# Natural Ordered Relation on Totally Ordered Ternary Semirings

**Koneti Rajani, Cheerla Meenakumari, Dr. G. Shobhalatha**

***Abstract*— In this paper we discussed about natural ordered relation in a totally ordered ternary semiring (T,+,.,,≤) in which(T,.) is semilattice defined as a ≤ b ⟺ a =xyb = xby = bxy; a= xya. And also we characterize different properties of a totally ordered semirings with natural ordered relation..**

**In section 2, the required preliminaries are presented. In Section 3, properties of totally ordered ternary semirings with natural ordered relation were discussed**

***Index Terms*— Totally ordered ternary semiring, R-Commutative, rectangular band, ,Normal, intra –regular, Semilattice.**

## I. INTRODUCTION

The algebraic theory of semigroups was widely developed by CLIFFORD[1,2] and PERTICH[11].The Theory of ternary algebraic systems were introduced by LEHMER[3] in 1932.LEHMER investigated certain algebraic system called triplexes which turn out to be commutative ternary groups. Ternary semigroups are universal algebras with one associative ternary operation. The notion of ternary semigroup was known to S.Banach who credited with the example of a ternary semigroup which cannot reduce to a semigroup. HEINZ MISTCH [4] defined the natural ordered relation on semigroup and proved it is totally ordered with respect to its natural partial order if and only if it is an idempotent semigroup. A band or an idempotent semigroup satisfying commutativity is called semilattice .The structure of these band is completely determined by NAOKI KIMURA [11] YAMUDA [11] and many others. In this paper we introduce the notion of natural order relation in totally ordered ternary semirings.

## II. PRELIMINARIES

This section contains preliminary definitions and results used in this paper

**Definition2.1**:A system (T,≤) is called partially ordered set if it satisfies the following axioms

(i) a ≤ a (Reflexivity)

(ii) a ≤ b, b ≤ a ⟹ a = b (Anti symmetry)

(iii) a≤ b, b ≤ c ⟹ a ≤ c (Transitivity)

For all a,b,c in T.

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**Definition 2.2:** A Ternary semigroup (T,.) is said to be a partially ordered ternary semigroup if T is partially ordered set such that for any a,b in T a ≤ b ⟹ [cad] ≤ [cbd], [acd] ≤ [bcd] , [cda] ≤ [cdb] for c,d in T.

**Definition 2.3:** A Ternary semiring is called Partial ordered ternary semiring if there exist partial order '≤' on T such that

(i) (T,+, ≤) is partially ordered semigroup

(ii) (T., ≤) is partially ordered semigroup. It is denoted by (T,+,. ≤)

**Example2.4**: Let T ={0,a,b,c} be a partially ordered ternary semiring with respect to addition and ternary multiplication and ordering such that a<b<c<0 and the operation is defined as follows

| + | a | b | c | 0 |
|---|---|---|---|---|
| a | a | b | c | 0 |
| b | b | b | c | 0 |
| c | c | c | c | 0 |
| 0 | 0 | 0 | 0 | 0 |

| . | a | b | c | 0 |
|---|---|---|---|---|
| a | 0 | 0 | 0 | 0 |
| b | 0 | 0 | 0 | 0 |
| c | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

**Definition 2.5**: A ternary semigroup (T,.) is band if every element in T is an idempotent .i.e. $a^3$ = a for all a in T.

**Definition 2.6:** A commutative band is called semilattice.

**Definition 2.7**:A ternary semigroup (T,.) is called rectangular band if ababa=a for all a,b in T.

**Definition2.8:**A semigroup (T,.) is called weakly seperative if $a^2$ = ab = $b^2$ implies a = b for all a,b in T.

**Definition 2.9**:A ternary semigroup (T,.) is called quasi -seperative if $a^3$ = aba =bab = $b^3$ implies that a = b for all a,b in T.

**Definition 2.10:** A ternary semigroup (T,.) is said to be left singular if it satisfies the identity $ab^2$ = a for all a,b in T.

