Interaction Solutions for the Sawada–Kotera Equation

Suzhen Lin, Huaitang Chen

Abstract— **In this paper, a new auxiliary equation method is presented. Analytical multiple function solutions including trigonometric functions, exponential functions, hyperbolic functions and other functions can be easily obtained. Using this method, we can obtain interaction solutions of the Sawada - Kotera equation. It is significant to help physicists to analyze special phenomena in their relevant fields accurately.**

***Index Terms*— interaction solution, Sawada–Kotera equation, auxiliary equation method.**

I. INTRODUCTION

In the past decades, many methods are proposed to obtain exact solutions of nonlinear partial differential equations: such as inverse scattering theory[1], Hirotas bilinear method[2], Darboux transformation[3] and so on. In recent years, a large number of powerful methods to solve nonlinear partial differential equations are considered. They are the homogeneous balance method[4], sine-cosine method[5], the hyperbolic tangent function method[6,7], the Jacobi elliptic function expansion method[8,9]. One of important methods is the auxiliary equation method. Ma[10] and Chen[11-13] are devoted to constructing special interaction soliton solutions by using combination of auxiliary equations and get great success. This method has attracted extensive attention as it's concise and understandable.

As we known, the complicated nature phenomena are often well described by nonlinear partial differential equations. The most representative nonlinear equation is the Sawada–Kotera equation.

$$u_{xxxxx}+5uu_{xxx}+5pu_xu_{xx}+5u^2u_x+u_t=0, \tag{1}$$

where p is a constant. Eq. (1) contains many famous equations. When $p=1$, it is the standard Sawada-Kotera equation[14].

II. THE NEW SOLUTIONS OF THE NOVEL SUB-EQUATION

The novel auxiliary equation reads:

$$f''(\xi)=2f^3(\xi). \tag{2}$$

Integrating once, we get:

$$f'(\xi)^2=f^4(\xi)+b. \tag{3}$$

Manuscript received April 08, 2022

Suzhen Lin, School of Mathematics and Statistics, Linyi University, Linyi, 276005,China

Huaitang Chen, School of Mathematics and Statistics, Linyi University, Linyi, 276005,China

We obtain new multiple solutions of eq. (2) in the following:

$$f_1=\left\{\ln\left[\frac{(\tanh(\xi)+\tan(\xi))e^{\xi}}{1+\tan(\xi)\coth(\xi)+\tanh(\xi)+\tan(\xi)}\right]\right\}^{-1}, \tag{4}$$

$$f_2=\left\{\ln\left[\frac{(\tanh(\xi)+\coth(\xi))e^{\xi}}{1+\tanh(\xi)\tan(\xi)+\tan(\xi)+\coth(\xi)}\right]\right\}^{-1}, \tag{5}$$

$$f_3=\left\{\ln\left[\frac{(\tanh(\xi)+\cot(\xi))e^{\xi}}{1+\cot(\xi)\coth(\xi)+\tanh(\xi)+\cot(\xi)}\right]\right\}^{-1}, \tag{6}$$

$$f_4=\left\{\ln\left[\frac{(\cot(\xi)+\coth(\xi))e^{\xi}}{1+\tanh(\xi)\cot(\xi)+\cot(\xi)+\coth(\xi)}\right]\right\}^{-1}, \tag{7}$$

$$f_5=\left\{\ln\left[\frac{(1+\tan(\xi)\coth(\xi))e^{\xi}}{1+\tanh(\xi)+\tan(\xi)+\tan(\xi)\coth(\xi)}\right]\right\}^{-1}, \tag{8}$$

$$f_6=\left\{\ln\left[\frac{(1+\cot(\xi)\coth(\xi))e^{\xi}}{1+\tanh(\xi)+\cot(\xi)+\cot(\xi)\coth(\xi)}\right]\right\}^{-1}, \tag{9}$$

$$f_7=\left\{\ln\left[\frac{(1+\tanh(\xi)\tan(\xi))e^{\xi}}{1+\tan(\xi)+\coth(\xi)+\tanh(\xi)\tan(\xi)}\right]\right\}^{-1}, \tag{10}$$

$$f_8=\left\{\ln\left[\frac{(1+\tanh(\xi)\cot(\xi))e^{\xi}}{1+\cot(\xi)+\coth(\xi)+\tanh(\xi)\cot(\xi)}\right]\right\}^{-1}. \tag{11}$$

