Synchronization of Chaotic Gyro System Based on Higher Order Sliding Mode Control

Jing Zhang

Abstract— **A chaotic gyro system to investigate the method for higher order sliding mode control design. The system is assumed to have uncertain parameters with known upper and lower bounds. We also design an optimal sliding surface for the sliding mode control. The control law is designed to guarantee the existence of the sliding mode around the nonlinear surface Simulations are carried out to demonstrate the utility of the control method.**

Index Terms—**Chaotic synchronization, Adaptive Sliding mode control, Finite time stability, Geometric homogeneity**

I. INTRODUCTION

Chaotic systems are dynamical systems and its response exhibits a lot of specific characteristics, including an excessive sensitivity to the initial conditions, fractal properties of the motion in phase space, broad spectrums of Fourier transform.

Today, chaos has been seen to have many useful applications in many engineering systems such as secure communications, optics, power converters, chemical and biological systems, neural networks and so on[1-3]. Synchronization in chaotic dynamic systems has received a great deal of interest due to its potential application in secure communications [4,5]. Several control methods have been successfully applied to chaotic motion control. For example Sliding mode control [6], adaptive control [7,8], backstepping control [9], etc. Many chaotic systems are inevitably affected by parameter variations and external disturbances. Sliding mode control (SMC) is a popular robust control approach for nonlinear systems operating under uncertainty conditions, as the controllers can be designed to compensate for the uncertainties or disturbances. In order to reduce the chattering, high order sliding mode (HOSM) approach has been recently proposed [10,11]. Keeping the main advantages of the standard sliding mode control, the chattering effect is reduced and finite-time convergence is provided. An interesting HOSM are proposed in [12] with the robustness of the system during the entire response. However, the knowledge of the upper bound of the system uncertainties is hard to be gotten accurately. In this paper, the approach in [12] is modified and applied Synchronization in chaotic dynamic systems, so that a continuous feedback is produced combining the robustness of high-order sliding modes and finite-time stabilization by continuous control. The aim of the modified method is to deal with unknown but bounded system uncertainties. The upper bounds of uncertainties are not required to be known in advance. System stability is proven by using the Lyapunov theory.

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Jing Zhang, School of Electronic and Information Engineering, Tianjin Vocational Institute, Tianjin, China

II. PRELIMINARIES AND MODEL FORMULATION

The chaotic gyro system equation in the sleep position is described as[1-3]:

$$\ddot{\phi}+c_1\dot{\phi}+c_2\dot{\phi}^3+\alpha^2\frac{(1-\cos\phi)^2}{\sin^3\phi}-\beta\sin\phi=-f\sin\omega t\sin\phi \quad (1)$$

Where ϕ is the angle with linear plus cubic damping and the term $f\sin\omega t$ is a harmonic parametric excitation. Defined the chaotic gyro syetem state variables as follows: $x_1=\phi, x_2=\dot{\phi}$. Then, the system (1) can be transformed the convenient first-order form:

$$\begin{aligned}\dot{x}_1&=x_2\\ \dot{x}_2&=-c_1x_1-c_2x_2^3-\alpha^2\frac{(1-\cos x_1)^2}{\sin^3 x_1}+(\beta-f\sin\omega t)\sin x_1\end{aligned} \quad (2)$$

Where The parameters are selected exactly $\alpha=10,\beta=1$, $c_1=0.5, c_2=0.05, \omega=2, f=35.5$. The nonlinear gyro system exhibits the chaotic behavior for these parameter values. Chaotic responses of system(2) is shown in Fig. 1without any control input. System (1) shows a chaotic behavior.

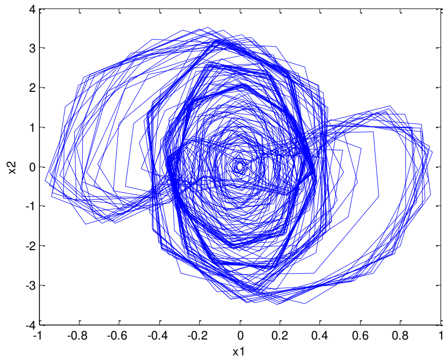


Fig. 1 Chaotic behavior of system x

The nonlinear gyro system, as master, is considered by (2). To control the system effectively we propose to add a control-input u . By adding this input, the nonlinear gyro slave system is described as follows:

$$\begin{aligned}\dot{y}_1&=y_2\\ \dot{y}_2&=-c_1y_1-c_2y_2^3-\alpha^2\frac{(1-\cos y_1)^2}{\sin^3 y_1}+\\ &(\beta-f\sin\omega t)\sin y_1+\Delta h(y_1,y_2)+u\end{aligned} \quad (3)$$

Where the $\Delta h(y_1,y_2)\in R$ is the system uncertainty and bounded. It is satisfied as follows: $|\Delta h(y_1,y_2)|\le H$. H is the upper-bounds of uncertainties with unknown bound.

