Hsu-Structure Manifold on Affine Connexions

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Abstract— — **In this paper we have studied some affine connexions in a Hsu-structure manifold. Certain theorems have also been proved, which are of great geometrical importance.**

***Index Terms*— C^∞-manifold, Hsu-structure, Hsu-metric structure, F-structure, π-structure**

I. Introduction

We consider a differentiable manifold V_n of differentiability class C^∞ and of dimension n. Let there exist in V_n a tensor field F of the type (1, 1), s linearly independent vector fields U_i, $i=1,2,..........\ s$ and s linearly independent 1-forms u^i such that for any arbitrary vector field X, we have

$$\overline{\overline{X}} = a^r X + cu^i(X)U_i, \tag{1.1}$$

$$\overline{U}_i = p_i^j U_j \tag{1.2}$$

Where

$$F(X) \overset{def}{=} \overline{X}$$ and a^r, c are constants

Then the structure $\{F, u^i, U_i, p_i^j;\ i, j=1,2,.........,s\}$ will be known as Hsu-structure and V_n will be known as Hsu-structure manifold of order s where s < n.

Definition: A structure on an n-dimensional manifold M of class C^∞ given by a non-null tensor field F satisfying

$$F^2 = a^r I$$

is called π - structure or Hsu-structure, where a is a non zero complex constant and I denotes the unit tensor field. Then M is called π - structure manifold or Hsu-structure manifold[8].

Agreement 1.1
All the equations which follow hold for arbitrary vector fields X, Y, Z ….. etc.

Now replacing X by $\overline{X}$ in (1.1), we get

$$\overline{\overline{\overline{X}}} = a^r \overline{X} + cu^i(\overline{X})U_i \tag{1.3}$$

Operating F in (1.1), we get

$$\overline{\overline{\overline{X}}} = a^r \overline{X} + cu^i(X)\overline{U}_i$$

Using (1.2) in above, we get

$$\overline{\overline{\overline{X}}} = a^r \overline{X} + cu^i(X)p_i^j U_j \tag{1.4}$$

From (1.3) and (1.4), we have

$$u^i(\overline{X}) = u^j(X)\, p_j^i \tag{1.5}$$

Further, operating F in (1.2) and using (1.1 and (1.2), we get

$${}^{(2)}p_i^j = a^r \delta_i^j + cu^j(U_i) \tag{1.6}$$

Where

$${}^{(r)}p_j^i = {}^{(r-1)}p_k^i\, p_j^k$$

On Hsu structure manifold V_n, let us introduce a metric tensor g such that $'F$ defined by $'F(X, Y) \overset{def}{=} g(\overline{X}, Y)$ is skew-symmetric, then V_n is called Hsu metric structure manifold.

We have on a Hsu metric structure manifold

$$g(\overline{X}, Y) + g(X, \overline{Y}) = 0.$$

Replacing Y by $\overline{Y}$ in above equation and using (1.1), we obtain

$$g(\overline{X}, \overline{Y}) + a^r g(X, Y) + cu^i(X)u^i(Y) = 0 \tag{1.7}$$

Where

$$u^i(X) = g(U_i, X) \tag{1.8}$$

Then V_n satisfying (1.7), (1.8) is called Hsu metric structure manifold[5].

Agreement 1.2: The Hsu metric structure manifold will always be denoted by V_n.

Definitons: [1][4]

Almost tangent metric manifold: A differentiable manifold M_n on which there exists a tensor field F of the type (1, 1) such that

$$F^2 = 0 \tag{1.9}$$

is called an almost tangent manifold and $\{F\}$ is called an almost tangent structure on M_n.

On almost tangent manifold, let us introduce a metric g such $'F$ defined by $'F(X, Y) \overset{def}{=} g(\overline{X}, Y)$ is alternating . Then M_n is called an almost tangent metric manifold and structure $\{F, g\}$ is called an almost tangent metric structure.

