Passivity Analysis of Time-Delayed Neural Networks with both Leakage Delay and Randomly Occurring Uncertainties

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Abstract—**This article studies the related issues of the robustness and passiveness of neural networks with time-varying delays and leakage delays with parameter uncertainties.The white noise sequence that obeys the relevant Bernoulli distribution enters the system in randomly form.By choosing the appropriate LKFs, and using methods such as Wirtinger inequality and free weight matrix to improve the delay standard, and express it in the form of linear matrix inequality.Sufficient conditions are established to ensure the robust random stability and passivity of the neural network under consideration.Finally, a simulation example is given using the LMI toolbox to prove the validity and conservativeness of the standard proposed in this article.**

Index Terms—**delayed neural networks, leakage delay, passivity, uncertainties.**

I. Introduction

In the past few decades, scientists have been working hard to study neural networks, and have been applied in many fields, such as signal processing, pattern recognition and optimization problems.At the same time, it has been widely used in practical fields such as industrial control, clinical medicine, commercial bank loans, and risk warning.The most important thing in these applications is to ensure the stability of the system.In actual hardware, due to the limited switching speed and communication time of the amplifier, a time delay will inevitably be caused.In addition, the image signal contains many uncertain factors in the transmission process, such as system shock, signal confusion and so on.Similarly, parameter uncertainty is also an important reason for system delay and poor performance, such as poor external or operating environment and switching loss. Therefore, it is necessary and not to be ignored to consider the robustness of the delay system. Greatly improve the quality of the system.And many scholars have made research and contributions [1-4].

On the other hand, in many large-scale projects, the stability and passivity of the system are inseparable.Passivity analysis is based on stability analysis and is derived from the theory of system dissipation. The main physical meaning of passivity is to truly reflect the attenuation of system energy.In essence, the theory of system stability can be analyzed from the passivity, which is a comprehensive analysis method of the passivity.The concept of passivity has been integrated with control theory for decades. After many scientists research and development, the concept of passivity has been widely used in fuzzy control, synchronization problems, network control and other fields.And it has become one of the valuable reference tools in the comprehensive research of control system.Analysis and research on the passivity of time-delay neural networks will also produce indispensable application value.Recently, passive research has provided some testing standards for the neural network theory of delay uncertainty [5-7].

In [8], several sufficient conditions are given to ensure the passivity of neural networks with discrete time-varying delays and distributed infinite delays.In [9,10,11], the authors studied the passivity of neural networks with discrete time-varying delay and distributed time-varying delay. In [12], the author obtained several sufficient conditions for neutral discrete and distributed time-delay neural networks to check the passivity of the neural network under con sideration.However, time delays may occur in a random manner, and sometimes delays that vary over time are indis tinguishable.In this case, the above methods may be difficult to apply, so it is necessary to further study the passive problem of neural networks with time-varying delay under the assumption of mild time-varying delay and small error. The authors in [13] study the passivity of neural networks with leakage delays. In [14], the passivity of uncertain neural networks with leakage delay and time-varying delay is further studied[19-23].

The purpose of this paper is to study the robustness and passivity of neural networks with time-varying delays and uncertain parameters of leakage delays.Uncertainties of white noise sequences (ROUs) that obey some uncorrelated Bernoulli distributions appear in a random form and enter the system.By combining appropriate Lyapunov-Krasovskii functionals, Wirtinger’s inequality and free weight matrix methods to improve the delay criterion, the generalized activation function is realized.Sufficient conditions are established to ensure that the considered neural network has robust random stability and passivity, and reduces the conservativeness of the system.Examples are given using the LMI toolbox to prove the validity and conservativeness of the standards proposed in this article.