III. THE NEW SUB-EQUATION METHOD

Step1: For a given nonlinear partial differential equation with independent variables x,t, **:**

$$P(u_t,u_x,u_{xx}\dots)=0. \tag{12}$$

We make a transformation as follow:

$$u(x,t)=u(\xi),\xi=k(x-ct). \tag{13}$$

Where k,c are constants to be determined

Step2: Inserting (13) into eq. (12), we get an ordinary differential equation:

$$P(u,u',u''...)=0. \tag{14}$$

Step3: We assume exact solutions of eq. (14) in the following:

$$u(\xi)=\sum_{i=0}^{n} a_i f^i(\xi). \tag{15}$$

Where n is a positive integer determined by the balance principle in eq. (14). $f(\xi)$ satisfies eq. (2). Substituting (15) into eq. (14). We obtain a set of algebra equations when we set all coefficients of $f^i(\xi)$ to zeroes. Therefore a_i will be determined by solving the set of algebra equations. We will apply the method to the Sawada－Kotera equation.

IV. New interaction solution of the Sawada-Kotera equation

The Sawada–Kotera Equation is:

$$u_{xxxxx}+5uu_{xxx}+5pu_x u_{xx}+5u^2 u_x+u_t=0, \tag{16}$$

Where p is a constant

We make the following transformations:

$$u(x,t)=u(\xi), \xi=k(x-ct). \tag{17}$$

Inserting (17) into eq. (16), we get an ordinary differential equation:

$$k^4 u^{(5)}(\xi)+5k^2 u(\xi)u'''(\xi)+5pk^2 u'(\xi)u''(\xi)+5u^2(\xi)u'(\xi)-cu'(\xi)=0. \tag{18}$$

We assume the solution of eq. (18) in the following:

$$u(\xi)=\sum_{i=0}^{n} a_i f^i(\xi), \xi=k(x-ct). \tag{19}$$

Where n is positive integer determined by balancing the linear term of $k^4 u^{(5)}(\xi)$ and the nonlinear term of $5u^2(\xi)u'(\xi)$. We get $n=2$.

$$u(\xi)=a_0+a_1 f(\xi)+a_2 f^2(\xi), \xi=k(x-ct), \tag{20}$$

Where a_0, a_1, a_2 are constants to be determined. We get the following result by using the auxiliary equation method:

$$a_0=0, a_1=0, a_2=a_2,$$

$$b=\frac{c}{a_2(4RootOf(72\_z^4+(6a_2p+12a_2)\_z^2+a_2)^2 p-12RootOf(72\_z^4+(6a_2p+12a_2)\_z^2+a_2^2)^2-a_2)}$$

$$c=c, k=RootOf(72\_z^4+(6a_2p+12a_2)\_z^2+a_2^2).$$

We obtain interaction solutions of eq. (16) in the following:

$$u_1=a_2\left\{\ln\left[\frac{(\tanh(\xi)+\tan(\xi))e^{\xi}}{1+\tan(\xi)\coth(\xi)+\tanh(\xi)+\tan(\xi)}\right]\right\}^{-2}, \tag{21}$$

$$u_2=a_2\left\{\ln\left[\frac{(\tanh(\xi)+\coth(\xi))e^{\xi}}{1+\tanh(\xi)\tan(\xi)+\tan(\xi)+\coth(\xi)}\right]\right\}^{-2}, \tag{22}$$

$$u_3=a_2\left\{\ln\left[\frac{(\tanh(\xi)+\cot(\xi))e^{\xi}}{1+\cot(\xi)\coth(\xi)+\tanh(\xi)+\cot(\xi)}\right]\right\}^{-2}, \tag{23}$$

$$u_4=a_2\left\{\ln\left[\frac{(\cot(\xi)+\coth(\xi))e^{\xi}}{1+\tanh(\xi)\cot(\xi)+\cot(\xi)+\coth(\xi)}\right]\right\}^{-2}, \tag{24}$$

