

The Jack-Up Leg, Wave-Particle Kinematics, and Its Interaction

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Abstract—Any structure installed in the sea will have to interact with the marine environment. The main source of interaction comes from wind, wave, and current. This interaction will produce an external force acting on the structure. In this paper, the motion of water particles was investigated. Several wave heights were selected to compare variation in the outcome. Results are presented in tabulated and graphical forms. Water particle motion is presented in terms of velocity and acceleration in the vicinity of the jack-up leg. It was found that the variation in the velocity is in a similar trend for all cases of wave height, where the velocities are higher near the sea surface. Similarly, the acceleration of water particles also shows a higher magnitude near the water surface. The highest water velocity is found to be 3.06 m/s for wave height $H=13$ m. A maximum acceleration of 1.38 m/s² occurred in the case of vertical wave-particle acceleration for $H=8$ m. Horizontal wave load-induced load of 23.5 kN occurred on the jack-up leg at a water level 3.4 m below mean water level (MWL).

Index Terms— jack-up leg, wave loading, water particle motion, wave-structure interaction.

I. INTRODUCTION

Wave loading accounts for the biggest contribution to the loading on offshore structures employed in the ocean. The source of this loading is due to the water particle motion during the interaction of wave as well as current with the structure. This paper focuses on the wave-structure interaction of a jack-up leg. The main feature of a typical jack-up structure is a self-floating body with three or four retractable legs at its corners to support the body during the operation. The kinematics of water particle motion in the vicinity of the jack-up structure's leg were investigated and results are presented in the following sections. The basic formulation of water particle motion and the modeling of the jack-up structure's legs, loading parameters, and subsequent results derived from the analysis are presented and discussed. The major finding is in the form of water particle velocity and acceleration are estimated for the location along the span of the leg of structure. These magnitudes were later plotted to represent the overall variation for the whole water depth of 70 m.

II. BACKGROUND

During its operational condition, the jack-up structure will lower its legs to the seabed thus supporting and lifting its main body clear from the water surface. In this condition, wave and structure interaction now occurred only on the legs. Due to its

operational limitation and the challenges in environmental loading as well as its technical capacity, the jack-up structure become suitable to be used in the region of shallow and medium water depth [1]. The typical size of a jack-up can have a leg of 130 m long with the capability to drill to a subsurface of up to 9 km [2]. The technical problems associated with their safe and economic operation become very great as the waters become more hostile as the depth increased [3]. The maximum water depth that the jack-up structure know to have operated is 175m by the rig name CJ80 with an air gap of 25 m [4]. The interaction of water particles with the jack-up leg will produce external induced forces on the structural members. The kinematics of water particles are investigated and the results are presented in form of velocity and acceleration.

III. STRUCTURAL MODELLING

The jacket leg considered in this study is adopted from an earlier study [5], [1]. The structure was modeled using finite element software to represent the lattice assembly of elements as shown in Fig. 1. It is assumed that the wave-particle kinematics is parallel to the rectangular sides of the leg's lattice assembly. Fig. 2 shows the cross-section assembly of the leg members which has a total length of 104.6 m. The outer tubular has a diameter of 324 mm and wall thickness of 19.1 mm while the inner bracing has a diameter of 224 mm and wall thickness of 9.5 mm.