**Definition 2.11**: A ternary semigroup (T,.) is said to be lateral singular if it satisfies the identity bab = a for all a,b in T.

**Definition 2.12:** A ternary semigroup (T,.) is said to be right singular if it satisfies the identity $b^2a$ = a for all a,b in T.

**Definition 2.13:** A ternary semigroup (T,.) is said to be two sided singular, if it is both left and right singular.

**Definition 2.14:**A ternary semigroup (T,.) is said to be singular if it is left, lateral and right singular.

**Definition 2.15:**A ternary semigroup (T,.) is said to be left regular, if it satisfies the identity a = $a^3$xy for all a,x,y in T.

**Definition2.16:** A ternary semigroup (T,.) is said to be right regular ,if it satisfies the identity a = $xya^3$ for all a,x,y in T.

**Definition 2.17**:A ternary semigroup (T,.) is said to be lateral regular, if it satisfies the identity a = x $a^3$y for all a,x,y in T.

**Definition 2.18:** A ternary semigroup (T,.) is said to be two sided regular if it is left as well as right regular.

**Definition2.19**: A ternary semigroup (T,.) is said to be regular if it is left, right and lateral regular.

**Definition2.20:** A system (T, ≤ ) where the relation ' ≤ ' on T satisfying the following axioms

(i)Reflexivity: a ≤ a

(ii) Anti symmetry : a ≤ b ,b ≤ a imply a = b

(iii) Transitivity a ≤ b, b ≤c imply a ≤ c
(iv) Linearity a ≤ b (or) b ≤ a
For all a,b,c in T is called a totally ordered set.

**Definition 2.21**: A ternary semiring (T,+,.) is said to be totally ordered ternary semiring if there exists a partially order "≤" on T such that
(i)(T,+, ≤) is a totally order semigroup
(ii) (T,., ≤) is totally ordered semigroup and it is denoted by (T,+,., ≤)

**EXAMPLE2.22:** Consider the set T={1,2,3,4} with the order 1 <2 <3 <4 and with the following binary operation '+' and ternary multiplication'.'

| + | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 4 |
| 2 | 3 | 4 | 4 | 4 |
| 3 | 4 | 4 | 4 | 4 |
| 4 | 4 | 4 | 4 | 4 |

| . | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 2 | 4 | 4 | 4 |
| 2 | 4 | 4 | 4 | 4 |
| 3 | 4 | 4 | 4 | 4 |
| 4 | 4 | 4 | 4 | 4 |

**Result 2.23[4]:**For an arbitrary semigroup T and its natural partial order the following are equivalent:
(i) a ≤ b
(ii) a =xb=by, ay=a for some x,y in $S^1$.
(iii) a = xb =by, xa=a=ay for some x,y in $S^1$.

**Result 2.24:**The order relation defined in the result 1.2.3 is total if and only if for every a,b in R ,a + b = a or b

## III. TOTALLY ORDERED TERNARY SEMIRINGS:

**Theorem3.1:**Suppose (T,+,.) is a ternary semiring in which(T,.) is band. If a relation defined by the rule a ≤ b ⇔ a=xyb= xby= bxy,a=xya for all a,b in T and x,y in $T^1$,where $T^1$ is ternary semigroup T with unity element '1'. If (T,.) is regular then (T,+,., ≤) is partially ordered ternary semiring.

**Proof:** Given that (T,+,.) is a ternary semiring in which (T,.) is semilattice and (T,.) is regular. Let a,b in T and x, y in $T^1$.Define ≤on T by rule a ≤ b ⇔ a=xyb=xby=bxy,a=xya

**Reflexivity:**
For a=1 a= a.1.1=1.a.1=1.1 .a, a=1.1.a
Where '1' is the identity element in (T,.)
⟹ a ≤ a
Therefore '≤' is reflexive.

**Anti symmetric:**
Let a ≤ b and b ≤ a then
a ≤ b ⇔ a = xyb = xby = bxy ,a = xya
b ≤ a ⇔ b = uva = uav = auv ,b = uvb
a = xyb
= xy(auv)
= (xya)uv
= auv
a = b
Therefore '≤' is antisymmetric.