Almost Hermite Manifold: A differentiable manifold M_n on which there exists a tensor field F of the type (1, 1) such that

$$F^2 = -I_n \tag{1.10}$$

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is called an almost complex manifold and $\{F\}$ is called an almost complex structure.

An almost complex manifold endowed with an almost complex structure and a metric g such that

$$g(\bar{X},\bar{Y})=g(X,Y) \tag{1.11}$$

is called an almost Hermite manifold and structure $\{F,g\}$ is called an almost Hermite structure.

Metric $\pi-$structure manifold:

A differentiable manifold M_n on which there exists a tensor field F of the type (1, 1) such that

$$F^2=-\lambda^r I_n \tag{1.12}$$

where λ is a non zero complex constant. Then $\{F\}$ is called a $\pi-$structure or Hsu structure and M_n is called $\pi-$structure manifold or Hsu structure manifold.

On almost tangent manifold, let us introduce a metric g such that $'F$ is defined by

$$'F(X,Y)\overset{def}{=}g(\bar{X},Y) \text{ is alternating.}$$

Then $\{F, g\}$ is called metric $\pi-$structure or H-structure and M_n is called metric $\pi-$ structure manifold or H-structure manifold[7][3].

F-structure Manifold: Let M_n be an n dimensional differentiable manifold of class C^∞ and let there be a tensor field of type (1, 1) and rank $r(1\le r\le n)$ everywhere such that

$$F^3+F=0 \tag{1.13}$$

Then $\{F\}$ is called an F-structure and M_n is called F-structure manifold[9].

Almost Grayan manifold: If on an differentiable manifold M_n (n = 2m+1) of differentiability class C^{r+1}, there exist a tensor field F of type (1, 1), a 1-form u and a vector field U, satisfying

$$F^2=-I_n+u\otimes U \tag{1.14}$$

and

$$\bar{U}=0 \tag{1.15}$$

Then M_n is called an almost contact manifold and the structure $\{F, U, u\}$ is said to give an almost contact structure to M_n.

On an almost contact manifold, let us introduce a metric g such that $'F$ defined by

$$'F(X,Y)\overset{def}{=}g(\bar{X},Y)$$ is skew symmetric. Then M_n is called an almost Grayan manifold and the structure $\{F,g,U,u\}$ is called an almost Grayan structure[10]. In this manifold it can be easily calculated

$$g(\bar{X},\bar{Y})=g(X,Y)-u(X)u(Y)=0 \tag{1.16}$$

Torsion tensor: A vector valued, skew-symmetric, bilinear function S defined by

$$S(X,Y)\overset{def}{=}D_XY-D_YX-[X,Y] \tag{1.17}$$

is called torsion tensor of a connexion D in a C^∞ manifold V_n.

For the symmetric or torsion free connexion D, the torsion tensor vanishes.

Curvature tensor: The tensor K of the type (1, 3) defined by

$$K(X,Y,Z)\overset{def}{=}D_XD_YZ-D_YD_XZ-D_{[X,Y]}Z \tag{1.18}$$

is called the curvature tensor of the connexion D.

Remark 1.1

It may be noted that V_n gives an almost tangent metric manifold, an almost Hermite manifold, metric $\pi-$structure manifold, F-structure manifold, an almost Grayan manifold and $\{F, g, u^1, u^2, U_1, U_2\}$ structure manifold according as $(a^r=0, c=0)$; $(a^r=-1, c=0)$; $(a^r=\lambda^r, c=0)$; $(a^r=-1, p_i^j=0)$; $(a^r=-1, c=1, i, j=1, p_1^1=0)$ and $(a^r=-1, c=1, p_i^j+p_j^i=0; i,j=1,2)$ respectively.

Affine Connexion D: Let us consider in V_n an affine connection D satisfying[2][6]

$$(D_XF)Y=0 \tag{2.1)a}$$

and we call it as F-connexion.