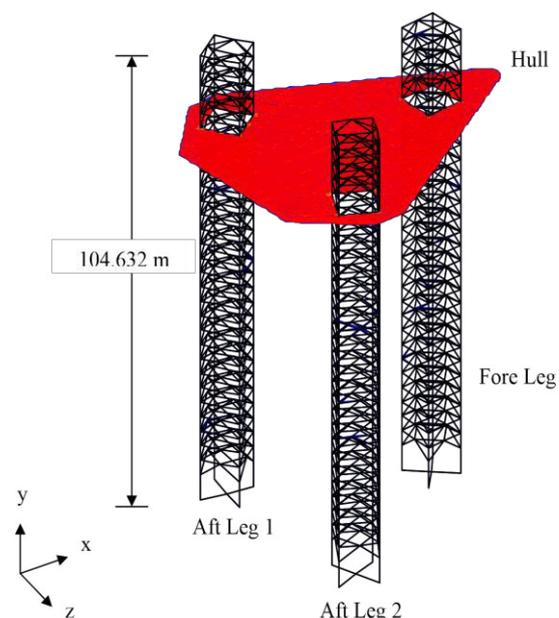


Fig. 1. Jack-up structural model

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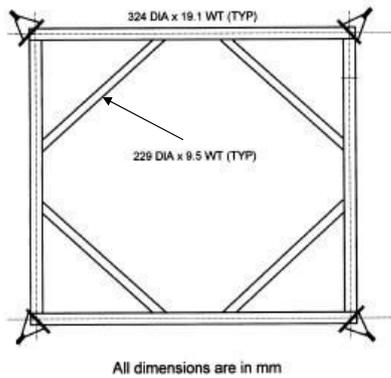


Fig. 2. Jack-up leg layout.

IV. WAVE KINEMATIC FORMULATION

The structure is analyzed for the operational condition at a water depth of 70 m. The effect of wave height on the variation of wave-particle velocity and acceleration during interaction with jack-up legs is estimated. Wave parameters are presented in TABLE 1.

TABLE 1. Wave Parameters

Wave Parameters						
	H=8m	H=9m	H=10m	H=11m	H=12m	H=13m
Water Depth, h (m)	70	70	70	70	70	70
Wave Height, H (m)	8	9	10	11	12	13
Wave Length, λ (m)	136.13	154.97	173.78	192.45	210.89	229.04
Wave number, k (m ⁻¹)	0.0462	0.0405	0.0362	0.0326	0.0298	0.0274
Frequency, ω (rad)	0.5867	0.5493	0.5179	0.4911	0.4677	0.4473
Period, T (sec)	10.71	11.44	12.13	12.80	13.43	14.05

There are several types of wave theories that can be used to represent wave behavior in the ocean. In this study, the Airy wave theory was adopted [6], [7]. Water particles velocities in the x-direction, *v* at any point of time, *t* is given as:

Horizontal direction;

$$u(x, t) = \frac{\pi H \cosh[k(z+h)]}{T \sinh(kh)} \cos(kx - \omega t) \quad (1)$$

Vertical direction;

$$v(x, t) = \frac{\pi H \sinh[k(z+h)]}{T \sinh(kh)} \sinh(kx - \omega t) \quad (2)$$

where H is the wave height, T is the wave period, z is the distance from MWL, h is water depth, k is wave number, ω is the wave frequency.

A similar procedure was adopted estimating water particle acceleration. The associated acceleration of water particles, *ū* and, *ṽ* at any point of time, *t* is defined as:

Horizontal acceleration;

$$\dot{u}(x, t) = \frac{2\pi^2 H \cosh[k(z+h)]}{T^2 \sinh(kh)} \sinh(kx - \omega t) \quad (3)$$

Vertical acceleration;

$$\dot{v}(x, t) = \frac{-2\pi^2 H \sinh[k(z+h)]}{T^2 \sinh(kh)} \cos(kx - \omega t) \quad (4)$$

Sea driven current velocity that accompanied the wave particle motion is estimated using the following relationship;

$$v_{ct} = v_{cto} \left(\frac{z+d}{d} \right)^{\frac{1}{7}} \quad (5)$$

where *v_{cto}* is current velocity at mean sea level, *z* is distance from the surface and *d* is water depth.

Wave loads on submerged section of the jack-up legs are estimated using the Morison equation;

$$F_{wave}(z, t) = \frac{1}{2} \rho C_D A u |u| + \frac{\pi}{4} \rho C_M D^2 \dot{u} \quad (6)$$

where ρ is water density, C_D and C_M are drag coefficient and inertial coefficient respectively.

TABLE 2. Hydrodynamic coefficients for circular cylinder [8].

