determined by using the auxiliary equation method. We get the following results:

$$a_0 = c, a_1 = \sqrt{\alpha+\beta^2}, a_2 = 0, b_0 = \delta b_2, b_1 = 0,$$
$$b_2 = b_2, b_3 = \beta v\sqrt{\alpha+\beta^2} - \beta^2 v - \alpha v - 2b_2 v, b_4 = 0. \qquad (21)$$

We obtain interaction solutions of eqs. (14) and (15) in the following:

$$u_j(\xi) = c + \sqrt{\alpha+\beta^2}\left(\frac{\phi_j'}{\phi_j}\right), \qquad (22)$$

$$H_j(\xi) = -bb_2 + b_2\left(\frac{\phi_j'}{\phi_j}\right)^2 + \left(\beta a\sqrt{\alpha+\beta^2} - \beta^2 a - \alpha a - 2b_2 a\right)\left(\frac{1}{\phi_j}\right), \qquad \textbf{(23)}$$

where φ_j (j=1,…,8) are solutions of eq. (1).

Conclusion

In this paper, we successfully obtained new interaction solutions of WBK equations by using the auxiliary equation method. Analytical multiple function solutions including trigonometric functions, exponential functions and hyperbolic functions can be easily obtained. Using this method，we obtain the interaction solutions of the Whitham–Broer–Kaup equations. It is significant to help physicists to analyze special phenomena in their relevant fields accurately.

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References

[1] Syed Tauseef Mohyud-Din, Ahmet Yıldırım, Gulseren Demirli. Traveling wave solutions of Whitham-Broer-Kaup equations by homotopy perturbation method. Journal of King Saud University (Science). 2010, 22:173-176.

[2] Fan EG, Hon Y C. Applications of extended tanh method to special types of nonlinear equations. Appl. Math. Comput, 2003, 141: 351-358.

[3] Wang M L, Zhou Y B, Li Z B. Applications of a homogeneous balance method to exact solutions of nonlinear equations in mathematical physics [J]. Phys. Lett. A, 1996, 216: 67-75.

[4] Wang M L, Li X Z. Applications of F-expansion to periodic wave solutions for a new Hamiltonian amplitude equation. Chaos, Solitons and Fractals, 2005, 24: 1257-1268.

[5] Wazwaz A M. A sine-cosine method for handling nonlinear wave equations. Math. Comput. Modelling, 2004,40 (5-6) : 499-508.

[6] Wazwaz A M. Generalized solitonary and periodic solutions for nonlinear partial differential equations by the Exp-function method. Nonlinear Dyn, 2008, 52: 1-9.

[7] Kudryashov N A. Simplest equation method to look for exact solutions of nonlinear differential equations. Chaos, Solitons and Fractals, 2005, 24: 1217-1231.

[8] Huaitang Chen, Li Yang and Xiaoling Wen. A new auxiliary equation method and novel interaction solutions of the generalized fisher equation. Journal of Scientific Research in Physical & Mathematical Science. 2016, 3(2):2349-7149.

[9] Sudao Bilige, Temuer Chaolu. An extended simplest equation method and its application to several forms of the fifth-order KdV equation. Appl. Math. Comput, 2010, 216: 3146-3153.

[10] YANG Qiongfen, YANG Lijuan. LUO Shoushuang. New Exact Solutions of the Whitham－Broer－Kaup equation. Journal of Mianyang Teachers'College, 2017, 36(11):8-10.