II. Problem analysis

A. *Problem formulation and preliminaries*

In this section, we consider the following time-delayed neural

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networks with both leakage delay and randomly occurring uncertainties:

$$\dot{x}(t) = (A + \alpha(t)\Delta A(t))x(t-\delta) + (W + \beta(t)\Delta W(t))\phi(x(t)) + (W_1 + \nu(t)\Delta W_1(t))\phi(x(t-h(t))) + u(t) \quad (1)$$

where $x_1 = [x_1(t) \cdot\cdot\cdot x_n(t)]^T \in R_n$ is the state vector of the system at time t, $A = diag\{-a_1, \dots, a_n\} \in R^{n\times n}(a_k > 0, k = 1 \cdot\cdot\cdot, n)$ is the self-feedback matrix, $W_1^{n\times n}$, $W^{n\times n} \in R^{n\times n}(i = 1, 2)$ is the connection weight matrix, $\phi(x(t)) = [\phi_1(x_1(t)) \cdot\cdot\cdot \phi_n(x_n(t))]^T \in R^n$ is the activation function of neurons, and $u(t) = [u_1(t) \dots u_n(t)]^T \in R_n$ is the control input vector. The symbol δ and h(t) are the leakage delay and the time-varying continuous and bounded function in the delayed neural networks, satisfying

$$0 \leqslant \tau(t) \le \tau, \quad 0 \le \delta$$

where and are constants.The sum of the theories used in this paper is summarized as:

[K1] activation function $\phi_i(\bullet)(i = 1, \dots, n)$ in (1)is bounded and satisfy the following inequality

$$\ell_i^- \le \frac{\phi_i(\kappa_1) - \phi_i(\kappa_2)}{\kappa_1 - \kappa_2} \le \ell_i^+, \kappa_1 \ne \kappa_2 \in \mathcal{R}^n$$

where $\ell_i \geqslant 0$ and $\ell_i^+ > 0$ are known real constants. Define $L^- = diag\{\ell_1, \ell_2, \dots, \ell_n\}$ and $L^+ = diag\{\ell_1^+, \ell_2^+, \dots, \ell_n^+\}$

[K2] The real-valued matrix △A(t), △W(t),and △W1(t)with appropriate dimensions represent the norm bounded pa rameter uncertainty of the following structure:

$$[\triangle A(t) \ \ \triangle W(t) \ \ \triangle W1(t)] = HF(t)\ [E1\ E2\ E3]$$

where $F(t) \in R^{l\times j}$ is an unknown time-varying matrix satisfying

$$F(t)^T F(t) \leqslant I$$

and H, E1, E2,and E3 are known constant matrices.

[K3] In order to illustrate the phenomena of randomly occurring uncertainties,we introduce the stochastic variables α(t), β(t),and ν(t), which satisfy the mutually independent Bernoulli-distributed white sequences in order to explain the parameter uncertainty in the paper. The natural supposes about α(t), β(t) and ν(t) are as follows:

$Pr\{\alpha(t) = 1\} = \alpha$, $Pr\{\alpha(t) = 0\} = 1-\alpha$
$Pr\{\beta(t) = 1\} = \beta$, $Pr\{\beta(t) = 0\} = 1-\beta$
$Pr\{\upsilon(t) = 1\} = \upsilon$, $Pr\{\upsilon(t) = 0\} = 1 - \upsilon$

α, β,and υ are known constants,where $\alpha \in [0, 1]$, $\beta \in [0, 1]$,and $\upsilon \in [0, 1]$.

First, the initial conditions related to system (1) are given $x(s) = g(s), s \in [\rho, 0]$.

where g(s) is bounded and continuously differential on [ρ, 0], ρ = max δ, τ. Suppose x(t, g) is the state trajectory of system (1) From the above initial conditions and the corresponding trajectories under the initial conditions of x(t, 0).

Lemma 1 ([15]).For any matrix $Q \in R^n$, $Q > 0$,a vector function $\omega : [d1, d2] \rightarrow R^n$, such that the integrations concerned are well defined, the following holds:

$$(\alpha-\beta)\int_\beta^\alpha x^T(s)Qx(s)ds \ge \left\{\int_\beta^\alpha x^T(s)ds\right\} Q \left\{\int_\beta^\alpha x(s)ds\right\}$$

Lemma 2 ([16])For given matrices H, E and F with $F^T F \leqslant I$ and a scalar ε > 0, the following holds:

$$HFE + (HFE)^T \leqslant \varepsilon HH^T + \varepsilon^T E^T E.$$

Lemma 3 ([17]). For any n × n constant matrix R ∈ R, and R is a symmetric positive definite matrix,a scalar function h := h(t) > 0,and a vector valued function $x : [-h, 0] \rightarrow R_n$, so the following related integrals can be easily defined and then the following inequality holds:

$$-h\int_{t-h}^{t} \dot{x}^T(s)R\dot{x}(s)ds \le \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} -R & R \\ R & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}$$

Lemma 4 ([18]). System (1) is called globally passive if there exists a scalar γ > 0 such that

$$2\int_0^{t_p} f^T(x(s))u(s)ds \ge -\gamma \int_0^{t_p} u^T(s)u(s)ds$$

for all $t_p \ge 0$ and for all x(t, 0).