Hydrodynamic Coefficients		
Surface condition	Drag coefficient, C _D	Inertia coefficient, C _M
Multiyear roughness	1.05	1.8
Mobile unit (cleaned)	1.0	1.8
Smooth member	0.65	2.0

V. RESULTS AND DISCUSSION

In this section, the results of the observation are presented in tabulated form and graphical plots. It can be used to evaluate the magnitude of hydrodynamic loading on the jack-up legs by incorporate it into the wave loading formulae. The estimation of wave velocity and acceleration are based on Airy wave theory. Tables 3 to 6 show the horizontal and vertical wave velocities in the location of the jack-up leg. Fig. 3 to Fig. 6 show the associated graphs of these tables.

TABLE 3. Horizontal wave particle velocity at jack-up leg.

Horizontal Wave Velocity at Fore Leg (m/s)						
Above Seabed (m)	H=8m	H=9m	H=10m	H=11m	H=12m	H=13m
0	0.181	0.285	0.408	0.548	0.701	0.864
6.0	0.188	0.293	0.418	0.559	0.712	0.876
9.4	0.198	0.306	0.432	0.575	0.729	0.893
12.8	0.214	0.324	0.453	0.597	0.753	0.918
16.2	0.234	0.348	0.481	0.627	0.785	0.951
19.6	0.260	0.380	0.516	0.665	0.825	0.992
23.0	0.293	0.418	0.558	0.711	0.873	1.042
26.4	0.333	0.464	0.609	0.765	0.930	1.101
29.8	0.381	0.519	0.670	0.830	0.997	1.169
33.2	0.439	0.584	0.740	0.904	1.074	1.248
36.6	0.507	0.661	0.822	0.989	1.162	1.338
40.0	0.588	0.749	0.916	1.087	1.262	1.439
43.4	0.684	0.852	1.024	1.198	1.375	1.553
46.8	0.796	0.971	1.148	1.324	1.502	1.680
50.2	0.928	1.109	1.288	1.467	1.644	1.822
53.6	1.083	1.268	1.449	1.627	1.804	1.980
57.0	1.264	1.451	1.631	1.807	1.981	2.155
60.4	1.477	1.661	1.838	2.010	2.180	2.349
63.8	1.726	1.904	2.073	2.237	2.400	2.563
67.2	2.018	2.182	2.339	2.493	2.646	2.799

70.6	2.360	2.502	2.640	2.778	2.918	3.060
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TABLE 4. Vertical wave particle velocity at jack-up leg.

Vertical Wave Velocity at Aft Leg (m/s)						
Above Seabed (m)	H=8m	H=9m	H=10m	H=11m	H=12	H=13m
0	0	0	0	0	0	0
6.0	0.012	0.014	0.016	0.017	0.018	0.019
9.4	0.019	0.023	0.025	0.027	0.029	0.029
12.8	0.027	0.031	0.035	0.038	0.039	0.040
16.2	0.035	0.041	0.045	0.049	0.050	0.051
19.6	0.044	0.051	0.056	0.060	0.062	0.063
23.0	0.054	0.062	0.068	0.072	0.074	0.075
26.4	0.065	0.075	0.081	0.085	0.087	0.088
29.8	0.079	0.088	0.095	0.099	0.101	0.102
33.2	0.094	0.104	0.111	0.115	0.116	0.116
36.6	0.111	0.121	0.128	0.131	0.133	0.132
40.0	0.131	0.141	0.147	0.150	0.150	0.149
43.4	0.154	0.163	0.168	0.170	0.169	0.167
46.8	0.181	0.189	0.192	0.192	0.190	0.186
50.2	0.213	0.218	0.219	0.217	0.213	0.207
53.6	0.250	0.251	0.249	0.244	0.238	0.230
57.0	0.293	0.289	0.283	0.275	0.265	0.255
60.4	0.343	0.333	0.321	0.309	0.295	0.282
63.8	0.402	0.383	0.364	0.346	0.328	0.312
67.2	0.470	0.440	0.413	0.388	0.365	0.344
70.6	0.551	0.506	0.468	0.435	0.405	0.379

