Dimensional And Sensitivity Analysis Of Flat Back Model

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***Abstract*: Dimensional analysis and sensitivity analysis conducted on blunt trailing edge airfoil with splitter plates at various angle of attack and velocities provides minimal conclusions on performance of flat back airfoils .The paper contributes ot the study and analysis of the same tested on an educational Low subsonic Wind Tunnel under ambient conditions. Performance through a sweep of angles was needed to determine the behavior of the airfoils at low angle of attack. Dimensional analysis is a practice of checking relations among physical quantities by identifying their dimensions. The sensitivity analysis is the study of how the uncertainty in the output of a mathematical model or system can be apportioned to different sources of uncertainty in its output. It also helps to understand the relationship between input and output variables in the system or model.**

***Index Terms*— Blunt Trailing Edge , Drag reduction, airfoil, experiment.**

I. INTRODUCTION

A. Blunt Trailing edge airfoil

In case of two dimensional blunt trailing edge airfoil the section selected is 200mm*200 mm sectional area with a chord length of 163 mm.A blunt trailing edge modifications is done to mitigate the drag . The addition of splitter plate to the trailing edge of the airfoil increases the pressure at the trailing edge of the airfoil thereby increasing the pressure .The airfoil is designed according to the specifications occupying the test section area of Low subsonic widn tunnel of 300mm*300 mm. The model is shown in fig.1 is having density of 590 grams and is fabricated in wooden material. The model consists of 13 pressure tapings which will simultaneously read the pressure on upper and lower surface of the airfoil respectively.

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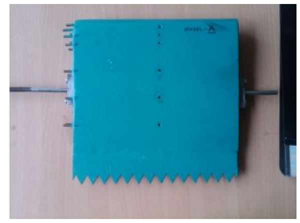

Fig. 1 Blunt Trailing Edge with splitter plate

The pressure readings are obtained by connecting the pressure tappings to multi tube manometer . The angle of attack are varied from 0, 5, 10, 13 and 15 degrees and the corresponding velocity changes are varied from 10, 15, 20, 22.5 and 25 m/s. The co-efficient of pressure is calculated from the formula

$$C_p[p_i, p_\infty, \rho, U_\infty] = \frac{2(p_i - p_\infty)}{\rho U_\infty^2} \quad \text{.......... (1)}$$

Where , pi is the static pressure at any pressure tap on the airfoil surface, p is the free stream pressure (measured on the Pitot static port), is air density, and U is the free-stream velocity, given by

$$U_\infty = \sqrt{\frac{2(p_{stagn} - p_\infty)}{\rho}} \quad \text{............... (2)}$$

The pressure readings , are evaluated for the center of pressure and the plot of co efficient of pressure versus no of pressure tappings with respect to the chord are plotted on the graph. The table 1.1 represents the readings and the plot is shown in fig 2

Velocity = 15 m/s					
	Angle of Attack				
x/c	0	5	10	15	20
0.67	1	1.4	1.3	1.1	0.9
0.505	0.9	1.5	1.4	1.2	1
0.34	0.9	1.6	1.5	1.3	1.1
0.175	0.4	1.7	1.6	1.5	1.2
0.125	-0.3	1.6	1.4	1	0.8
0.075	-0.2	1.6	1.4	0.9	0.7
0	2.4	2.2	2.2	2.2	2.2
0.075	-0.1	0.4	0.8	0.7	0.9
0.125	0.2	0.4	0.9	1	1.2
0.175	0.8	0.9	1.1	1.2	1.3
0.34	0.7	1	1	1	1.1
0.505	0.8	1	1	1	1
0.67	0.8	0.9	0.9	0.9	0.8

Table 1.1. Coefficient of pressure at various angle of attack with respect to the position of pressure tappings with respect to the chord.

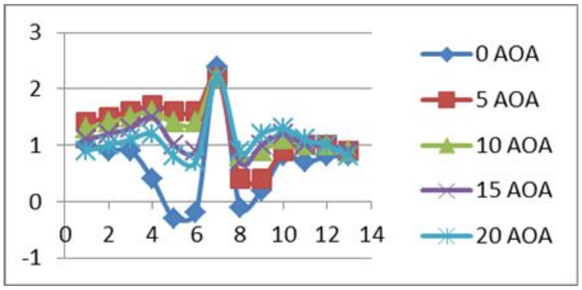

Figure. 2 co-efficient of pressure versus no of pressure tappings

B : Dimensional Analysis

The data of the independent and dependent parameters of the system has been gathered during experimentation. In this case there are two dependent and independent pi terms for each processing operations. It is necessary to correlate quantitatively various independent and dependent pi terms involved in this airfoil model. This correlation is nothing but a mathematical model as design tool for such workstation.