B. Main results

To facilitate reading, we set up the matrix:

$$L_1 = diag(L_1^-, L_2^-, \dots, L_n^-),$$
$$L_2 = diag(L^+, L_2^+, \dots, L_n^+),$$
$$L_1 = diag(L_1^- L_1^+, L_2^- L_2^+, \dots, L_n^- L_n^+),$$
$$L_1 = diag(\tfrac{L_1^- + L_1^+}{2}, \tfrac{L_2^- + L_2^+}{2}, \dots, \tfrac{L_n^- + L_n^+}{2}).$$

Theorem 1 Under satisfying (k1)-(k3), if the system (1) has a scalar γ > 0, it is passive in definition 1.six positive constants $\varepsilon_i > 0(i = 1, 2, 3, 4, 5, 6)$, five symmetric positive definite matrices Pi(i = 1, 2, 3, 4, 5), four positive diagonal matrices D, H, R and S, and four matrices $Q_i(i = 1, 2, 3, 4)$ such that the following LMI holds:

$$\Omega = \begin{bmatrix} \Omega_1 & \Omega_2 \\ * & \Omega_3 \end{bmatrix} < 0, \quad (2)$$

$$\Omega_1 = \begin{bmatrix} \Omega_{11} & \Omega_{12} & L_3Z & 0 & P_1A & AP_1A & Q_2W+L_4R+L_4Z & Q_2W_1-L_4Z & \alpha Q_2 \\ * & \Omega_{22} & 0 & 0 & 0 & P_1C & D-H+Q_1W & Q_1B & Q_1 \\ * & * & \Omega_{33} & Q_3 & 0 & 0 & 0 & L_4S+L_4Z & 0 \\ * & * & * & -P_4-P_5 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -P_2 & -AP_1A & 0 & 0 & 0 \\ * & * & * & * & * & -P_3 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Omega_{77} & Z & -\beta I \\ * & * & * & * & * & * & * & \Omega_{88} & 0 \\ * & * & * & * & * & * & * & * & -\gamma I \end{bmatrix},$$

$$\Omega_2 = \begin{bmatrix} Q_2H_1 & Q_2H_2 & Q_2H_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_1H_1 & Q_1H_2 & Q_1H_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Omega_3 = diag\{-\varepsilon_4 I, -\varepsilon_5 I, -\varepsilon_6 I, -P_5, -\varepsilon_1 I, -\varepsilon_2 I, -\varepsilon_3 I, -P_5\}$$

in which

Ω_{11} =- P_1A-AP_1+P_2+ δ^2P_3+P_4- Q_2A-AQ_2- P_5-L_3Z+(ε_1+ ε_4)E^T_1 E_1-L_3R, Ω_{12} = P_1- L_1D+L_2H-AQ^T_1 -Q_2+P_5, Ω_{22} = τ^2P_5-Q_1 -Q_1^T , Ω 33 =- L_3S - L_3Z, Ω_{77} = (ε_2 + ε_5)E^T_2 E_2 -R -Z, Ω_{88} = (ε 3 + ε 6)E^T_3 E_3- S - Z.

proof: From definition (K1) we can get

$$\int_0^{x_i(t)} (\phi_i(s) - L_i^- s)ds \geq 0,$$

$$\int_0^{x_i(t)} (L_i^+(s) - \phi_i s)ds \geq 0, i = 1, 2, ..., n$$

Let D = diag(d_1, d_2, ..., d_n), H = diag(h_1, h_2, ..., h_n), and consider the following Lyapunov-Krasovskii functional:

V(t) = V1(t) + V2(t) + V3(t) + V4(t)   (3)