**Transitivity:**
Let a ≤ b ⇔a = xyb = xby = bxy, a=xya
b ≤ c⇔b = uvc = ucv = cuv, b = uvb for some x,y,u,v in $T^1$
To prove a ≤ c
let a = xyb
= xy(uvc)
= (xy)(uv)c x,y,u,v in $T^1$ then xyuv in T
a = stc for s=xy and t=uv
Similarly we can prove a=sct= cst
Also sta = (xy) (uv) a
= (xy) (uv) (bxy)
= (xy) (uvb) (xy)
= (xy) b (xy)
= (xy) (bxy)
= (xy) a
= a
a= stc =sct = cst , a=sta
a ≤ c
Therefore '≤' is transitive
Hence (T, ≤) is partially ordered set.

**Compatibility:**
First we prove compatibility with multiplication
a ≤ b⟹ a=xyb =xby = bxy ;xya=a
acd
= (xyb)cd = (xby)cd = (bxy)cd; xy(acd)=acd
= xy (bcd) = (xby) cd = (bxy) cd ;
= xy (bcd) = (xyb) cd = (xyb) cd;
= xy(bcd) = bcd = bcd;
= xy(bcd) =$(xb^3y)$cd = bc$(d^3xy)$; ( T is regular)
= xy(bcd) = (xby)cd = bc(dxy);
= xy(bcd) = xb(ycd) =(bcd)xy ;
⟹xy(bcd) =x(bcd)y =(bcd)xy; xy(acd)=acd
⟹acd ≤ bcd
Similarly we can prove cad ≤ cbd and cda ≤ cdb
Therefore '≤' is compatible with respect to multiplication.
Now we prove '≤' is compatible with addition
Let a≤ b ⟹ a = xyb =xby = xyb; xya=a
⟹ a + c= xyb + c= xby + c= bxy + c; xya+c =a+c
⟹ a + c = xyb + $xyc^3$=xby +$xc^3y$ =bxy + $c^3xy$; xya+$xyc^3$=a+c
⟹a+c = $xy(b+c^3)$ =$x(b+c^3)y$=$(b+c^3)xy$;
$xy(a+c^3)$ = a+c
⟹ a + c ≤ b + c
Similarly we can prove c + a ≤ c + b
Therefore '≤' is compatible with respect to addition.
Hence (T,+,., ≤) is partial ordered ternary semiring.

**Theorem3.2**: If (T,.) be a ternary semigroup which is band .Define '≤' on T by a ≤ b ⇔ a=xyb=xby=bxy; a=xya for all a,b in T and for some x,y in $T^1$ where $T^1$is ternary semigroup T with unity element '1'.then (T,≤) is a totally ordered semigroup.

**Proof:** As given (T,.) is a ternary semigroup which is band .i.e. T = E(T) , where E(T) denotes the set of idempotents of T.
Let a,b in T, relation '≤' on T is given by a ≤ b ⇔ a=xyb=xby=bxy; a=xya for all a,b in T and for some x,y in $T^1$.
From theorem 3.1. it is proved that (T,≤) is partially ordered ternary semigroup.
Again from the result 2.26[4] we conclude that (T,≤) is a totally ordered ternary semigroup.

**Remark**:1.(T,.,≤) is totally ordered if (T,.) is band.
2.Through out this paper $T^1$is a totally ordered ternary semigroup with unity element '1'.

**Definition3.3**:A ternary semigroup (T,.) is said to be R-commutative for all a,b in T and x, y in $T^1$ abc = xy(bac)

**Theorem 3.4:** Let (T,.,≤) be a totally ordered ternary semigroup where '≤' on T defined by a ≤ b ⇔ a =xyb=xby=bxy; a=xya for all a,b in T and for some x,y in $T^1$

.The necessary and sufficient condition is that (T,.) is commutative if and only if (T,.) is R-commutative.