(2.1)a is equivalent to

$$D_X\bar{Y}=\overline{D_XY} \tag{2.1)b}$$

Replacing Y by $\bar{Y}$ and using (1.1), (2.1)a in above, we get

$$c[u^i(Y)D_XU_i+(D_Xu^i)(Y)U_i]=0 \tag{2.1)c}$$

Theorem 2.1

In V_n, we have

$$cu^i(Y)u^j(D_XU_i)=-(a^r\delta_i^j-{}^{(2)}p_i^j)(D_Xu^i)(Y) \tag{2.2}$$

$$cu^j(D_XU_i)U_j=-(a^r\delta_i^j-{}^{(2)}p_i^j)(D_XU_j) \tag{2.3}$$

Proof

Operating u^j in (2.1)c and using (1.6), we get (2.2).

Putting U_i for Y in (2.1)c and using (1.6), we obtain (2.3).

Theorem 2.2

In V_n, we have

$$S(\bar{X},\bar{Y})+a^rS(X,Y)+cu^i(S(X,Y))U_i-\overline{S(\bar{X},Y)}-\overline{S(X,\bar{Y})}$$

$$=-[\bar{X},\bar{Y}]-a^r[X,Y]-cu^i([X,Y])U_i+\overline{[\bar{X},Y]}+\overline{[X,\bar{Y}]} \tag{2.5}$$

Proof

From (2.1)b, we get

$$D_{\bar{X}}\bar{Y}=\overline{D_{\bar{X}}Y}, D_{\bar{Y}}\bar{X}=\overline{D_{\bar{Y}}X}, D_X\bar{\bar{Y}}=\overline{\overline{D_XY}}, D_Y\bar{\bar{X}}=\overline{\overline{D_YX}} \tag{2.6}$$

Now in view of (1.1), we get

$$S(\bar{X},\bar{Y})+a^rS(X,Y)+cu^i(S(X,Y))U_i-\overline{S(\bar{X},Y)}-\overline{S(X,\bar{Y})}$$

$$=S(\bar{X},\bar{Y})+\overline{\overline{S(X,Y)}}-\overline{S(\bar{X},Y)}-\overline{S(X,\bar{Y})}$$

Using (1.17) and (2.6) in right hand side of above, we get (2.5). Now, we consider in V_n a scalar valued bilinear function μ, vector valued linear function v and a 1 –form σ given by,

$$\mu(X,Y) \overset{def}{=} (D_Y u^i)(\overline{X}) - (D_X u^i)(\overline{Y}) + (D_{\overline{Y}} u^i)(X) - (D_{\overline{X}} u^i)(Y) \quad (2.7)$$

$$v(X) \overset{def}{=} (D_{U_i} F)(X) - (D_X F)(U_i) - D_{\overline{X}} U_i \quad (2.8)$$

and

$$\sigma(X) \overset{def}{=} (D_X u^j)(U_i) - (D_{U_i} u^j)(X) \quad (2.9)$$

$$i, j = 1, 2, \ldots\ldots\ldots\ldots\ldots\ldots, s.$$

Theorem 2.3

In V_n, we have

$$(a^r \delta_i^j - {}^{(2)}p_i^j)\mu(X,Y) = c[u^i(X)u^j(D_{\overline{Y}} U_i) - u^i(Y)u^j(D_{\overline{X}} U_i) - u^i(\overline{X})(D_Y u^j)U_i + u^i(\overline{Y})(D_X u^j)U_i] \quad (2.10)a$$

$$(a^r \delta_i^j - {}^{(2)}p_i^j)\mu(X,Y) = -c[u^i(X)u^j(v(Y)) - u^i(Y)u^j(v(X)) + u^i(\overline{X})(D_Y u^j)U_i - u^i(\overline{Y})(D_X u^j)U_i] \quad (2.10)b$$

and

$$(a^r \delta_i^j - {}^{(2)}p_i^j)\mu(X,Y) = -c[u^i(X)\{\sigma(\overline{X}) + (D_{U_i} u^j)(\overline{X})\} - u^i(Y)\{\sigma(\overline{Y}) + (D_{U_i} u^j)(\overline{Y})\} - u^i(\overline{X})(D_Y u^j)U_i + u^i(\overline{Y})(D_X u^j)U_i] \quad (2.10)c$$