TABLE 5. Horizontal wave particle acceleration at jack-up leg

Horizontal Wave Acceleration at Fore Leg (m/s ²)						
Above Seabed (m)	H=8m	H=9m	H=10m	H=11m	H=12m	H=13m
0	0.025	0.032	0.038	0.043	0.047	0.050
6.0	0.026	0.033	0.039	0.044	0.048	0.051
9.4	0.027	0.034	0.040	0.045	0.049	0.052
12.8	0.029	0.036	0.042	0.047	0.050	0.053
16.2	0.032	0.039	0.045	0.049	0.053	0.055
19.6	0.036	0.042	0.048	0.052	0.055	0.057
23.0	0.040	0.047	0.052	0.056	0.058	0.060
26.4	0.046	0.052	0.057	0.060	0.062	0.064
29.8	0.052	0.058	0.062	0.065	0.067	0.068
33.2	0.060	0.065	0.069	0.071	0.072	0.072
36.6	0.070	0.074	0.076	0.078	0.078	0.077
40.0	0.081	0.084	0.085	0.085	0.084	0.083
43.4	0.094	0.095	0.095	0.094	0.092	0.090
46.8	0.109	0.109	0.107	0.104	0.100	0.097
50.2	0.127	0.124	0.120	0.115	0.110	0.105
53.6	0.149	0.142	0.135	0.127	0.121	0.114
57.0	0.174	0.162	0.151	0.142	0.133	0.124
60.4	0.203	0.186	0.171	0.157	0.146	0.136
63.8	0.237	0.213	0.192	0.175	0.161	0.148
67.2	0.277	0.244	0.217	0.195	0.177	0.162
70.6	0.324	0.280	0.245	0.218	0.195	0.177

TABLE 6. Vertical wave particle acceleration at jack-up leg

Vertical Wave Acceleration at Fore Leg (m/s ²)						
Above Seabed (m)	H=8m	H=9m	H=10m	H=11m	H=12m	H=13m
0	0.000	0.000	0.000	0.000	0.000	0.000
6.0	0.030	0.039	0.046	0.053	0.059	0.064
9.4	0.048	0.061	0.074	0.084	0.093	0.101
12.8	0.067	0.085	0.102	0.116	0.128	0.139
16.2	0.087	0.110	0.131	0.149	0.165	0.178
19.6	0.110	0.138	0.163	0.185	0.203	0.218
23.0	0.135	0.168	0.197	0.222	0.243	0.261
26.4	0.164	0.201	0.234	0.262	0.286	0.305
29.8	0.197	0.239	0.275	0.306	0.331	0.353
33.2	0.235	0.280	0.320	0.353	0.380	0.403
36.6	0.278	0.327	0.369	0.404	0.433	0.457
40.0	0.328	0.381	0.425	0.461	0.491	0.515
43.4	0.387	0.441	0.486	0.523	0.553	0.577
46.8	0.455	0.510	0.555	0.592	0.621	0.645
50.2	0.534	0.589	0.633	0.668	0.696	0.718
53.6	0.626	0.679	0.720	0.752	0.777	0.797
57.0	0.734	0.782	0.818	0.846	0.867	0.883
60.4	0.860	0.899	0.928	0.950	0.965	0.977
63.8	1.007	1.034	1.052	1.065	1.074	1.079
67.2	1.179	1.188	1.193	1.194	1.193	1.191
70.6	1.381	1.366	1.351	1.338	1.325	1.313

Fig. 3 shows the graph of variation of horizontal wave velocity near the reference points on the jack-up leg as presented in Table 3. Horizontal wave velocity increases exponentially from the seabed to the MWL for all wave heights under consideration. The increase in velocity is almost at a similar rate reaching the maximum magnitudes of 2.36 ms⁻¹ and 3.06 ms⁻¹ for wave height H = 8 m and H = 13 m respectively. It was expected that the wave-particle velocity is at the highest near the MWL as shown translated by equation (1) where the higher the water depth, *h*, the higher the horizontal wave particle velocity, *u*.