**Proof:** As given (T,.,≤) is a totally ordered ternary semigroup and a ≤ b ⇔ a=xyb=xby=bxy; a=xya for all a,b in T and for some x,y in $T^1$.

Suppose (T,.) is commutative .i.e. abc= acb = bac = bca=cab=cba for all a,b,c in T

Since (T,.,≤) is totally ordered for any a,b in T either a ≤ b (or) b ≤ a. . To prove (T,.) is R-commutative

**Case (i):** consider a ≤ b

a =xyb = xby= bxy; a = xya

xy (bac) = xy(abc) (since (T,.) is commutative)

= (xya)bc

xy(bac) = abc

**Case(ii):** consider b ≤ a then

b=xya=xay=axy; b=xyb

xy(bac) = (xyb)ac

= bac

xy(bac) = abc (since (T,.) is commutative)

∴ (T,.) is R-commutative.

Conversely assume that (T,.) is R-commutative To have (T,.) is commutative consider

**case(i):** a ≤ b a=xyb=xby=bxy; a=xya

for abc = (xya)bc

= xy(abc)

= bac (since (T,.) is R-commutative)

Similarly we can prove abc=bca=cba=cab=acb

**case(ii):** b ≤ a b= xya= xay = axy ; b=xyb

for bac = (xyb)ac

= xy(bac)

= abc (since (T,.) is R-commutative)

Similarly we can prove abc=bca=cba=cab=acb

∴ (T,.) is commutative.

**Definition 3.6:** A ternary semigroup (T,.) is called left medial if xyzuv=yzxuv= zxyuv for every x,y,z,u,v in T.

**Definition 3.7:** A ternary semigroup (T,.) is called right medial if xyzuv=xyuvz=xyvzu for every x,y,z,u,v in T.

**Definition 3.8**:A ternary semigroup (T,.) is lateral medial if xyzuv=xzuyv=xuyzv for every x,y,z,u,v in T.

**Definition3.9**: A ternary semigroup T is called two sided (or) semimedial if it is both left and right medial.

**Definition 3.10:** A ternary semigroup T is called medial if it is left, right and lateral medial.

**Theorem 3.11:** Suppose (T,.≤) is a totally ordered ternary semigroup such that '≤' on T defined by a ≤ b ⇔ a=xyb=xby=bxy; a=xya =xay =axy for all a,b in T and for some x,y in $T^1$ .If (T,.) is medial , regular ternary semigroup then (T,.) is commutative.

**Proof:** Given that (T,.≤) is a totally ordered ternary semigroup and a ≤ b ⇔ a=xyb=xby=bxy; a=xya =xay =axy for all a,b in T and for some x,y in $T^1$ .

Also (T,.) is medial and regular we have to prove (T,.) is commutative. For this consider the following cases

**Case(i):** If a ≤ b ⇔ a=xyb=xby=bxy; a=xya =xay =axy for all a,b in T and for some x,y in $T^1$ .

By taking (T,.) as left regular for all 'a' there exists x,y in T such that

$a^3xy$ = a

$a^3xybc$ = abc

a a a x y b c = abc

(x y b)a(a x y b c) = abc

(x y b a)(a b x y c) = abc (since xyzuv=xuyzv)

(x y b a)(a b c x y) = abc (since xyzuv=xyvzu)

(x y b a)( c a b x y) = abc(since xyzuv=zxyuv)

(x y b a)( c a b x y) = abc(since xyzuv=zxyuv)

(x y b a c)(a b x y) = abc

(x y a c b)(a b x y) = abc(since xyzuv=xyuvz)

a c b a a = abc

b a c a a = abc(since xyzuv=zxyuv)

b a a a c = abc(since xyzuv=xyvuz)

b $a^3$c = abc

bac = abc

**Case(ii):**If b ≤ a then b=xya=xay=axy;

a= xyb = bxy = xby

Again by left regularity of T

$b^3xy$ = b

$b^3xyac$ = bac

b b b x y a c = bac

(a x y b)(b x y a c) =bac

(a x y b b)(x y a c) =bac

( y a x b b)(x y a c) =bac(since zuv=zxyuv)