The synchronization problem considered in this paper is to design a sliding mode controller u based on finite time stabilization, which synchronize the states of the master nonlinear gyro system (1) and the slave system (2) in spite of the unknown nonlinear parameter vector. In other words, the aim of synchronization is to make as follows:

$$\lim_{t\to\infty}\|Y-X\|=0 \tag{4}$$

Let us define the tracking error as:

$$\begin{aligned}E(t)&=Y(t)-X(t)=[y_1-x_1,y_2-x_2]^T\\&=[e(t),\dot e(t)]^T=[e_1(t),e_2(2)]^T\end{aligned} \tag{5}$$

$$X=[x_1,x_2]^T:=[x_1,\dot x_1]^T$$

Subtract (1) from (2) and the real error dynamics would be obtained as:

$$\begin{aligned}&\dot e_1=e_2\\&\dot e_2=-c_1e_1-\alpha^2[\Psi_1(y_1)-\Psi_1(x_1)]\\&-c_2[\Psi_2(y_2)-\Psi_2(x_2)]+(\beta-f\sin\omega t)\\&\times[\Psi_3(y_1)-\Psi_3(x_1)]+\Delta h(y_1,y_2)+u\end{aligned} \tag{6}$$

Where these nonlinear functions are defined as follows:

$$\Psi_1(z)=\frac{(1-\cos z)^2}{\sin^3 z},\Psi_2(z)=z_2^3,\Psi_3(z)=\sin z$$

This paper proposes a new adaptive sliding mode controller for chaotic gyro system to not only preserve the advantages of variable structure control but also reduce the chattering phenomenon and the synchronization of the states of slave chaotic systems converges to the states of master system. An adaptive higher order sliding mode control for chaotic gyro system is established in the next section.

III. ADAPTIVE HOSMC FOR GYRO SYSTEM

In practical terms, the resolution of the finite time stabilization is a delicate task which has generally been studied for homogeneous systems of negative degree with respect to a flow of a complete vector field. Indeed, for this kind of systems, finite time stability is equivalent to asymptotic stability [13-15].A constructive feedback control law for finite time stabilization of all-dimension chain of integrators without uncertainty has been proposed in [13]. Before designing our robust finite time controller, we introduce the algorithm given in [13] and show its problem in terms of robustness.

A. Finite Time stabilization of an integrator chain system

Consider the nominal system (6), which is represented by SISO independent integrator chains, is defined as follows:

$$\begin{cases}\dot z_1=z_2\\ \vdots\\ \dot z_{n-1}=z_n\\ \dot z_n=\omega_{nom}\end{cases} \tag{7}$$

Lemma 1: [13] Let $k_1,\cdots,k_n>0$ be such that the polynomial $\lambda^n+k_n\lambda^{n-1}+\cdots+k_2\lambda+k_1$ is Hurwitz. Consider the system (7), There exists $\varepsilon\in(0,1)$ such that, for every $\alpha\in(1-\varepsilon,1)$, the origin is a globally finite-time stable equilibrium for the system under the feedback

$$\omega_{nom}(z)=-k_1sgn(z_1)|z_1|^{\alpha_1}-\cdots-k_nsgn(z_n)|z_n|^{\alpha_n} \tag{8}$$

where $\alpha_1,\cdots,\alpha_n$ satisfy $\alpha_{i-1}=\dfrac{\alpha_i\alpha_{i+1}}{2\alpha_{i+1}-\alpha_i}$ $i=2,\cdots,n$ with $\alpha_{n+1}=1$ and $\alpha_n=\alpha$.

In order to design adaptive sliding mode controller for uncertain chaotic systems with unknown bounded uncertainties, there exist two major phases, First, an integral sliding manifold should be selected such that the sliding motion on the manifold has the desired properties. Second, an adaptive continuous control law should be determined such that the existence of the sliding mode can be guaranteed without knowing the upper-bounds of uncertainties from lemma 1.

B. Design of Adaptive HOSMC for Gyro system

Define the integral sliding manifold for system (8):

$$s=e_2-e_2(0)-\int_0^t\omega_{nom}dv \tag{9}$$

It is obviously $\dot s(t)=\dot e_2-\omega_{nom}$ (10)

where the construction of control law ω_{nom} is given in Lemma 1**.** In order to stabilize in finite time system (6) with uncertainties, we define the following control law:

$$u=u_0+u_1 \tag{11}$$

where

$$\begin{aligned}u_0&=c_1e_1+\alpha^2[\Psi_1(y_1)-\Psi_1(x_1)]\\&+c_2[\Psi_2(y_2)-\Psi_2(x_2)]\\&-(\beta-f\sin\omega t)[\Psi_3(y_1)-\Psi_3(x_1)]\end{aligned} \tag{12}$$

$$u_1=\hat\rho sign(s) \tag{13}$$

The adaptive law is designed as follows:

$$\dot{\hat\rho}=\frac{1}{\gamma}\|s\| \tag{14}$$

Where parameter $\gamma>0$ is the gain of adaptation determining the adaptive process.