$$u_5=a_2\left\{\ln\left[\frac{(1+\tan(\xi)\coth(\xi))e^{\xi}}{1+\tanh(\xi)+\tan(\xi)+\tan(\xi)\coth(\xi)}\right]\right\}^{-2}, \tag{25}$$

$$u_6=a_2\left\{\ln\left[\frac{(1+\cot(\xi)\coth(\xi))e^{\xi}}{1+\tanh(\xi)+\cot(\xi)+\cot(\xi)\coth(\xi)}\right]\right\}^{-2}, \tag{26}$$

$$u_7=a_2\left\{\ln\left[\frac{(1+\tanh(\xi)\tan(\xi))e^{\xi}}{1+\tan(\xi)+\coth(\xi)+\tanh(\xi)\tan(\xi)}\right]\right\}^{-2}, \tag{27}$$

$$u_8=a_2\left\{\ln\left[\frac{(1+\tanh(\xi)\cot(\xi))e^{\xi}}{1+\cot(\xi)+\coth(\xi)+\tanh(\xi)\cot(\xi)}\right]\right\}^{-2}, \tag{28}$$

Where $\xi=k(x-ct)$. k, c and a_2 are arbitrary nonzero constants.

Conclusion and discussion

In this paper, we obtain the interaction solutions of the Sawada－ Kotera equation by using the auxiliary equation method. These solutions include trigonometric functions, hyperbolic functions and exponential functions, which can help physicists to analyze special phenomena in their relevant fields accurately.

References

1. C. S. Gardner, J. M. Greene, M. D. Kruskal, R. M. Miura, Method for solving the KdV equation, Phys. Rev. Lett, 1967, 19, 19, 1095-1097.
2. R. Hirota, Exact Solution of the KdV Equation for Multiple Collisions of Solutions, Physical Review Letters, 1971, 27, 18,1192-1194.
3. Y. S. Li, J. E. Zhang, Darboux transformations of classical Boussinesq system and its multisoliton solutions, Physics. Letters. A, 2001, 284, 6, 253-258.

4. M. Wang, Y. Zhou and Z. Li, Applications of a homogeneous balance method to exact solutions of nonlinear equations in mathematical physics, Physics. Letters. A, 1996, 216, 1-5, 67-75.
5. C. Yan, A simple transformation for nonlinear waves, Physics. Letters. A, 1996, 224, 1-2, 77-84.
6. W. X. Ma and B. Fuchssteiner, Explicit and exact solutions to a Kolmogorov-Petrovskii Piskunov equation, International Journal of Non-Linear Mechanics, 1996, 31, 3,329–338.
7. T Blickle，BG Lakatos ，C Mihálykó Z Ulbert, The hyperbolic tangent distribution family, Powder Technology,1998, 97, 2, 100-108.
8. Z. T. Fu, S. K. Liu, S. D. Liu and Q. Zhao, New Jacobi elliptic function expansion and new periodic solutions of nonlinear wave equation, Phys. Lett. A, 2001, 290, 72-76.
9. H. T. Chen and H. Q. Zhang, Improved Jacobin elliptic function method and its applications, Chaos, Solitions and Fractal, 2003,15, 585-591.
10. W. X. Ma and B. Fuchssteiner, Explicit and exact solutions to a Kolmogorov-Petrovskii- Piskunov equation, International Journal of Non-Linear Mechanics, 1996, 31, 3, 329–338.
11. Y. Chen, Q Wang, A unified rational expansion method to construct a series of explicit exact solutions to nonlinear evolution equations, Applied Mathematics and Computation, 2006, 177, 1, 396-409.
12. L. L. Xu, H. T. Chen, New three-soliton solutions to (2+1)-dimensional Nizhnik-Novikov Vesselov equations with variable coefficients, Acta Physica Sinica, 2013, 62, 9, 090204209.
13. H. T. Chen, S. H. Yang, W. X. Ma, Double sub-equation method for complexiton solutions of nonlinear partial differential equations, Applied Mathematics and Computation, 2013, 219, 9, 4775-4781.
14. Sawada K, Kotera T. A method for finding N-soliton solutions for the KdV equation and KdV -like equation, progress of theoretical Physics, 1974, 51(5):1355-1367.