Where(4)-(7)

$$V_1(t) = (x(t) - A\int_{t-\delta}^t x(s)ds)^T P_1(x(t) - A\int_{t-\delta}^t x(s)ds)$$

$$V_2(t) = 2\sum_{i=1}^n d_i \int_0^{x_i(t)} (\phi_i(s) - L_i^- s)ds + 2\sum_{i=1}^n h_i \int_0^{x_i(t)} (L_i^+ s - \phi_i(s))ds$$

$$V_3(t) = \int_{t-\delta}^t x^T(s)P_2x(s)ds + \delta\int_{-\delta}^0\int_{t+\theta}^t x^T(s)P_3x(s)dsd\theta$$

$$V_4(t) = \int_{t-\tau}^t x^T(s)P_4x(s)ds + \tau\int_{-\tau}^0\int_{t+\theta}^t \dot{x}^T(s)P_5\dot{x}dsd\theta$$

Calculating the time derivative of V_i(t)(i = 1, 2, 3, 4), we obtain (8)-(11)

$$\begin{aligned} \dot{V}_1(t) &= 2(x(t) - A\int_{t-\delta}^t x(s)ds)^T P_1(\dot{x}(t) - Ax(t) + Ax(t-\delta)) \\ \dot{V}_2(t) &= 2\dot{x}^T(t)D(\phi(x(t)) - L_1x(t)) + 2\dot{x}^T(t)H(L_2x(t) - \phi(x(t))) \\ \dot{V}_3(t) &= x^T(t)(P_2 + \delta^2P_3)x(t) - x^T(t-\delta)P_2x(t-\delta) \\ &\quad -\delta\int_{t-\delta}^t x^T(s)P_3x(s)ds \\ &\leq x^T(t)(P_2 + \delta^2P_3)x(t) - x^T(t-\delta)P_2x(t-\delta) \\ &\quad -(\int_{t-\delta}^t x(s)ds)^T P_3(\int_{t-\delta}^t x(s)ds) \\ \dot{V}_4(t) &= x^T(t)P_4x(t) - x^T(t-\tau)P_4x(t-\tau) + \tau^2\dot{x}^T(t)P_5\dot{x}(t) \\ &\quad -\tau\int_{t-\tau}^t \dot{x}^T(s)P_5\dot{x}(s)ds \\ &\leq x^T(t)P_4x(t) - x^T(t-\tau)P_4x(t-\tau) + \tau^2\dot{x}^T(t)P_5\dot{x}(t) \\ &\quad + \begin{bmatrix} x^T(t) \\ x^T(t-\tau) \end{bmatrix}^T \begin{bmatrix} -P_5 & P_5 \\ * & -P_5 \end{bmatrix} \begin{bmatrix} x^T(t) \\ x^T(t-\tau) \end{bmatrix} \end{aligned}$$

From the system (1), we have (12)

$$0 = 2(\dot{x}^T(t)Q_1 + x^T(t)Q_2)[-\dot{x}(t) - (A + \alpha(t)\Delta A(t))x(t)$$
$$+(W_1 + \beta(t)\Delta W_1(t))\phi(x(t)) + (W_2 + \upsilon(t)\Delta W_2(t))\phi(x(t-\tau(t))) + u(t)$$

for positive diagonal matrices R > 0, Z > 0 and S > 0, we can get from assumption (K1) that [4]: (13)-(15)

$$\begin{bmatrix} x(t) \\ \phi(x(t)) \end{bmatrix}^T \begin{bmatrix} L_3R & -L_4R \\ * & R \end{bmatrix} \begin{bmatrix} x(t) \\ \phi(x(t)) \end{bmatrix} \leq 0$$

$$\begin{bmatrix} x(t-\tau(t)) \\ \phi(x(t-\tau(t))) \end{bmatrix}^T \begin{bmatrix} L_3S & -L_4S \\ * & S \end{bmatrix} \begin{bmatrix} x(t-\tau(t)) \\ \phi(x(t-\tau(t))) \end{bmatrix} \leq 0$$

$$\begin{bmatrix} x(t) \\ \phi(x(t)) \\ x(t-\tau(t)) \\ \phi(x(t-\tau(t))) \end{bmatrix}^T \begin{bmatrix} L_3Z & -L_4Z & -L_3Z & L_4Z \\ * & Z & L_4Z & -Z \\ * & * & L_3Z & -L_4Z \\ * & * & * & Z \end{bmatrix} \begin{bmatrix} x(t) \\ \phi(x(t)) \\ x(t-\tau(t)) \\ \phi(x(t-\tau(t))) \end{bmatrix} \leq 0$$