Proof

Replacing Y by $\overline{Y}$ in (2.2), we get

$$cu^i(\overline{Y})u^j(D_X U_i) = -(a^r \delta_i^j - {}^{(2)}p_i^j)(D_X u^i)(\overline{Y}) \quad (2.11)$$

Replacing X by $\overline{X}$ in (2.2), we get

$$cu^i(Y)u^j(D_{\overline{X}} U_i) = -(a^r \delta_i^j - {}^{(2)}p_i^j)(D_{\overline{X}} u^i)(Y) \quad (2.12)$$

Further by using (2.11), (2.12) in (2.7), we get (2.10)a.
Using (2.1)a in (2.8), we get

$$v(X) = -(D_{\overline{X}} U_i) \quad (2.13)$$

Using (2.13) in (2.10)a, we get (2.10)b. Replacing X by $\overline{X}$ in (2.9), we get

$$-u^j(D_{\overline{X}} U_i) = \sigma(\overline{X}) + (D_{U_i} u^j)(\overline{X}) \quad (2.14)$$

Using (2.14) in (2.10)a, we get (2.10)c.
In V_n, we have

$$\overline{K(X,Y,\overline{Z})} = a^r K(X,Y,Z) + cu^i(K(X,Y,Z))U_i \quad (2.16)a$$

$$p_i^j u^i(K(X,Y,\overline{Z})) = a^r u^j(K(X,Y,Z)) + ({}^{(2)}p_i^j - a^r \delta_i^j)u^i(K(X,Y,Z)) \quad (2.16)b$$

and

$$a^r\left[K(\overline{X},\overline{Y},Z) + K(\overline{Y},\overline{Z},X) + K(\overline{Z},\overline{X},Y)\right] = -c[u^i(Z)K(\overline{X},\overline{Y},U_i) + u^i(X)K(\overline{Y},\overline{Z},U_i) + u^i(Y)K(\overline{Z},\overline{X},U_i)] \quad (2.16)c$$

Proof

Replacing Z by $\overline{Z}$ in (1.18) and using (2.1)b, we get

$$K(X,Y,\overline{Z}) = \overline{K(X,Y,Z)} \quad (2.17)$$

Operating F in (2.17) and using (1.1), we obtain (2.16)a.

Operating u^j on both sides of (2.16)a and using (1.5) and (1.6), we get (2.16)b. Bianchi's first identity of symmetric connexion D is given by

$$K(X,Y,Z) + K(Y,Z,X) + K(Z,X,Y) = 0 \quad (2.18)$$

Operating F in (2.18), we get

$$\overline{K(X,Y,Z)} + \overline{K(Y,Z,X)} + \overline{K(Z,X,Y)} = 0 \quad (2.19)$$

Using (2.17) in (2.19), we get

$$K(X,Y,\overline{Z}) + K(Y,Z,\overline{X}) + K(Z,X,\overline{Y}) = 0 \quad (2.20)$$

Replacing X by $\overline{X}$, Y by $\overline{Y}$ and Z by $\overline{Z}$ in (2.20) and using (1.1), we get (2.16)c.