Lift (L) = $k_1 \times (\pi_1)^{a1} \times (\pi_2)^{b1} \times (\pi_3)^{c1} \times (\pi_4)^{d1}$ ------ (1.1)

The values of exponential a_1, b_1, c_1, d_1 are established, considering exponential relationship between dependent pi term lift and Independent π terms $\pi_1, \pi_2, \pi_3, \pi_4$, independently taken one at a time, on the basic of data collected through classical experimentation.

There are four unknown terms in the equation curve fitting constant K_1 and indices a_1, b_1, c_1, d_1, to get the values of these unknown we need minimum four sets of values of $(\pi_1, \pi_2, \pi_3, \pi_4)$.

Solution to establish the relationship for π_{11} is as follows:

Model of dependent pi term π_{11} for lift obtained for this model is

$(F_L / ds^*\rho) = k_1 \times (\pi_1)^{a1} \times (\pi_2)^{b1} \times (\pi_3)^{c1} \times (\pi_4)^{d1}$ ------ (1.2)

$\pi_{11} = k_1 \times (\pi_1)^{a1} \times (\pi_2)^{b1} \times (\pi_3)^{c1} \times (\pi_4)^{d1}$ ------ (1.3)

Taking log on the both sides of equation for π_{11}, ,

$\text{Log } \pi_{11} = \log k1 + a_1 \log \pi_1 + b_1 \log \pi_2 + c_1 \log \pi_3 + d_1 \log \pi_4$ ----- (1.4)

Let, $Z_1 = \log \pi_{11}$, $K_1 = \log k1$, $A = \log \pi_1$, $B = \log \pi_2$, $C = \log \pi_3$, $D = \log \pi_4$,

Putting the values in equations 1.4, the same can be written as

$Z_1 = K_1 + a_1 A + b_1 B + c_1 C + d_1 D$ -----(1.5)

Equation 1.5 is a regression equation of Z on A, B, C, and D. in an n dimensional co-ordinate system.

This represents a regression hyper plane .To determine the regression hyper plane, determines a_1, b_1, c_1, and d_1 in equation 1.5 so that

$\Sigma Z_1 = nK_1 + a_1{*}\Sigma A + b_1{*}\Sigma B + c_1{*}\Sigma C + d_1{*}\ \Sigma D$

$\Sigma Z_1{*}A = K_1{*}\Sigma A + a_1{*}\Sigma A{*}A + b_1{*}\Sigma B{*}A + c_1{*}\Sigma C{*}A + d_1{*}\ \Sigma D{*}A$

$\Sigma Z_1{*}B = K_1{*}\Sigma B + a_1{*}\Sigma A{*}B + b_1{*}\Sigma B{*}B + c_1{*}\Sigma C{*}B + d_1{*}\ \Sigma D{*}B$

$\Sigma Z_1{*}C = K_1{*}\Sigma C + a_1{*}\Sigma A{*}C + b_1{*}\Sigma B{*}C + c_1{*}\Sigma C{*}C + d_1{*}\ \Sigma D{*}C$

$\Sigma Z_1{*}D = K_1{*}\Sigma D + a_1{*}\Sigma A{*}D + b_1{*}\Sigma B{*}D + c_1{*}\Sigma C{*}D + d_1{*}\ \Sigma D{*}D$ -------------- (1.6)

In the above set of equations the values of the multipliers K_1, a_1, b_1, c_1, and d_1 are substituted to compute the values of the unknowns (viz. K_1, a_1, b_1, c_1, and d_1). The values of the terms on L H S and the multipliers of . K_1, a_1, b_1, c_1, and d_1 in the set of equations are calculated and tabulated in the Table 1.1 to Table 1.4. After substituting these values in the equations 1.6 one will get a set of 5 equations, which are to be solved simultaneously to get the values of K_1, a_1, b_1, c_1, and d_1. The above equations can be verified in the matrix form and further values of K_1, a_1, b_1, c_1, and d_1 can be obtained by using matrix analysis.