In addition,by assumption (K3) and Lemma 2, we get (16)-(21)

$$\begin{aligned} &-2\dot{x}^T(t)Q_1\Delta Ax(t) \leq \\ &\qquad \varepsilon_1^{-1}\dot{x}^T(t)Q_1H_1H_1^TQ_1^T\dot{x}(t) + \varepsilon_1x^T(t)E_1^TE_1x(t) \\ &2\dot{x}^T(t)Q_1\Delta W\phi(x(t)) \leq \\ &\qquad \varepsilon_2^{-1}\dot{x}^T(t)Q_1H_2H_2^TQ_1^T\dot{x}(t) + \varepsilon_2\phi^T(x(t))E_2^TE_2\phi(x(t)) \\ &2\dot{x}^T(t)Q_1\Delta W_1\phi(x(t-\tau(t))) \leq \\ &\qquad \varepsilon_3^{-1}\dot{x}^T(t)Q_1H_3H_3^TQ_1^T\dot{x}(t) + \varepsilon_3\phi^T(x(t-\tau(t)))E_3^TE_3\phi(x(t-\tau(t))) \\ &-2x^T(t)Q_2\Delta Ax(t) \leq \\ &\qquad \varepsilon_4^{-1}x^T(t)Q_2H_1H_1^TQ_2^Tx(t) + \varepsilon_4x^T(t)E_1^TE_1x(t) \\ &2x^T(t)Q_2\Delta W\phi(x(t)) \leq \\ &\qquad \varepsilon_5^{-1}x^T(t)Q_2H_2H_2^TQ_2^Tx(t) + \varepsilon_5\phi^T(x(t))E_2^TE_2\phi(x(t)) \\ &2x^T(t)Q_2\Delta W_1\phi(x(t-\tau(t))) \leq \\ &\qquad \varepsilon_6^{-1}x^T(t)Q_2H_3H_3^TQ_2^T\dot{x}(t) + \varepsilon_6\phi^T(x(t-\tau(t)))E_3^TE_3\phi(x(t-\tau(t))) \end{aligned}$$

it follows from (8)-(21) that

$$\dot{V}(t) - 2\phi^T x(t))u(t) - \gamma u^T(t)u(t) = \Gamma^T\Xi\Gamma(t) \qquad (22)$$

where

$$\Gamma(t) = (x^T(t), \dot{x}^T(t), x^T(t-\tau(t)), x^T(t-\tau), x^T(t-\delta), \int_{t-\delta}^{t} x^T(s)ds, \phi^T(x(t)), \phi^T(x(t-\tau(t))), u^T(t))^T$$

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & L_3Z & 0 & P_1A & AP_1A & Q_2W + L_4R + L_4Z & Q_2W_1 - L_4Z & \alpha Q_2 \\ * & \Xi_{22} & 0 & 0 & 0 & P_1C & D - H + Q_1W & Q_1B & Q_1 \\ * & * & \Xi_{33} & Q_3 & 0 & 0 & 0 & L_4S + L_4Z & 0 \\ * & * & * & -P_4 - P_5 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -P_2 & -AP_1A & 0 & 0 & 0 \\ * & * & * & * & * & -P_3 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Xi_{77} & Z & -\beta I \\ * & * & * & * & * & * & * & \Xi_{88} & 0 \\ * & * & * & * & * & * & * & * & -\gamma I \end{bmatrix}$$

in which

$\Xi_{11} = -P_1A - P_1 + P_2 + \delta^2 P_3 + P_4 - Q_2A - AQ_2 - P_5 - L_3Z + (\varepsilon_1 + \varepsilon_4)E^T_1 E_1 - L_3R + \varepsilon^{-1}_4 Q_2H_1H^T_1 Q^T_2 + \varepsilon^{-1}_5 Q_2H_2H^T_2 Q^T_2 + \varepsilon^{-1}_6 Q_2H_3H^T_3 Q^T_2$

$\Xi_{12} = P_1 - L_1D + L_2H - AQ^T_1 - Q_2 + P_5$

$\Xi_{22} = \tau^2 P_5 - Q_1 - Q^T_1 + \varepsilon^{-1}_1 Q_1H_1H^T_1 Q^T_1 + \varepsilon^{-1}_2 Q_1H_2H^T_2 Q^T_1 + \varepsilon^{-1}_3 Q_1H_3H^T_3 Q^T_1$