Theorem 1 Consider the error system (6) with parametric uncertain and disturbances. If the controller is designed as (11)-(13). The positive adaptive feedback gains are updated according to the adaptation law (14), and can ensures the establishment error state trajectory converges to the sliding manifold (9) $s=0$ in finite time.

Proof : The Lyapunov candidate function is selected to be:

$$V=\frac{1}{2}s^2+\frac{1}{2}\beta(\hat\rho\text{-}\rho*)^2 \tag{15}$$

ρ is a bounded function. So, its estimate value $\hat\rho$ is also bounded . Suppose $\hat\rho\le\rho*$, it exists $\rho*\ge\rho_d$.

$$\begin{aligned}&\dot V=s\dot s+\beta(\hat\rho\text{-}\rho*)\dot{\hat\rho}\\&=s[\lambda(c-b)z_1-(b+\lambda+c\lambda)z_2-3x_1^2\lambda z_1\\&-\lambda z_1^2(z_1+3x_1)-cu-c\rho-\omega_{nom}]+\beta(\hat\rho\text{-}\rho*)\dot{\hat\rho}\\&=s[-\hat\rho|c|sign(s)\text{-}c\rho]+\beta(\hat\rho\text{-}\rho*)\dot{\hat\rho}\\&\le-\hat\rho|c|\|s\|+|c\rho|\cdot\|s\|+\beta(\hat\rho\text{-}\rho*)\dot{\hat\rho}\\&\le-\hat\rho\|s\|+|c\rho_d|\cdot\|s\|+\beta(\hat\rho\text{-}\rho*)\dot{\hat\rho}\\&\le-\hat\rho|c|\|s\|+\rho*|c|\|s\|\text{-}\rho*|c|\|s\|+|c\rho_d|\cdot\|s\|+\beta(\hat\rho\text{-}\rho*)\dot{\hat\rho}\\&\le-(\rho*-\hat\rho)|c|\|s\|\text{-}\rho*|c|\|s\|+|c\rho_d|\cdot\|s\|+\beta(\hat\rho\text{-}\rho*)\dot{\hat\rho}\\&\le-(|c|+1)\|s\|\cdot(\rho*-\hat\rho)\text{-}(\rho*-\rho_d)|c|\cdot\|s\|\end{aligned}$$

$$\le -\sqrt{2}\frac{(|c|+1)}{\beta}\|s\|\cdot\left[\beta\cdot\frac{(\rho^*-\hat{\rho})}{\sqrt{2}}\right]-\sqrt{2}(\rho^*-\rho_d)|c|\cdot\frac{\|s\|}{\sqrt{2}}$$

$$\le -\min\left\{\sqrt{2}\frac{(|c|+1)}{\beta}\|s\|,\sqrt{2}(\rho^*-\rho_d)|c|\right\}*$$

$$\left[\beta\cdot\frac{(\rho^*-\hat{\rho})}{\sqrt{2}}+\frac{\|s\|}{\sqrt{2}}\right]$$

$$\le -\min\left\{\sqrt{2}\frac{(|c|+1)}{\beta}\|s\|,\sqrt{2}(\rho^*-\rho_d)|c|\right\}*$$

$$\left[\left(\beta\cdot\frac{\rho^*-\hat{\rho}}{\sqrt{2}}\right)^2+\left(\frac{\|s\|}{\sqrt{2}}\right)^2\right]^{\frac{1}{2}}$$

$$\le -\min\left\{\sqrt{2}(|c|+1)\|s\|,\sqrt{2}(\rho^*-\rho_d)|c|\right\}V^{\frac{1}{2}}$$

$$\eta\ge\min\left\{\sqrt{2}(|c|+1)\|s\|,\sqrt{2}(\rho^*-\rho_d)|c|\right\}>0$$

$$V(0)\ge V(t)+\int_0^t\eta[V(\tau)]^{\frac{1}{2}}d\tau\ge\int_0^t\eta[V(\tau)]^{\frac{1}{2}}d\tau$$

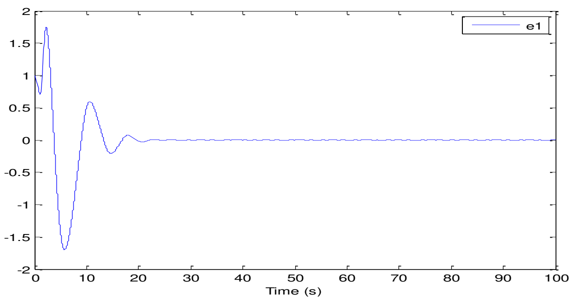
One obtain:

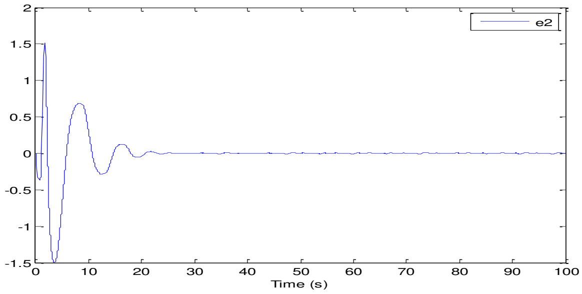
$$t_f\le\frac{2[V(0)]^{\frac{1}{2}}}{\min\left\{\sqrt{2}(|c|+1)\|s\|,\sqrt{2}(\rho^*-\rho_d)|c|\right\}}$$

IV. SIMULATION

In this section, the presented control algorithm is demonstrateed. In these numerical simulations, the fourth-order Runge－Kutta method is used to solve Lur’ e-like system with time step size 0.001 in Matlab/Simulink. The parameters are selected as follow:

A perturbation $\beta(\cdot)=0.5\sin(2y_1)+10\sin(t)$ is considered, where $n=3$ $\alpha_3=\frac{3}{4}$, $\alpha_2=\frac{3}{5}$, $\alpha_1=\frac{1}{2}$, $k_1=3$, $k_2=2.5$, $k_3=1$ η=1.5. The simulation results are illustrated in Fig. 2. From the figure, we can see that the synchronization error e_1, e_2, e_3 will converge to zero in the finite time. Figs.3 and 4 show the control input and the corresponding sliding manifold S(t). In particular, it is worthy of note that, no information of upper-bounds of uncertainties is used in our control design. Estimate value of adaptive gain $\hat{G}$ is described in Fig.5. The adaptation gain parameter and initial value are set as $q=2$ and $\hat{G}(0)=0$. Fig.5 shows that the adaptation parameter tends to a constant value.





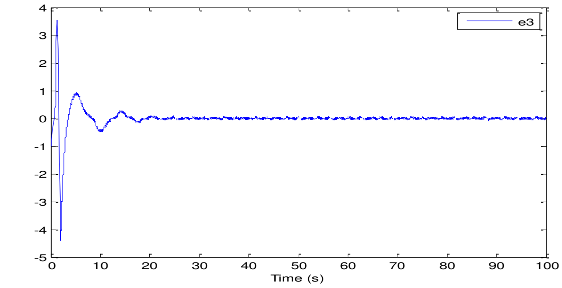


Fig. 2 Time responses of error states

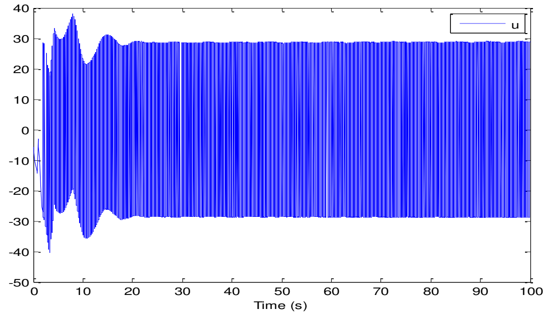


Fig.3 Time response of control input u(t).

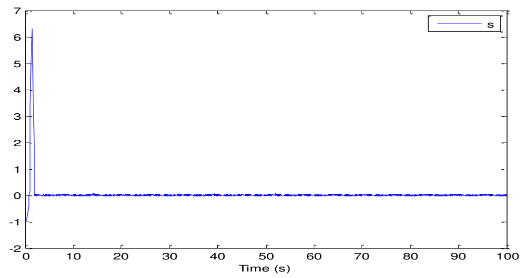


Fig.4 Time response of the corresponding sliding manifold S(t)

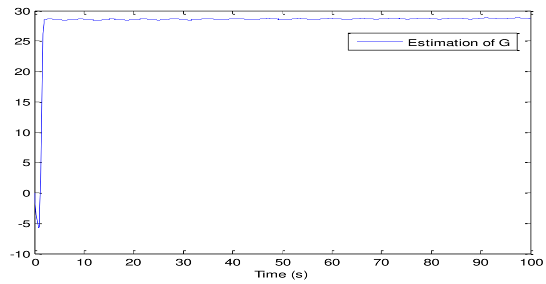


Fig.5 Time response of parameter estimation value G

V. CONCLUSION

This work proposes an adaptive SMC controller for nonlinear systems with parametric uncertainties. This method can be viewed as the finite time stabilization based on geometric homogeneity and integral sliding mode control. The knowledge of the upper bound of the system uncertainties is not prior required. Simulation results demonstrate that the proposed control method is effective.

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