$\mathbf{X_1 = inv\ (W) \times P_1}$ ....(1.7)

The matrix method of solving these equations using 'MATLAB' is given below.

W = 5 x 5 matrix of the multipliers of K_1, a_1, b_1, c_1, and d_1

P_1 = 5 x 1 matrix of the terms on L H S and

X_1 = 5 x 1 matrix of solutions of values of K_1, a_1, b_1, c_1, and d_1

Then,

The matrix obtained is given by,

$$Z \times \begin{vmatrix} 1 \\ A \\ B \\ C \\ D \end{vmatrix} = \begin{vmatrix} n & A & B & C & D \\ A & A^2 & BA & CA & DA \\ B & AB & B^2 & CB & DB \\ C & CA & CB & C^2 & DC \\ D & DA & DB & DC & D^2 \end{vmatrix} = \begin{vmatrix} K \\ a \\ b \\ c \\ d \end{vmatrix}$$

$$\mathbf{P_1 = W_1 \times X_1}$$

$$\begin{vmatrix} 58.06181 \\ 267.2039 \\ 42.18899 \\ -236.312 \\ 151.4745 \end{vmatrix} = \begin{vmatrix} 20 & 92.04119983 & 14.67743 & -81.1457 & 52.17698 \\ 92.0412 & 423.5791233 & 67.54643 & -373.437 & 240.1216 \\ 14.67743 & 67.54643466 & 11.79383 & -59.5505 & 38.29121 \\ -81.1457 & -373.4372064 & -59.5505 & 329.6421 & -211.697 \\ 52.17698 & 240.1215712 & 38.29121 & -211.697 & 136.1218 \end{vmatrix} X \begin{vmatrix} K \\ a1 \\ b1 \\ c1 \\ d1 \end{vmatrix}$$

$$[P_1] = [W_1]\,[X_1]$$

Using Mat lab, $X_1 = W_1 \backslash P_1$, after solving X_1 matrix with K_1 and indices a_1, b_1, c_1, d_1 are as follows

K	-1.6644
a1	-0.3426
b1	-0.4117
c1	-1.7968
d1	-0.3234

But K_1 is log value so convert into normal value take antilog of K_1

Antilog (-1.6644) = 0.021657084

Hence the model for dependent term π_{11}

$\pi_{11} = k_1 \text{ x } (\pi_1)^{a1} \text{ x } (\pi_2)^{b1} \text{ x } (\pi_3)^{c1} \text{ x } (\pi_4)^{d1}$

$(F_L / ds{*}\rho) = 0.021657084\ (\pi_1)^{-.3426}\ (\pi_2)^{-0.4117}\ (\pi_3)^{-1.7968}\ (\pi_4)^{-0.3234}$ Hence , $Fl = 0.021657084 \left(\frac{L*C}{ds2}\right)^{-0.3426} \left(\frac{\theta}{\alpha}\right)^{-0.4117} \left(\frac{\mu}{ds*v*\rho}\right)^{-1.7968} \left(\frac{w}{ds*\rho}\right)^{-0.3234}$

SENSITIVITY ANALYSIS

The influence of the various independent π terms has been studied by analyzing the indices of the various π terms in the models. Through the technique of sensitivity analysis, the change in the value of a dependent π term caused due to an introduced change in the value of individual π term is evaluated. In this case, of change of ± 10 % is introduced in the individual independent π term independently (one at a time).Thus, total range of the introduced change is 20 %. The effect of this introduced change on the change in the value of the dependent π term is evaluated .The average values of the change in the dependent π term due to the introduced change of 20 % in each independent π term. This is defined as sensitivity. The total % change in output for ±10% change in input are shown in Table .Nature of variation in response variables due to increase in the values of independent pi terms. sequence of influence of independent pi terms on dependent pi terms for all models .