$\Xi_{33} = -L_3S - L_3Z$, $\Xi_{77} = (\varepsilon_2 + \varepsilon_5)E^T_2 E_2 - R - Z$

$\Xi_{88} = (\varepsilon_3 + \varepsilon_6)E^T_3 E_3 - S - Z$

Using Schur complement lemma, it is easy to verify $\Pi < 0$ and equivalence of $\Omega < 0$

$$\dot{V}(t) - 2\phi^T x(t))u(t) - \gamma u^T(t)u(t) \le 0 \qquad (23)$$

From (23) and the definition of V(t), we can have

$$2\int_0^{t_p} \phi^T(x(s))u(s)ds \ge -\gamma \int_0^{t_p} u^T(s)u(s)ds$$

For all $t_p \ge 0$ and all x(t, 0). According to Lemma 4, we know that the neural network (1) is globally passive.The proof is complete.

When the system has no uncertainty, the system (1) becomes:

$$\dot{x}(t) = Ax(t-\delta) + W\phi(x(t)) + W_1\phi(x(t-h(t))) + u(t)) \qquad (24)$$

When the system has no leakage delay, the system (1) becomes:

$$\dot{x}(t) = (A + \alpha(t)\Delta A(t))x(t) + (W + \beta(t)\Delta W(t))\phi(x(t)) + (W_1 + \nu(t)\Delta W_1(t))\phi(x(t-h(t))) + u(t)) \qquad (25)$$

When the system has no leakage delay and uncertainty, the system (1) becomes:

$$\dot{x}(t) = Ax(t) + W\phi(x(t)) + W_1\phi(x(t-h(t))) + u(t)) \qquad (26)$$

The theory in this article is still valid in the above system

C. Figures

In this section, in order to illustrate the advantages of the results of the newly proposed passivity standard, we will give numerical examples commonly used in his papers.

A.Example

Consider NN(1) with time-varying delay and borrow the matrix parameters given below[14]

$$A = \begin{bmatrix} -0.9 & 0 \\ 0 & 0.8 \end{bmatrix}, W = \begin{bmatrix} 1.3 & -1.7 \\ -1.6 & 1.2 \end{bmatrix}, W_1 = \begin{bmatrix} 1.4 & 1.9 \\ 0.6 & -1.2 \end{bmatrix}$$

$\phi(t) = 0.1(|x+1| - |x-1|)$, $\tau(t) = 2 + 0.15\sin(t)$, $\alpha = \beta = \upsilon = 0$, $\delta = 0$.

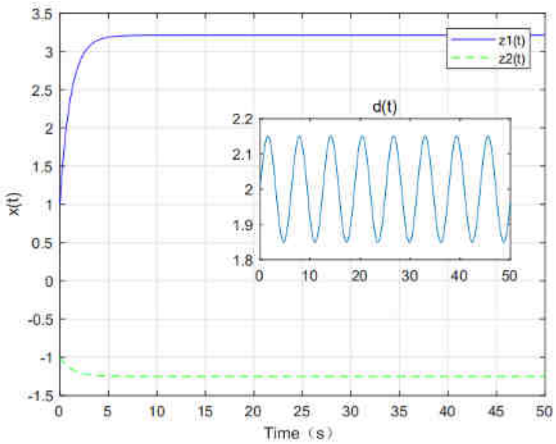

Figure 1: State trajectory of the system of Example

In order to solve the linear matrix inequality in Theorem 1 and verify our reasoning, the method of MATLAB LMI Toolbox is used. Applying Theorem 1 in this article to this example, it can be concluded from Figure 1 that this article has obtained relatively stable parameter values. In this example, we get that the neural network with time-varying delay can still maintain its stability in a more general system.

III. Conclusions

This paper mainly discusses the problems related to the robustness and passivity of neural networks with time varying delays and uncertain leakage delays. In the case of uncertain random parameters, the leakage delay and time are solved. For the stability and passivity of the delayed neural network system, better methods are used to improve the conservativeness of the system, and examples are used to verify the validity and conservativeness of the standards proposed in this article.

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