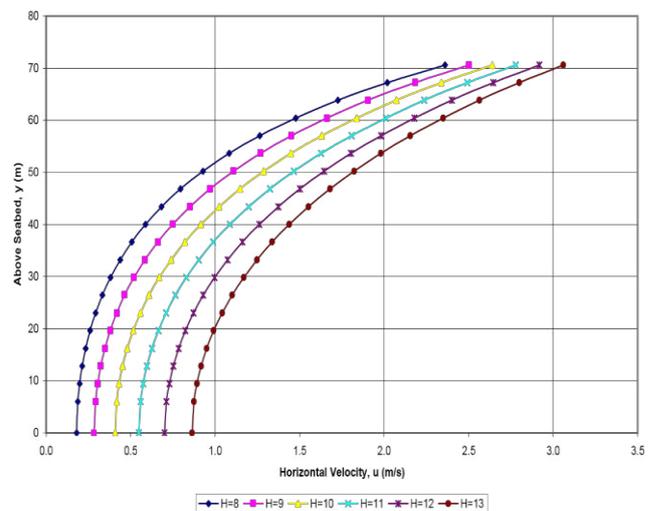


Fig. 3. Variation of horizontal wave particle velocity at jack-up leg

Variation in vertical wave particle velocity is shown in Fig. 4 as presented in Table 4. The trend shows that lower

wave height will have a higher vertical velocity near the MWL. It is in good agreement with the water velocity particle motion which is more circular near the surface. Fig. 3 and Fig. 4 inclined to confirm that the velocity profiles resembled the case for intermediate water depth where $\frac{1}{20} < \frac{d}{L} < \frac{1}{2}$, [9].

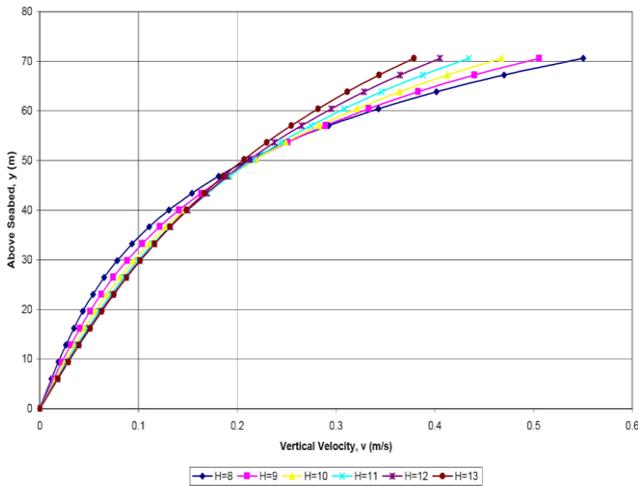


Fig. 4. Variation of vertical wave particle velocity at jack-up leg

Fig. 5 and Fig. 6 show the variation in horizontal and vertical wave-particle acceleration during interaction with the jack-up leg. For both cases, the water particles kinematics show an exponential relationship with the variation in water depth. Horizontal wave acceleration rates are higher for a smaller wave height where it reached a magnitude of 0.324 ms^{-2} for $H = 8 \text{ m}$ at MWL. Maximum acceleration occurred near the MWL where it was associated with maximum horizontal wave velocities. There is a large variation between different wave-height as shown in Fig. 5. There is a much higher magnitude of vertical wave acceleration as shown in Fig. 6. The maximum magnitude is 1.4 ms^{-2} near MWL. Vertical wave-particle acceleration reached quite a similar magnitude near the MWL due to the restriction of wave vertical motion at the surface. The maximum variation of vertical acceleration occurred at mid-water-depth of about 35 m above the seabed.