II. CONCLUSION

Dimensional analysis helps to find the relation of the various dependent and independent variables for the particular airfoil model. The results clear that the impact of the geometric variable of airfoils , angle of attack and weight of the airfoil model are the governing parameters which effect on the aerodynamic forces i.e lift and drag force .i.e the term $\pi_1 = (1{*}c/ds^2)$ which represents the geometric variable of the airfoil model, $\pi_2 = (\theta / \alpha)$, which represents the dependency on angle of attack of the airfoil, $\pi_3 = (\mu / ds{*}v{*}\rho)$ which is a relation of viscosity of fluid medium, and $\pi_4 = (w/ds{*}\rho)$ which competes with the weight or density of the airfoil section. The research and calculations clear the effect and impact of the various components on designing of the airfoil model. The dimensional analysis helps to get the relation of dependent and independent parameters to be considered on checking the performance of the section or model.

APPENDIX

Table 1.1 Sample calculations for pi terms for calculating lift of airfoil section

Sr. no	Π1=L s*C/d s^2	Π2=θ/α	Π3=μ/ds*v*ρ	Π4=W/ds* ρ	Π01=FL/ ds*ρ
1	40000	11.992909	0.000154229	406.301824	162.6866
2	40000	5.9964545	0.000154229	406.301824	325.3731
3	40000	3.9976364	0.000154229	406.301824	366.0448
4	40000	2.9982273	0.000154229	406.301824	317.2388
5	40000	11.992909	0.000102819	406.301824	406.7164

Table 1.2 sample calculations for log of pi terms for lift of airfoil section

Sr.no	logZ	Logπ1	Logπ2	Logπ3	Logπ4
	2.211352	4.60206	1.078925	-3.81183	2.608849
2	2.512382	4.60206	0.777895	-3.81183	2.608849
3	2.563534	4.60206	0.601803	-3.81183	2.608849
4	2.501386	4.60206	0.476865	-3.81183	2.608849
5	2.609292	4.60206	1.078925	-3.98793	2.608849

Table 1.3 Sample Calculations of Multipliers of the R.H. S. terms of equation 1.6 for formulation of model for calculating lift of airfoil model

A	AA	A	AA	AB	AC	AD	B	AB	BB	BC	BD
4.60	21.18	4.60	21.18	4.97	-17.54	12.01	1.08	4.97	1.16	-4.11	2.81
4.60	21.18	4.60	21.18	3.58	-17.54	12.01	0.78	3.58	0.61	-2.97	2.03
4.60	21.18	4.60	21.18	2.77	-17.54	12.01	0.60	2.77	0.36	-2.29	1.57
4.60	21.18	4.60	21.18	2.19	-17.54	12.01	0.48	2.19	0.23	-1.82	1.24
4.60	21.18	4.60	21.18	4.97	-18.35	12.01	1.08	4.97	1.16	-4.30	2.81

A	AA	A	AA	AB	AC	AD	B	AB	BB	BC	BD
4.60	21.18	4.60	21.18	4.97	-17.54	12.01	1.08	4.97	1.16	-4.11	2.81
4.60	21.18	4.60	21.18	3.58	-17.54	12.01	0.78	3.58	0.61	-2.97	2.03
4.60	21.18	4.60	21.18	2.77	-17.54	12.01	0.60	2.77	0.36	-2.29	1.57
4.60	21.18	4.60	21.18	2.19	-17.54	12.01	0.48	2.19	0.23	-1.82	1.24
4.60	21.18	4.60	21.18	4.97	-18.35	12.01	1.08	4.97	1.16	-4.30	2.81

Table 1.4 Sample Calculations of Multipliers of the L.H. S. terms of equations 1.6 for formulation of model for lift

L.H.S Multipliers for Lift			
ZA	ZB	ZC	ZD
-144.49438	1.2559525	-8.4293064	5.7690822
-144.49438	0.4707195	-9.5767829	6.5544239
-144.49438	0.2179534	-9.7717678	6.6878731
-144.49438	0.1084389	-9.5348703	6.5257386

Table 1.5 Sample calculations for pi terms for lift

Π1=Ls * C/ds^2	Π2=θ/α	Π3=μ/ds* v*ρ	Π4=W/ds* ρ	Π01=FL/ ds*ρ
40000	11.992909	0.0001542	406.3018	162.687
40000	5.9964545	0.0001542	406.3018	325.373
40000	3.9976364	0.0001542	406.3018	366.045
40000	2.9982273	0.0001542	406.3018	317.239
40000	11.992909	0.0001028	406.3018	406.716

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