( y a x b)( b x y a c) =bac

(y a x b)(b a x y c) =bac(since xyzuv=xuyzv)

(y a x b)( a x b yc) = bac(since xyzuv=yzxuv)

(y a x b)(a b y x c) =bac(since xyzuv=xzuyv)

(y a x b)(a b x y c) =bac(since xyzuv=xyvzu)

(y a x b a)(b x y c) = bac

(x y a b a)(b x y c) = bac(since xyzuv=zxyuv)

(a x y b a)( b x y c) = bac

( a x y b)( a b x y c) = bac

( a xy b)( a xy b c) = bac

a b b b c =bac

a $b^3$c = bac

abc =bac

Similarly we can prove abc = bca = acb = cab = cba

∴(T,.) is commutative

**Definition 3.12:** A ternary semigroup (T,.) is said to be intra regular for all 'a' in T there exists x, y in T such that $xa^5y$= a .

**Theorem 3.13:** Assume that (T,.,≤) is a totally ordered ternary semigroup such that '≤' on T defined by a ≤ b ⇔ a=xyb=xby=bxy; a=xya for all a,b in T and for some x,y in $T^1$ .If (T,.) is medial then(T,.) is intra regular.

**Proof:** Given that (T,.≤) is a ternary semigroup which is band and a ≤ b ⇔ a = xyb=xby=bxy; a=xya for all a,b in T and for some x,y in $T^1$and also (T,.) is medial.

To prove (T,.) is intra regular consider the following cases

**Case (i):** If a ≤ b ⇔ a = xyb = xby = bxy; a = xya for all a,b in T and for some x,y in $T^1$

$xa^5y$ = x a $a^3$ a y

= x a a a y

= a x a a y (since (T,.) is medial)

= a x y a a (since (T,.) is medial)

= a (x y a) a

= a a a

= $a^3$

$xa^5y$ = a

**Case (ii) :** If b ≤ a ⇔ b=xya=xay=axy; b=xyb for all a,b in T and for some x,y in $T^1$

$xb^5y$ = x b $b^3$ b y

= x b b b y

= b x b b y (since (T,.) is medial)

= b x y b b (since(T,.) is medial)

= b(x y b) b

= b b b

= $b^3$

$xb^5y$ = b

∴ (T,.) is intra regular.

**Theorem 3.14:** If (T,.≤) is a totally ordered ternary semigroup such that '≤' on T defined by a ≤ b ⇔ a=xyb=xby=bxy; a=xya for all a,b in T and for some x,y in $T^1$ .then (T,.) is right regular.

**Proof:** As given (T,.≤) is a ternary semigroup which is band and a ≤ b ⇔ a=xyb=xby=bxy; a=xya for all a,b in T and for some x,y in $T^1$ .

To prove (T,.) is right regular we have to show

$xya^3$ =a for all a in T and for some x,y in T

**Case(i):** If a ≤ b ⇔ a=xyb=xby=bxy; a=xya for all a,b in T and for some x,y in $T^1$ .

Consider $xya^3$ = xya

= a

**Case(ii):**If b ≤ a ⇔ b=xya=xay=axy; b=xyb for all a,b in T and for some x,y in $T^1$ .

Consider $xyb^3$ = xyb = b

∴ (T,.) is right regular

**Note:** By taking commutativity we can prove (T,.) is regular

**Theorem 3.15:** Suppose (T,.,≤ ) be a ternary semigroup such that '≤' on T defined by a ≤ b ⇔ a=xyb=xby=bxy; a=xya for all a,b in T and for some x,y in $T^1$ .If (T,.) is cancellative then (T,+) is weakly seperative ternary semigroup.