Affine connexion $\tilde{D}$: Let us consider in V_n an affine connexion $\tilde{D}$ satisfying

$$u^i(Y)(\tilde{D}_X U_i) + cu^j(\tilde{D}_X u^i)(Y)U_i = 0 \quad (3.1)$$

Theorem 3.1

In V_n, we have

$$u^i(Y)\left[a^r(\tilde{D}_X U_i) + cu^j(\tilde{D}_X U_i)U_j\right] + (\tilde{D}_X u^i)(Y)p_i^j p_j^k U_k = 0 \quad (3.2)a$$

$$({}^{(2)}p_i^j - a^r \delta_i^j) div\, U_j = cu^j(\tilde{D}_{U_j} U_i) \quad (3.2)b$$

Where

$$div\, X \overset{def}{=} (C_1^1 \nabla X) \quad (3.3)$$

$$(\nabla X)Y \overset{def}{=} (\tilde{D}_Y X) \quad (3.4)$$

Proof

Operating F^2 in (3.1) and using (1.1) and (1.2), we get (3.2)a. Now contracting (3.1) with respect to X and using (3.3) and (3.4), we get

$$u^i(Y) div\, U_i + (\tilde{D}_{U_i} u^i)(Y) = 0 \quad (3.5)$$

Replacing i by j, then Y by U_i in (3.3) and using (1.6), we get (3.2)b.

Theorem 3.2

In V_n, we have

$$cu^i(Y)u^j(\tilde{D}_X U_i) + ({}^{(2)}p_i^j - a^r \delta_i^j)(\tilde{D}_X u^i)(Y) = 0 \quad (3.6)a$$

$$({}^{(2)}p_i^j - a^r \delta_i^j)(\tilde{D}_X u^i)(Y)u^j(\tilde{D}_Z U_i) = cu^i(Y)u^j(\tilde{D}_Z U_j)(\tilde{D}_X u^j)(U_j) \quad (3.6)b$$

Proof

By operating u^j on (3.1) and using (1.6), we obtain (3.6)a.
Multiplying (3.2)a with $u^j(\tilde{D}_Z U_j)$, we get (3.6)b.

Affine connexion $\overset{\circ}{D}$

Let us consider in V_n an affine connexion $\overset{\circ}{D}$ satisfying

$$u^i(Y)(\overset{\circ}{D}_X U_i) + (\overset{\circ}{D}_X u^i)(Y)U_i = 0 \qquad (4.1)\text{a}$$

and

$$(\overset{\circ}{D}_X F)(Y) + (\overset{\circ}{D}_Y F)(X) = 0 \qquad (4.1)\text{b}$$

It may be noted that all the results of the section above hold for $\overset{\circ}{D}$. In addition we have the following results:

Theorem 4.1

In V_n, we have

$$\overline{\overset{\circ}{D}_X \bar{Y}} + \overline{\overset{\circ}{D}_Y \bar{X}} - a^r(\overset{\circ}{D}_X Y + \overset{\circ}{D}_Y X) = c\left[u^i(\overset{\circ}{D}_X Y) + u^i(\overset{\circ}{D}_Y X)\right]U_i \qquad (4.2)\text{a}$$

$$\overset{\circ}{D}_{\bar{Y}} \bar{X} - a^r(\overset{\circ}{D}_Y X) = \overline{\overset{\circ}{D}_{\bar{Y}} X} - \overline{\overset{\circ}{D}_Y \bar{X}} + cu^i(\overset{\circ}{D}_Y X)U_i \qquad (4.2)\text{b}$$

and

$$\overset{\circ}{D}_{\bar{Y}} \bar{X} + a^r(\overset{\circ}{D}_Y \bar{X} - \overline{\overset{\circ}{D}_Y X} - \overset{\circ}{D}_{\bar{Y}} X) = c\left[\left\{u^i(\overset{\circ}{D}_{\bar{Y}} X) - u^i(\overset{\circ}{D}_Y \bar{X})\right\}U_i + u^i(\overset{\circ}{D}_Y X)\overline{U_i}\right] \qquad (4.2)\text{c}$$

Proof

The equation (4.1)b is equivalent to

$$\overset{\circ}{D}_X \bar{Y} + \overset{\circ}{D}_Y \bar{X} = \overline{\overset{\circ}{D}_X Y} - \overline{\overset{\circ}{D}_Y X} \qquad (4.3)$$

Operating F in (4.3) and using (1.1), we get (4.2)a. Replacing Y by $\bar{Y}$ in (4.3) and using (1.1), (4.3), we get (4.2)b. Further, Operating F in (4.2)b and using (1.1), we get (4.2)c.