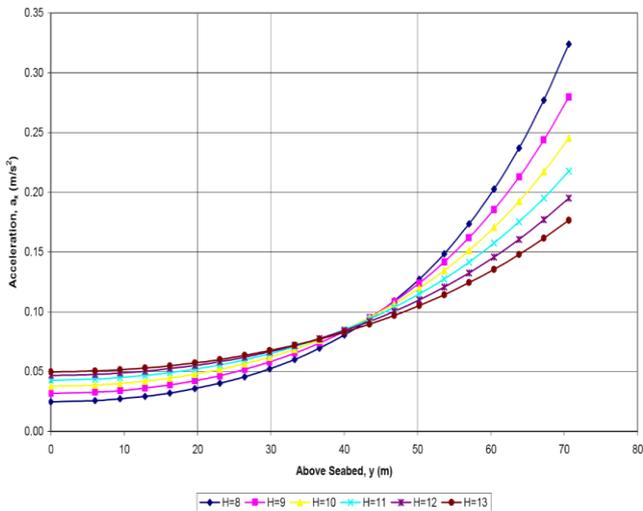


Fig. 5. Variation of horizontal wave particle acceleration at jack-up leg

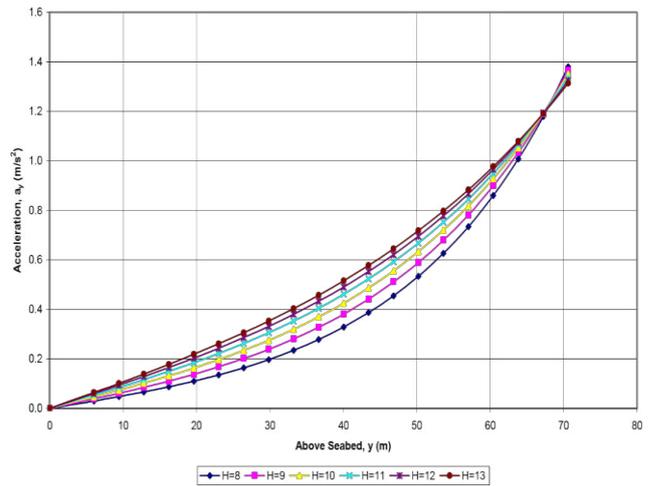


Fig. 6. Variation of vertical wave particle acceleration at the jack-up leg

Fig. 7 shows an external load acting on the jack-up leg due to wave-particle interaction with the structural members for $H=8\text{m}$. Joints of the tubular leg members that are at the corner of the leg's assembly received a higher load as expected. Similarly, middle points of the horizontal braces that are in a perpendicular direction to the wave-particle motion also experienced high wave load. Maximum induced load occurred at corner joints and middle of transverse members are found to be 23.5 kN which is located near the mean water level (MWL). These local loading at each structural level may further be analyzed to estimate the total base shear loading as well as its associated overturning moment experienced by the structure.

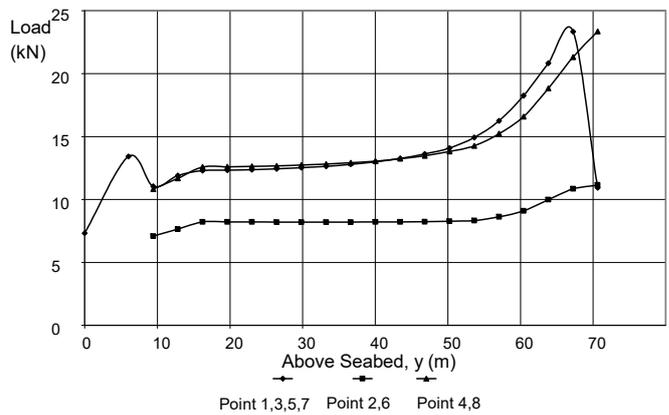


Fig. 7. Horizontal load on the jack-up leg (case $H=8\text{m}$)

VI. CONCLUSIONS

The following conclusions may be drawn from the study on the wave-particle kinematics and interaction with the jack-up leg.

1. Results presented in this paper are outcomes of a study on wave particles kinematics near the jack-up leg for cases of

wave height, H, between 8 m and 13m. The corresponding water depth, $d = 70$ m.

2. Wave-particle velocities are in exponential increment for all water depth with a magnitude of 2.35 m/s to 3.06 m/s for $H = 8$ m and $H = 13$ m respectively.
3. Maximum acceleration of 1.38 m/s^2 which occurred in the case of vertical wave article acceleration for $H=8$ m.
4. Variation wave height has some effects on the variation in final wave-particle velocities and acceleration.
5. Maximum induced load due to wave-particle motion interaction with the leg structure is found to be 23.5 kN at a location near 3.4 m below mean water level.

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