Proof: Given that (T,.,≤) is ternary semigroup which is band and

a ≤ b⇔ a= xyb=xby=bxy;a=xya for all a,b in T and x, y in $T^1$ and (T,.) is cancellative

we have to prove (T,+) is weakly seperative, for this consider the following cases

case(i): if a ≤ b ⇔ a=xyb=xby=bxy;a=xya

Assume a + a = a + b = b + b

a + a = a + b

xya + a = xyb + b

(xy + 1)a = (xy +1)b

a = b (since (T,.) is cancellative)

similarly

b + b = a + b

xy(b + b) = xy(a + b)

xyb + xyb = xya +xyb

2xyb = xya + a

2xyb = xya + xya

2xyb = 2xya

b = a (since (T,.) is cancellative)

Therefore a + a = a + b = b + b ⇒ a = b

case(ii) : b ≤ a ⇔ b= xya= xay= axy; b=xyb

Consider

a + a = a + b

xy(a + a) = xy(a + b)

xya + xya = xya + xyb

2xya = b + xyb

2xya = xyb + xyb

2xya = 2xyb

a = b (since (T,.) is cancellative)

Similarly

b + b = a + b

b + xyb = a + xya

b(1 + xy) = a(1 + xy)

b = a (since (T,.) is cancellative)

Therefore a + a = a + b = b + b ⇒ a = b

Hence (T,+) is weakly seperative ternary semigroup.

**Definition 3.16**:A ternary semigroup (T,.) is said to be normal if x(abc)x =x(acb)x= x(bca)x= x(bac)x= x(cab)x=x(cba)x for all x,a,b in T.

**Theorem 3.17:** Assume that (T,.,≤) is a totally ordered ternary semigroup where '≤' is defined by a ≤ b ⇔ a = xyb=xby=bxy;a= xya for all a,b in T and for some x,y in $T^1$.If T is R-commutative then it is normal.

**Proof** : As given (T,.,≤) is a totally ordered ternary semigroup and T is R-commutative ⇒ xy(bac) = abc for all a,b in T and x,y in $T^1$.

Since (T,.,≤) is totally ordered for any a,b in T either a ≤ b (or) b ≤ a

**Case(i):** If a ≤ b ⇔ a = xyb=xby=bxy;a= xya

To prove T is normal

x(abc)x = x(xya)bcx

= x(xy(abc))

= x(bac)x (since (T,.) is R-commutative)

**Case(ii):** If b ≤ a ⇔ b= xya=xay=axy ;b= xyb

let x(abc)x = xa(xyb)cx

= x(axy)bcx

= x(xya)bcx

= x (xy(abc))x

= x(bac)x(since (T,.) is R-commutative)

by the theorem 3.4 if (T,.) is R-commutative then (T,.) is commutative

Therefore x(abc)x=x(abc)x=x(cab)x= x(bca)x = x(cab)x =x(cba)x

∴(T,.) is normal.

**Theorem 3.18**:Let (T,.,≤) be a totally ordered ternary semigroup where '≤' is defined by a ≤ b⇔ a =xyb=xby=bxy; a=xya for all a,b in T and x,y in $T^1$. If T is rectangular band then it is R-commutative.

**Proof:** Given that (T,.,≤) is a totally ordered ternary semigroup where '≤' is defined by

a ≤ b⇔ a =xyb=xby=bxy; a=xya for all a,b in T and x,y in $T^1$.and also (T,.) is rectangular band. we have to prove (T,.) is R-commutative

**Case(i):** Consider a ≤b then a=xyb=xby=bxy ;a=xya

xy(bac) = (xyb)ac

= a a c

= (x b y)(x y a)c

= (x b y)x y (x y b)c

= x b (y x y x y) b c

= xbybc(since (T,.) is rectangular band)

= (xby)bc

xy(bac) = abc

Similarly we can prove abc=bca=cba=cab=acb

**Case (ii):** consider b ≤ a then b=xya=xay=axy; b=xyb

xy(abc) = (xya)bc

= b b c

= (x a y)(x y b)c

= (x a y) xy (xya)c

= x a (y x y xy)a c

= x a y a c

xy(abc) = b a c (since (T,.) is rectangular band)

∴(T,.) is R-commutative.

## Conclusion

In this paper we mainly discussed about the natural ordered relation in totally ordered ternary semirings.

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