Affine connexion $\overset{*}{D}$

Let us consider in V_n an affine connexion $\overset{*}{D}$ satisfying

$$u^i(Y)(\overset{*}{D}_X U_i) + (\overset{*}{D}_X u^i)(Y)U_i = 0 \qquad (5.1)\text{a}$$

and

$$(\overset{*}{D}_X F)(Y) + (\overset{*}{D}_{\bar{X}} F)(\bar{Y}) = 0 \qquad (5.1)\text{b}$$

It may be noted that all the results of the section three hold for $\overset{*}{D}$. In addition we have the following results:

Theorem 5.1

In V_n, we have

$$\overline{\overset{*}{D}_X Y} + \overline{\overset{*}{D}_{\bar{X}} \bar{Y}} - \overset{*}{D}_X \bar{Y} = a^r(\overset{*}{D}_{\bar{X}} Y) + cu^i(\overset{*}{D}_{\bar{X}} Y)U_i \qquad (5.2)\text{a}$$

$$\overline{\overset{*}{D}_{U_j} \bar{Y}} + (\overset{*}{D}_{U_j} F)\bar{Y} = a^r(\overset{*}{D}_{U_j} Y) + cu^i(\overset{*}{D}_{U_j} Y)U_i \qquad (5.2)\text{b}$$

Proof

(5.1)b is equivalent to

$$\overline{\overset{*}{D}_X Y} + \overline{\overset{*}{D}_{\bar{X}} \bar{Y}} = \overset{*}{D}_X \bar{Y} + \overset{*}{D}_{\bar{X}} \bar{\bar{Y}} \qquad (5.3)$$

Using (1.1) in (5.3), we get (5.2)a. Replacing X by U_i in (5.3), we get

$$(\overset{*}{D}_{U_i} \bar{Y} + \overline{\overset{*}{D}_{U_i} Y}) + p_i^j\left[\overset{*}{D}_{U_j} a^r Y + cu^i(Y)U_i\right] = p_i^j(\overline{\overset{*}{D}_{U_j} \bar{Y}}) \qquad (5.4)$$

Replacing X by U_i in (5.2)b, we get

$$(\overset{*}{D}_{U_i} \bar{Y} - \overline{\overset{*}{D}_{U_i} Y}) = -p_i^j(\overset{*}{D}_{U_i} F)\bar{Y} \qquad (5.5)$$

From (5.4) and (5.5), we get

$$-p_i^j(\overset{*}{D}_{U_i} F)\bar{Y} + p_i^j\left[\overset{*}{D}_{U_j} a^r Y + cu^i(Y)U_i\right] = p_i^j(\overline{\overset{*}{D}_{U_j} \bar{Y}}) \qquad (5.6)$$

Using (5.1)a in (5.6), we get (5.1)b.

Theorem 5.2

In V_n, we have

$$\overset{*}{D}_{\bar{X}} \bar{Y} - a^r(\overline{\overset{*}{D}_X \bar{Y}} + a^r(\overset{*}{D}_X Y) = \overline{\overset{*}{D}_{\bar{X}} Y} - ca^r u^i(\overset{*}{D}_X Y)U_i + cu^i(X)\left[a^r\left\{(\overset{*}{D}_{U_i} Y) + u^j(\overset{*}{D}_{U_i} Y)U_j\right\} + (\overline{\overset{*}{D}_{U_i} \bar{Y}})\right] \qquad (5.7)$$

Proof

Replacing X by $\bar{X}$ in (5.3) and using (1.1), (5.1), we get (5.7).

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